Chapter 9:

Maxwell's Equations:

Differential Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of isolated magnetic charge*
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{S}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$	Ampère's circuit law

Time-Harmonic Maxwell's Equations: (assuming time factor $e^{j\omega t}$)

Point Form	Integral Form
$\nabla \cdot \mathbf{D}_s = \rho_{vs}$	$\oint \mathbf{D}_s \cdot d\mathbf{S} = \int \rho_{vs} \, dv$
$\nabla \cdot \mathbf{B}_s = 0$	$\oint \mathbf{B}_s \cdot d\mathbf{S} = 0$
$\nabla \times \mathbf{E}_s = -j\omega \mathbf{B}_s$	$\oint \mathbf{E}_s \cdot d\mathbf{l} = -j\omega \int \mathbf{B}_s \cdot d\mathbf{S}$
$\nabla \times \mathbf{H}_s = \mathbf{J}_s + j\omega \mathbf{D}_s$	$\oint \mathbf{H}_s \cdot d\mathbf{l} = \int (\mathbf{J}_s + j\omega \mathbf{D}_s) \cdot d\mathbf{S}$

Faraday's law:	$V_{\text{emf}} = -\frac{d\lambda}{dt} = -N \frac{d\Psi}{dt}$	
Transformer emf:	$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$	
Motional emf:	$V_{\text{emf}} = \oint_L \mathbf{E}_m \cdot d\mathbf{l} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$	Stokes's theorem gives: $\nabla \times \mathbf{E}_m = \nabla \times (\mathbf{u} \times \mathbf{B})$
Moving loop in time-varying field:	$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$	
Displacement current density:	$\mathbf{J}_d = \frac{\partial \mathbf{D}}{dt}$	Displacement current: $I_d = \int_S \mathbf{J}_d \cdot d\mathbf{S} = \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$
Lorentz force equation:	$\mathbf{F} = Q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$	

EM2 equations: Compiled by Mohammad Sanad AlTaher

Magnetic Vector Potential (A):	$\mathbf{A} = \int_v \frac{\mu \mathbf{J}\, dv}{4\pi R}$	$\mathbf{B} = \nabla \times \mathbf{A}$ $\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$	Lorenz condition for potentials: $\nabla \cdot \mathbf{A} = -\mu\varepsilon \frac{\partial V}{\partial t}$
Wave Equations: ($\boldsymbol{\nabla}^2$: vector Laplacian)	$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\varepsilon}$	$\nabla^2 \mathbf{A} - \mu\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}$	
	$\boldsymbol{\nabla}^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$	$\boldsymbol{\nabla}^2 \mathbf{E_s} - \gamma^2 \mathbf{E_s} = 0$ (also valid for $\mathbf{H}_s$)	$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

Chapter 10:

Type of medium	σ	ε	μ	Condition
Lossy dielectric:	$\neq 0$	$\varepsilon_r\varepsilon_0$	$\mu_r\mu_0$	N/A
Lossless dielectric:	$\simeq 0$	$\varepsilon_r\varepsilon_0$	$\mu_r\mu_0$	$\sigma << \omega\varepsilon$
Good dielectric:				$\tan\theta << 1$ (< 0.2)
Good conductor:	$\simeq \infty$	ε_0	$\mu_r\mu_0$	$\tan\theta >> 1$
Free space:	$= 0$	ε_0	μ_0	N/A

	Lossy Medium **Exact**	Lossless Medium	Free Space	Conductor
$\alpha =$ Np/m (or m^{-1}) *attenuation constant*	$\omega\left[\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]\right]^{1/2}$	0	0	$\sqrt{\pi f\mu\sigma}$
$\beta =$ rad/m *phase constant*	$\omega\left[\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]\right]^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu_o\varepsilon_o}$ $\frac{1}{\sqrt{\mu_0\varepsilon_0}} = c = 3\times10^8$	$\sqrt{\pi f\mu\sigma}$
$\eta =$ Ω	$\sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}} = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{\mu}{\varepsilon_c}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu_o}{\varepsilon_o}}$ = 120π $\simeq 377$	$(1+j)\frac{\alpha}{\sigma}$
$v = \frac{\omega}{\beta}$, m/s $\lambda = \frac{2\pi}{\beta}$ m				

AC Resistance due to skin effect:	$R_{ac} = \frac{\ell}{\sigma\delta w} = \frac{R_s\ell}{w}$	$R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f\mu}{\sigma}}$	l: length of conductor w: width R_S: resistance of conductor with unit length and width
Loss tangent and angle:	$\tan\theta = \frac{\sigma}{\omega\varepsilon}$	$\theta = 2\theta_\eta$	θ_η: angle of intrinsic impedance θ: loss angle, δ in Prof. Nihad's notes
		$\tan\theta = \frac{\varepsilon''}{\varepsilon'}$	
Skin depth: (δ_C in Prof. Nihad's notes)	Conductors: $\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}$	Exact: $\delta = \frac{1}{\alpha}$	δ (in meters) is the distance a wave travels in a medium whereby its amplitude decreases by e^{-1}
Direction of propagation of TEM:	$\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k$		$\mathbf{a}_E$: vector components of **E** (electric field intensity) $\mathbf{a}_H$: vector components of **H** (magnetic field intensity) $\mathbf{a}_k$: direction of wave propagation
Complex permittivity:	$\varepsilon_c = \varepsilon' - j\varepsilon''$	$\varepsilon' = \varepsilon = \varepsilon_r\varepsilon_0$ $\varepsilon'' = \sigma/\omega$ $\varepsilon''/\varepsilon'$ = loss tangent	$\varepsilon_c = \varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right] = \varepsilon[1 - j\tan\theta]$

Propagation Constant:	$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$	$\gamma = \alpha + j\beta$	$\gamma = j\omega\sqrt{\mu\varepsilon_c}$
	$\beta^2 - \alpha^2 = \omega^2\mu\varepsilon$ $2\alpha\beta = \omega\mu\sigma$		$\frac{\pi}{2} = \theta_\eta + \theta_\gamma$
	$\eta\gamma = j\omega\mu$	$\tan\theta_\gamma = \frac{\beta}{\alpha}$	$0 \le \theta_\eta \le \frac{\pi}{4}$ $\frac{\pi}{4} \le \theta_\gamma \le \frac{\pi}{2}$
Good dielectric: (low-loss)	$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\beta = \omega\sqrt{\mu\varepsilon}$	$\eta = \sqrt{\frac{\mu}{\varepsilon}}$
Field intensity:	$\boldsymbol{H} = \frac{1}{\eta}(\boldsymbol{a}_k \times \boldsymbol{E})$		$\boldsymbol{E} = -\eta(\boldsymbol{a}_k \times \boldsymbol{H})$
Intrinsic impedance:	$\tan\theta_\eta = \frac{\alpha}{\beta}$		

Poynting's Vector:	$\mathcal{P} = \mathbf{E} \times \mathbf{H}$		
Time Average Poynting Vector:	Integral form: $\mathcal{P}_{\text{ave}}(z) = \frac{1}{T}\int_0^T \mathcal{P}(z,t)\,dt$	Wave traveling in +z: $\mathcal{P}_{\text{ave}}(z) = \frac{E_o^2}{2\lvert\eta\rvert} e^{-2\alpha z}\cos\theta_\eta\,\mathbf{a}_z$	Phasor form: $\mathcal{P}_{\text{ave}}(z) = \frac{1}{2}\text{Re}(\mathbf{E}_s \times \mathbf{H}_s^*)$
Total Time Average Power:	$P_{\text{ave}} = \int_S \mathcal{P}_{\text{ave}} \cdot d\mathbf{S}$ crossing a surface **S**		
Reflection Coefficient:	$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$	$-1 \le \Gamma \le +1$ real or complex	E_{r0}: Magnitude of reflected E E_{i0}: Magnitude of incident E
	$\lvert\Gamma\rvert^2 = \frac{P_{\text{reflected}}}{P_{\text{incident}}}$	If $\eta_2 < \eta_1$ (or $\varepsilon_{r2} > \varepsilon_{r1}$), then Γ is negative If $\eta_2 > \eta_1$ (or $\varepsilon_{r2} < \varepsilon_{r1}$), then Γ is positive	
Transmission Coefficient:	$\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$	$\tau = 1 + \Gamma$ real or complex	E_{t0}: Magnitude of Transmitted E
Standing Wave Ratio (SWR):	$\frac{\lvert\mathbf{E}_1\rvert_{\max}}{\lvert\mathbf{E}_1\rvert_{\min}} = \frac{\lvert\mathbf{H}_1\rvert_{\max}}{\lvert\mathbf{H}_1\rvert_{\min}} = \frac{1 + \lvert\Gamma\rvert}{1 - \lvert\Gamma\rvert}$		

Wave traveling from medium 1 to 3: (medium 2: coating)	$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{Z - \eta_1}{Z + \eta_1}$	$Z = \eta_2 \cdot \frac{\eta_3 \cdot \cos(\beta_2 \cdot d) + j \cdot \eta_2 \cdot \sin(\beta_2 \cdot d)}{\eta_2 \cdot \cos(\beta_2 \cdot d) + j \cdot \eta_3 \cdot \sin(\beta_2 \cdot d)}$	
		$Z = \eta_2 \cdot \frac{\eta_3 + j \cdot \eta_2 \cdot \tan(\beta_2 \cdot d)}{\eta_2 + j \cdot \eta_3 \cdot \tan(\beta_2 \cdot d)}$	Z:Input Impedance d: thickness of coating (distance from interface 1-2 to interface 2-3)
Zero reflection coating:	$\Gamma = 0 \rightarrow Z = \eta_1$ $j \cdot \tan(\beta_2 \cdot d) = \frac{\eta_1\eta_2 - \eta_3\eta_2}{\eta_2{}^2 - \eta_1\eta_3}$	Half-wave section: (n: integer) $\eta_1 = \eta_3$ & $d = n\frac{\lambda_2}{2}$	
		Quarter-wave transformer: (n: integer) $d = (2n+1)\frac{\lambda_2}{4}$ & $\eta_2 = \sqrt{\eta_1\eta_3}$	
Index of Refraction: (μ_r=1)	$n = \sqrt{\varepsilon_r}$	if $n_1 > n_2$:	$SWR = {}^{n_1}/_{n_2}$
		if $n_1 < n_2$:	$SWR = {}^{n_2}/_{n_1}$

General form of Plane Waves:	$\boldsymbol{E}(x,y,z) = (\boldsymbol{a_x}E_{0x} + \boldsymbol{a_y}E_{0y} + \boldsymbol{a_z}E_{0z})e^{-j(k_x x + k_y y + k_z z)}$		
	$\boldsymbol{E}(x,y,z) = \boldsymbol{E_0}e^{-j \cdot k \cdot \boldsymbol{a_k} \cdot \boldsymbol{R}} = \boldsymbol{E_0}e^{-j \cdot \boldsymbol{k} \cdot \boldsymbol{R}}$		$\lvert E_0 \rvert = \sqrt{E_{0x}{}^2 + E_{0y}{}^2 + E_{0z}{}^2}$
Propagation Vector: (Wave Number Vector)	$\mathbf{k} = k_x\mathbf{a}_x + k_y\mathbf{a}_y + k_z\mathbf{a}_z$	$k^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\varepsilon$ $\therefore k = \beta$ for lossless media Where $k = \lvert \boldsymbol{k} \rvert$	$\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H}$ $\mathbf{k} \times \mathbf{H} = -\omega\varepsilon\mathbf{E}$ $\mathbf{k} \cdot \mathbf{H} = 0$ $\mathbf{k} \cdot \mathbf{E} = 0$
Snell's Laws:	Reflection: $\theta_r = \theta_i$	Refraction: $n_1 \sin\theta_i = n_2 \sin\theta_t$	
Parallel Polarization: (parallel to the plane of incidence, **xz-plane will always be the plane of incidence**)	Fresnel's Equations for Parallel Polarization: $\Gamma_{\parallel} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$ $\tau_{\parallel} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$	$1 + \Gamma_{\parallel} = \tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$	
		Brewster Angle: θ_i *when* $\Gamma_{\parallel} = 0$ $\sin^2\theta_{B_{\parallel}} = \frac{1 - \mu_2\varepsilon_1/\mu_1\varepsilon_2}{1 - (\varepsilon_1/\varepsilon_2)^2}$	
		For non-magnetic media: $\sin\theta_{B_{\parallel}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}}$ or $\tan\theta_{B_{\parallel}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1}$	

<table>
<tr><td rowspan="2">Perpendicular Polarization: (Perpendicular to the plane of incidence)</td><td colspan="2">Fresnel's Equations for Perpendicular Polarization:
$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$ $\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$</td><td>$1 + \Gamma_\perp = \tau_\perp$</td></tr>
<tr><td>Brewester angle:
$\sin^2\theta_{B_\perp} = \frac{1 - \mu_1\varepsilon_2/\mu_2\varepsilon_1}{1 - (\mu_1/\mu_2)^2}$
Only exists for magnetic media</td><td colspan="2">If $\varepsilon_1 = \varepsilon_2$, then the Brewester angle for perpendicular polarization can be found from:
$\sin\theta_{B_\perp} = \sqrt{\frac{\mu_2}{\mu_1 + \mu_2}}$ or $\tan\theta_{B_\perp} = \sqrt{\frac{\mu_2}{\mu_1}}$</td></tr>
<tr><td rowspan="3">Incident field components based on type of polarization:</td><td>Parallel Polarization:</td><td colspan="2">Perpendicular Polarization:</td></tr>
<tr><td>E$_i$ will have components along a$_x$ and a$_z$, whose magnitudes are:
$E_{i0x} = E_{i0}\cos(\theta_i)$ (positive x)
$E_{i0z} = E_{i0}\sin(\theta_i)$ (negative z)</td><td colspan="2">E$_i$ will only have a component along a$_y$, whose magnitudes is:
E_{i0}</td></tr>
<tr><td>H$_i$ will only have a component along a$_y$, whose magnitude is:
$H_{i0} = \frac{E_{i0}}{\eta}$</td><td colspan="2">H$_i$ will have components along a$_x$ and a$_z$, whose magnitudes are:
$H_{i0x} = \frac{E_{i0}}{\eta}\cos(\theta_i)$ (negative x)
$H_{i0z} = \frac{E_{i0}}{\eta}\sin(\theta_i)$ (positive z)</td></tr>
<tr><td rowspan="3">Critical angle: (wave traveling from medium 1 to medium 2)</td><td colspan="3">$\theta_t = \frac{\pi}{2} \to \theta_i = \theta_c$</td></tr>
<tr><td>If $\varepsilon_1 < \varepsilon_2 \to \theta_t < \theta_i \ \ \forall\, \theta_i$
Critical angle does not exist.</td><td colspan="2">If $\varepsilon_1 > \varepsilon_2 \to \theta_t > \theta_i \ \ \forall\, \theta_i$
$\therefore \sin(\theta_c) = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1} = \tan(\theta_{B\parallel})$</td></tr>
<tr><td colspan="3">If $\varepsilon_1 > \varepsilon_2 \ \wedge \theta_t = \frac{\pi}{2}$:
$\Gamma_\perp = 1 \ \wedge \ \tau_\perp = 2 \ \wedge \ \Gamma_\parallel = -1 \ \wedge \ \tau_\parallel = \frac{2\eta_2}{\eta_1}$</td></tr>
<tr><td>Linear Polarization:</td><td colspan="2">$\Delta\phi = \phi_y - \phi_x = n\pi, \quad n = 0, 1, 2, \ldots$</td><td>Or if either E_{x0} or E_{y0} is equal to zero</td></tr>
<tr><td>Circular Polarization:</td><td colspan="3">$\Delta\phi = \phi_y - \phi_x = \pm(2n + 1)\pi/2, \quad n = 0, 1, 2, \ldots$ and $E_{ox} = E_{oy} = E_o$</td></tr>
<tr><td rowspan="2">Elliptical Polarization:</td><td colspan="3">$\Delta\phi = \phi_y - \phi_x = \pm(2n + 1)\pi/2, \quad n = 0, 1, 2, \ldots$ and $E_{ox} \neq E_{oy}$</td></tr>
<tr><td colspan="3">Or if $\Delta\phi \neq \frac{n\pi}{2}$ $\forall\, n: integer$, tilted ellipse</td></tr>
<tr><td rowspan="2">Power:</td><td colspan="3">$P_{avg} = \frac{|E_{x0}|^2 + |E_{y0}|^2 + |E_{z0}|^2}{2\eta}$</td></tr>
<tr><td colspan="3">Isotropic: power is distibuted evenly over the surface,
$P_{avg} = \frac{P_{total}}{4\pi r^2}$</td></tr>
</table>

Chapter 11:

Parameters	Coaxial Line	Two-Wire Line	Planar Line
$R\ (\Omega/\text{m})$	$\frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right]$ $(\delta \ll a, c-b)$	$\frac{1}{\pi a\delta\sigma_c}$ $(\delta \ll a)$	$\frac{2}{w\delta\sigma_c}$ $(\delta \ll t)$
$L\ (\text{H/m})$	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\frac{d}{2a}$	$\frac{\mu d}{w}$
$G\ (\text{S/m})$	$\frac{2\pi\sigma}{\ln\frac{b}{a}}$	$\frac{\pi\sigma}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\sigma w}{d}$
$C\ (\text{F/m})$	$\frac{2\pi\varepsilon}{\ln\frac{b}{a}}$	$\frac{\pi\varepsilon}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\varepsilon w}{d}$ $(w \gg d)$

*$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}}$ = skin depth of the conductor; $\cosh^{-1}\frac{d}{2a} \simeq \ln\frac{d}{a}$ if $\left[\frac{d}{2a}\right]^2 \gg 1$.

Voltage for Δz increment of a T.L.:	$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$	Phasors: $-\frac{dV_s}{dz} = (R + j\omega L)I_s$ Series impedance per meter: $Z = R + j\omega L$
Current for Δz increment of a T.L.:	$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}$	Phasors: $-\frac{dI_s}{dz} = (G + j\omega C)V_s$ Shunt admittance per meter: $Y = G + j\omega C$
Wave equations for Voltage and Current:	$\frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0$	$\frac{d^2I_s}{dz^2} - \gamma^2 I_s = 0$
Propagation constant:	$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$	α: attenuation constant β: phase constant
	$\gamma = \sqrt{ZY}$	Setting R = 0: $\gamma = j\omega\sqrt{LC}\left[1 - j\frac{G}{\omega C}\right]^{\frac{1}{2}} \rightarrow \sqrt{LC} = \sqrt{\mu\varepsilon} \wedge \frac{G}{\omega C} = \frac{\sigma}{\omega\varepsilon}$
Wavelength and Phase velocity:	$\lambda = \frac{2\pi}{\beta}$	$u = \frac{\omega}{\beta} = f\lambda$
Characteristic Impedance (in Ω)	$Z_0 = R_0 + jX_0 = \sqrt{\frac{Z}{Y}}$	Characteristic Admittance: $Y_0 = \frac{1}{Z_0}$
	R_0: Characteristic Resistance	X_0: Characteristic Reactance

Case	Propagation Constant $\gamma = \alpha + j\beta$	Characteristic Impedance $Z_o = R_o + jX_o$
General	$\sqrt{(R + j\omega L)(G + j\omega C)}$	$\sqrt{\frac{R + j\omega L}{G + j\omega C}}$
Lossless	$0 + j\omega\sqrt{LC}$	$\sqrt{\frac{L}{C}} + j0$
Distortionless	$\sqrt{RG} + j\omega\sqrt{LC}$	$\sqrt{\frac{L}{C}} + j0$

Solutions to Wave equations:	$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$ ($\longrightarrow +z \quad -z \longleftarrow$)	$I_s(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}$ ($\longrightarrow +z \quad -z \longleftarrow$) or $I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}$
For a T.L. of infinite length:	Reflected voltage vanishes: $V_s(z) = V_o^+ e^{-\gamma z}$	Reflected Current vanishes: $I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z}$
Lossless Line: R = G = 0	Z_0 is pure real/resistive: $Z_0 = \sqrt{\frac{L}{C}}$	Phase velocity is constant: $u_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}$ No distortion, no dispersion.
	$\alpha = 0$	$\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon} = \omega \frac{L}{Z_0}$
Distortionless Line: $\frac{R}{L} = \frac{G}{C}$	Z_0 is pure real/resistive: $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$	Phase velocity is constant: $u_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}$ No distortion, no dispersion
	$\alpha = R\sqrt{\frac{C}{L}} = \sqrt{RG} = \frac{R}{R_0}$	$\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon} = \omega \frac{L}{Z_0}$
Lossless or Distortionless Coaxial line:	$Z_0 = \frac{\eta}{2\pi}\ln\left(b/a\right) = 60\sqrt{\frac{\mu_r}{\varepsilon_r}}\ln\left(b/a\right)$	b: radius of outer circle a: radius of inner circle

Wave equations continued:	At z = 0: $V_o^+ = \frac{1}{2}(V_o + Z_o I_o)$ $V_o^- = \frac{1}{2}(V_o - Z_o I_o)$	At z = l: $V_o^+ = \frac{1}{2}(V_L + Z_o I_L)e^{\gamma\ell}$ $V_o^- = \frac{1}{2}(V_L - Z_o I_L)e^{-\gamma\ell}$
Impedance at distance z' from load: (z' = l-z)	Lossy: $Z(z') = Z_0 \frac{Z_L + Z_0 \tanh(\gamma z')}{Z_0 + Z_L \tanh(\gamma z')}$	Lossless: $Z(z') = Z_0 \frac{Z_L + jZ_0 \tan(\beta z')}{Z_0 + jZ_L \tanh(\beta z')}$

General input impedance:	$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_o(V_o^+ + V_o^-)}{V_o^+ - V_o^-}$		$Z_{in} = Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}}$	
Input impedance of transmission line:	Lossy: $Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell}\right]$		Lossless: $Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan \beta \ell}{Z_o + jZ_L \tan \beta \ell}\right]$	
	Where the electrical length is βl (in radians)			
Voltage reflection coefficient:	$\Gamma(z) = \frac{V_o^-}{V_o^+} e^{2\gamma z}$		$\Gamma(z') = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma z'}$	
Reflection coefficients at load and elsewhere:	$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \lvert\Gamma_L\rvert e^{j\theta_{\Gamma_L}}$	$\Gamma(z') = \Gamma_L e^{-2\gamma z'}$	$\lvert\Gamma(z')\rvert = \lvert\Gamma_L\rvert e^{-2\alpha z'}$ $\theta_{\Gamma(z')} = \theta_{\Gamma_L} - 2\beta z'$	
	For a lossless line: $\Gamma(z') = \Gamma_L e^{-j2\beta z'} \rightarrow \lvert\Gamma(z')\rvert = \lvert\Gamma_L\rvert \wedge \theta_{\Gamma(z')} = \theta_{\Gamma_L} - 2\beta z'$		Current reflection coefficient: $-\Gamma(z')$	
	$Z(z) = Z_0 \frac{1+\Gamma(z)}{1-\Gamma(z)}$		$\Gamma(z) = \frac{Z(z) - Z_0}{Z(z) + Z_0}$	
Standing Wave Ratio:	$s = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + \lvert\Gamma_L\rvert}{1 - \lvert\Gamma_L\rvert}$		$\lvert\Gamma_L\rvert = \frac{S-1}{S+1}$	
Max & Min input impedances:	$\lvert Z_{in}\rvert_{max} = \frac{V_{max}}{I_{min}} = sZ_o$		$\lvert Z_{in}\rvert_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_o}{s}$	
	$z'_{max} = \frac{\theta_{\Gamma_L} + 2n\pi}{2\beta}$		$z'_{min} = \frac{\theta_{\Gamma_L} + (2n+1)\pi}{2\beta}$	
	$V_{max} = \lvert V_0^+\rvert \cdot (1 + \lvert\Gamma\rvert)$		$I_{min} = \frac{\lvert V_0^+\rvert}{Z_0} \cdot (1 - \lvert\Gamma\rvert)$	
	$V_{min} = \lvert V_0^+\rvert \cdot (1 - \lvert\Gamma\rvert)$		$I_{max} = \frac{\lvert V_0^+\rvert}{Z_0} \cdot (1 + \lvert\Gamma\rvert)$	
Open-Circuited line:	$Z_L = \infty$	$\Gamma_L = 1$	$\theta_{\Gamma_L} = 0$	$s = \infty$
	$\lvert V(z)\rvert = 2\lvert V_0^+\rvert \cdot \lvert\cos(\beta z')\rvert$ $2\lvert V_0^+\rvert = \lvert V_L\rvert$		$Z(z') = -jZ_0 \cot(\beta z')$ $\lvert I(z)\rvert = \frac{2\lvert V_0^+\rvert}{Z_0} \cdot \lvert\sin(\beta z')\rvert$	
Short-circuited line:	$Z_L = 0$	$\Gamma_L = -1$	$\theta_{\Gamma_L} = \pi$	$s = \infty$
	$\lvert V(z)\rvert = \lvert I_L\rvert Z_0 \cdot \lvert\sin(\beta z')\rvert$		$Z(z') = jZ_0 \tan(\beta z')$ $\lvert I(z)\rvert = \lvert I_L\rvert \cdot \lvert\cos(\beta z')\rvert$	
Open circuit & Short circuit impedances:	$Z_{oc} Z_{sc} = Z_0^2$		$\tan(\beta l) = \sqrt{-\frac{Z_{sc}}{Z_{oc}}}$	

Matched load:	$Z_L = Z_0$	$\Gamma_L = 0$	$s = 1$			
	$\lvert V(z)\rvert = \lvert V_0^+\rvert$	$V(z) = V_0^+ e^{-j\beta z}$	$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z}$	$Z(z) = Z_0$		
Resistive load: (lossless line)	$Z_L = R_L$	Γ_L is a real number				
	If $R_L > R_0$:			If $R_L < R_0$:		
	$\Gamma_L > 0$	$\theta_{\Gamma_L} = 0$	$s = \frac{R_L}{R_0}$	$\Gamma_L < 0$	$\theta_{\Gamma_L} = \pi$	$s = \frac{R_0}{R_L}$
	$Z_{max} = R_L$	$Z_{min} = \frac{R_0^2}{R_L}$		$Z_{max} = \frac{R_0^2}{R_L}$	$Z_{min} = R_L$	
	V_{max} & I_{min} occur together when: $2\beta z_{max} = 2n\pi \rightarrow z_{max} = \frac{n\lambda}{2}$ Where n = 0, 1, 2,.. Hence, starts at max			V_{min} & I_{max} occur together at: $z_{min} = \frac{n\lambda}{2}$ Where n = 0, 1, 2,.. Hence, starts at min		
Reactive load:	$\lvert\Gamma_L\rvert = 1$	$s = \infty$	$\theta_{\Gamma_L} = 2\tan^{-1}\left(\frac{R_0}{X_L}\right)$	$\lvert V(z)\rvert = 2\lvert V_0^+\rvert \cdot \left\lvert cos\left(\beta z' - \frac{\theta_{\Gamma_L}}{2}\right)\right\rvert$		
	$X_L > 0$, inductive $\rightarrow 0 < \theta_{\Gamma_L} < \pi$			$X_L < 0$, capacitive $\rightarrow \pi < \theta_{\Gamma_L} < 2\pi$		
	V_{max}: $2\beta z'_{max} = \theta_{\Gamma_L} + 2n\pi$ n = 0, 1, 2,…			V_{min}: $2\beta z'_{min} = \theta_{\Gamma_L} + n\pi$ n = 1, 3, 5,…		
Quarter wave section:	$l = (2n+1)\frac{\lambda}{4}$, n = 0, 1, 2,…			$\beta l = (2n+1)\frac{\pi}{2}$, n = 0, 1, 2,…		
	$Z_{in} = \frac{Z_0^2}{Z_L}$			If $Z_L = \infty$ (o.c) $\rightarrow Z_{in} = 0$		
				If $Z_L = 0$ (s.c) $\rightarrow Z_{in} = \infty$		
Half wave section:	$l = n\frac{\lambda}{2}$, n = 1, 2, 3,…	$\beta l = n\pi$	$Z_{in} = Z_L$			
Power for a lossless line:	$P_{inc} = \frac{1}{2}\frac{\lvert V_0^+\rvert^2}{Z_0}$	$P_{ref} = \frac{1}{2}\cdot\frac{\lvert V_0^-\rvert^2}{Z_0} = \frac{1}{2}\cdot\frac{\lvert V_0^+\rvert^2}{Z_0}\cdot\lvert\Gamma\rvert^2 = \lvert\Gamma\rvert^2\cdot P_{inc}$				
	Average power dissipated in the load: $P_{ave} = \frac{\lvert V_o^+\rvert^2}{2Z_o}(1 - \lvert\Gamma\rvert^2)$					

Chapter 12:

Propagation:	$k = \omega\sqrt{\mu\varepsilon}$	$-k_x^2 - k_y^2 + \gamma^2 = -k^2$	
WG Electric Field Distribution: (z)	$E_{zs}(x, y, z) = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)e^{-\gamma z}$		
WG Magnetic Field Distribution:	$H_{zs}(x, y, z) = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y)e^{-\gamma z}$		
WG Electric & Magnetic Fields Distribution: (in terms of E_z & H_z)	$E_{xs} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}$	$E_{ys} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x}$	
	$H_{xs} = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial x}$	$H_{ys} = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial y}$	
	$h^2 = \gamma^2 + k^2 = k_x^2 + k_y^2$	$k_x = \frac{m\pi}{a}$	$k_y = \frac{n\pi}{b}$
Cutoff Frequency:	$(f_c)_{mn} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$	$\frac{1}{\sqrt{\mu\varepsilon}} = u = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$	
Phase Constant:	$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$	$\beta = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$	
Wavelengths:	Cutoff: $\lambda_c = \frac{u}{f_c} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$	Guide: $\lambda_g = \frac{2\pi}{\beta} = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$ s.t. λ: wavelength in unbounded medium, $\lambda = \frac{2\pi}{k} = \frac{u}{f}$	
	$\frac{1}{\lambda^2} = \frac{1}{{\lambda_c}^2} + \frac{1}{{\lambda_g}^2}$		
Phase Velocity:	$u_p = \frac{\omega}{\beta} = \frac{u}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$	$u_p = \frac{\lambda_g}{\lambda} \cdot u$	

Transverse Magnetic Mode: TM_{mn}	$E_{zs} = E_o \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$H_z = 0$ $m = 1, 2, 3, \ldots$; $n = 1, 2, 3, \ldots$ $m \,\&\, n \neq 0$
	$E_{xs} = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$E_{ys} = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
	$H_{xs} = \frac{j\omega\varepsilon}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$H_{ys} = -\frac{j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
	Cutoff case: $k^2 = \omega^2\mu\varepsilon = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$	$\gamma = 0$ or $\alpha = 0 = \beta$ $f = f_c$ Subscript "c": cutoff
	Evanescent case: $k^2 = \omega^2\mu\varepsilon < \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$	$\gamma = \alpha, \quad \beta = 0$ $\alpha = \sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2 - k^2}$
	Propagating case: $k^2 = \omega^2\mu\varepsilon > \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$	$\gamma = j\beta, \quad \alpha = 0$ $\beta = \sqrt{k^2 - \left[\frac{m\pi}{a}\right]^2 - \left[\frac{n\pi}{b}\right]^2}$
Transverse Electric Mode: TE_{mn}	$H_{zs} = H_o \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$E_z = 0$ $m = 0, 1, 2, \ldots$; $n = 0, 1, 2, \ldots$ m or n can be 0 but not simultaneously
	$E_{xs} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$E_{ys} = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
	$H_{xs} = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$H_{ys} = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
	Same cases as for TM but lowest f_c mode (dominant) is TE_{10} (if $\boldsymbol{a} > \boldsymbol{b}$) or TE_{01} (if $\boldsymbol{a} < \boldsymbol{b}$)	
Wave Impedances:	TM_{mn}:	TE_{mn}:
	$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\beta}{\omega\varepsilon}$	$\eta_{TE} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\omega\mu}{\beta}$
	$\eta_{TM} = \frac{k}{\omega\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$; $\eta = \frac{k}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}$	$\eta_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$
	$\eta_{TM} \cdot \eta_{TE} = \eta^2$	
Group Velocity:	$u_g = \frac{\partial\omega}{\partial\beta} = u \cdot \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$	$u_g \cdot u_p = u^2$

Attenuation:	$\alpha = \alpha_c + \alpha_d$	
	$\alpha_c\|_{TE_{10}} = \frac{2 \cdot R_s}{b \cdot \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \cdot \left(\frac{1}{2} + \frac{b}{a}\left(\frac{f_c}{f}\right)^2\right)$ Only valid for TE_{10}, R_s as defined in chapter 10	$\alpha_d = \frac{\sigma_d \cdot \eta}{2\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$ Holds for any waveguide mode