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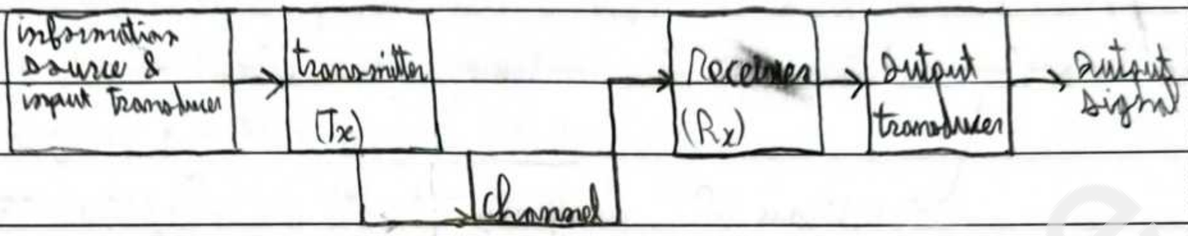
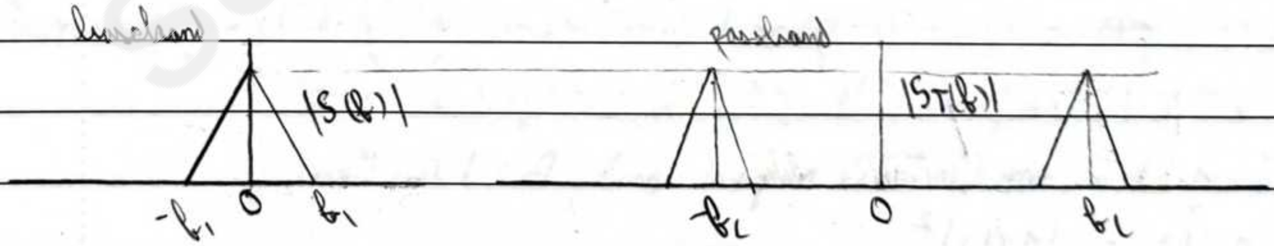
+ function diagram of a communication system:



- transmitter: processes the message signal to make it suitable for transmission over the channel
- Channel: the physical medium that connects the (Tx) & (Rx) together
- Receiver: reconstructs the transmitted signal and delivers it to the user destination

+ modulation is done:
- to reduce the antenna size: as wavelength increases, so does antenna size, hence higher frequencies allow for smaller antennas.
- to isolate signals: allows the transmission of signals simultaneously without interference.
- to increase the range of the signal: longer wavelengths can traverse longer distances.

- baseband signals are transmitted without modulation while passband are modulated



- analog signals are continuous in time and amplitude while digital signals are discrete in time and amplitude

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signal classes → periodic & non-periodic signals ①
→ deterministic & random signals ②
→ energy & power signals ③

① periodic & non-periodic signals:
- if $g(t)$ is a periodic signal then $g(t) = g(t + T_0)$
where t: time & T_0: period of $g(t)$ (duration to complete one cycle).
- if a signal does not repeat, $g(t) \neq g(t+T_0)\ \forall\ T_0$, then the signal $g(t)$ is said to be aperiodic or non-periodic.

② deterministic & random signals:
- a deterministic signal holds no uncertainty with respect to its value at any given time (hence its value can be determined for any t in the past, present, or future).
- a random signal holds some degree of uncertainty before it occurs.

③ energy & power signals:
an electrical signal can be a voltage signal or a current signal.
The instantaneous power is given by $P(t) = \frac{|V(t)|^2}{R} = |i(t)|^2 \cdot R$
hence, $P(t) \propto |V_{max}|^2$ where V_{max} is the amplitude of the signal
(or $P(t) \propto |I_{max}|^2$)
if $g(t)$ is an electrical signal and $R = 1\,\Omega$, then:
$P(t) = |g(t)|^2$
→ total energy, $\boxed{E = \lim_{T\to\infty}\left[\int_{-T}^{T} |g(t)|^2\, dt\right]} = \int_{-\infty}^{\infty} |g(t)|^2\, dt$
→ average power, $\boxed{P = \lim_{T\to\infty}\left[\frac{1}{2T}\int_{-T}^{T} |g(t)|^2\, dt\right]}$

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$\therefore$ $g(t)$ is an energy signal iff $0 < E < \infty$

& $g(t)$ is a power signal iff $0 < P < \infty$

- an energy signal's average power is zero.
- a power signal's energy is infinite.
- power signals are either periodic or random.
- energy signals are deterministic and aperiodic.

- fourier analysis is used to resolve signals into their sinusoidal components.
- If a system is linear and time-invariant then its response to a sinusoidal signal is another sinusoidal signal with the same frequency but a different amplitude

(Reminders): a system is linear if it satisfies superposition:

If $x_1(t)$ outputs $y_1(t)$ & $x_2(t)$ outputs $y_2(t)$, then the system satisfies superposition if $a\,x_1(t) + b\,x_2(t)$ output $a\,y_1(t) + b\,y_2(t)$

Where a & b are constants.

a system is time-invariant if $x(t + t_0)$ outputs $y(t + t_0)$

* modulation: the process in which a signal is modified to a form suitable for transmission over the channel.
  - some parameters of the carrier wave are changed to house the message signal.
* demodulation: the process in which the original signal is recovered from a degraded version after transmission through the channel.

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* modulation is divided into two part:
+ Continuous wave modulation:
- the carrier is a sine wave   amplitude modulation: AM
- subdivided into amplitude and angle modulation.
- angle modulation includes frequency and phase modulation. FM & PM
+ pulse modulation:
- the carrier is a periodic signal of rectangular pulses
- subdivided into analog pulse modulation and digital pulse modulation
- APM (analog pulse modulation) includes amplitude modulation (PAM), duration modulation (PDM) and position modulation (PPM).
- the standard digital form of pulse modulation is pulse code modulation
- pulse code modulation (PCM) starts as PAM then the amplitude of each modulated pulse is quantized or rounded to be coded.

* primary communication resources:
- transmitted power: the average power of the transmitted signal
- channel bandwidth: the band of frequencies allocated for the transmission of the message
- Communication channels: classified as power limited or band limited.

* Shannon's information capacity theorem:
+ design objectives of a communication system:
- must be reliable → low bit error rate (BER)
- must be efficient
+ design constraints of a communication system:
- allowable power transmission
- available channel bandwidth
- affordable cost

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+ hence, the channel capacity is defined as the maximum rate at which information can be transmitted across the channel without error.

$\therefore \quad C = B \log_2(1 + SNR)$ (in bits/second)

where C: channel capacity | B: channel bandwidth | SNR: signal to noise ratio

fourier transform Review

If $g(t)$ is a nonperiodic deterministic signal, then its fourier transform is:

$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt$$

The inverse fourier transform of $G(f)$ is:

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, dt$$

- all energy signals are fourier transformable : $g(t) \rightleftharpoons G(f)$ fourier transform pair
- $G(f)$ is generally a complex of frequency :

$$G(f) = |G(f)| \cdot e^{j\theta f}$$

where $|G(f)|$ is the continuous amplitude spectrum of $g(t)$ and $e^{j\theta f}$ is the continuous phase spectrum of $g(t)$

- for a real valued function $g(t)$: $G(-f) = G^*(f)$ | $*$: complex conjugate
  - amplitude spectrum is an even function: $|G(-f)| = |G(f)|$
  - phase is an odd function: $\theta(-f) = -\theta(f)$

+ dirichlet's conditions : sufficient (but unnecessary) conditions for the existence of FT

1- $g(t)$ is single-valued and has a finite number of maxima and minima in any finite time interval.

2- $g(t)$ has a finite number of discontinuities in any finite time interval.

3- $g(t)$ is absolutely integrable :

$$\int_{-\infty}^{\infty} |g(t)|\, dt < \infty$$

- tan is an odd function : $\tan(-x) = -\tan(x)$
- $\text{sinc}(fT) = \sin(\pi f T)/\pi T \rightarrow \text{sinc}(t) = \dfrac{\sin(\pi t)}{\pi t}$
- euler's formula : $e^{-jx} = \cos(-x) + j\sin(-x)$

$= \cos(x) - j\sin(x)$

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*properties of fourier transforms:

1. linearity: (Superposition) if $g_1(t) \rightleftharpoons G_1(f)$ ∧ $g_2(t) \rightleftharpoons G_2(f)$

$\therefore \boxed{c_1 g_1(t) + c_2 g_2(t) \rightleftharpoons c_1 G_1(f) + c_2 G_2(f)}$

2. time scaling: if $g(t) \rightleftharpoons G(f)$ $\therefore \boxed{g(at) \rightleftharpoons \frac{1}{|a|} G(f/a)}$

3. **duality**: if $g(t) \rightleftharpoons G(f)$ then $\boxed{G(t) \rightleftharpoons g(-f)}$

4. Time shifting: if $g(t) \rightleftharpoons G(f)$ then $\boxed{g(t-t_0) \rightleftharpoons G(f)\cdot e^{-j2\pi f t_0}}$

5. frequency shifting: if $g(t) \rightleftharpoons G(f)$ then $\boxed{e^{j2\pi f_0 t}\cdot g(t) \rightleftharpoons G(f-f_0)}$

6. Area under $g(t)$: $\int_{-\infty}^{\infty} g(t)\,dt = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\Big|_{f=0} dt = G(0)$

7. Area under $G(f)$: $g(0) = \int_{-\infty}^{\infty} G(f)\,df$

8. differentiation in the time domain: $\frac{d^n}{dt^n}[g(t)] \rightleftharpoons (j2\pi f)^n \cdot G(f)$

9. integration in the time domain: $\int_{-\infty}^{t} g(\tau)\,d\tau \rightleftharpoons \frac{1}{j2\pi f} G(f)$

10. Conjugate functions: $g^*(t) \rightleftharpoons G^*(-f)$, if $g(t)$ is a complex valued time function

11. multiplication in the time domain: $g_1(t)\cdot g_2(t) \rightleftharpoons G_1(f) * G_2(f)$

$\therefore g_1(t)\cdot g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)\, G_2(f-\lambda)\, d\lambda$

12. Convolution in the time domain: $g_1(t) * g_2(t) \rightleftharpoons G_1(f)\cdot G_2(f)$

$\therefore \int_{-\infty}^{\infty} g_1(\tau)\, g_2(t-\tau)\, d\tau \rightleftharpoons G_1(f)\cdot G_2(f)$

- it is sometimes easier to transform a pair to the frequency domain and multiply instead of convolving in the time domain

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* hilbert transform:

- The hilbert transform of a signal $x(t)$ is a signal $\hat{x}(t)$ whose frequency components lag the frequency components of $x(t)$ by $90°$
- hilbert transform is a $\pm 90°$ phase shifter.

→ If $\hat{g}(t)$ is the hilbert transform of $g(t)$

$$\rightarrow \hat{g}(t) = g(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau}\, d\tau$$

and the inverse hilbert transform is:

$$g(t) = \frac{-1}{\pi}\int_{-\infty}^{\infty} \frac{\hat{g}(\tau)}{t-\tau}\, d\tau$$

$\because F\{\frac{1}{t}\} = -j\pi\, sgn(f) \rightarrow F\{\frac{1}{\pi t}\} = -j\, sgn(f)$

where $sgn(f) = \begin{cases} 1, & f>0 \\ 0, & f=0 \\ -1, & f<0 \end{cases}$

$$\therefore \hat{g}(t) = g(t) * \frac{1}{\pi t} \rightarrow \hat{G}(f) = G(f)\cdot[-j\, sgn(f)]$$

example: if $g(t) = \cos(2\pi f_c t)$, find $\hat{g}(t)$.

1- $g(t) \rightleftharpoons G(f)$, $F\{\cos(2\pi f_c t)\} = \frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$

2- $F\{\frac{1}{\pi t}\} = -j\, sgn(f)$

3- $\because g(t) * \frac{1}{\pi t} = \hat{g}(t) \wedge \hat{G}(f) = G(f)\cdot[-j\, sgn(f)]$

$\rightarrow \cos(2\pi f_c t) * \frac{1}{\pi t} = \hat{g}(t) \wedge \hat{G}(f) = \frac{1}{2}\cdot[\delta(f-f_c)+\delta(f+f_c)]\cdot[-j\, sgn(f)]$

4- $\therefore \hat{G}(f) = \frac{1}{2j}[\delta(f-f_c) - \delta(f+f_c)]$

$\because \delta(f-f_c) \neq 0$ when $f = f_c \rightarrow sgn(f) = 1$

$\wedge\ \delta(f+f_c) \neq 0$ when $f = -f_c \rightarrow sgn(f) = -1$

5- $\because \hat{G}(f) = F\{\hat{g}(t)\} \rightarrow \hat{g}(t) = F^{-1}\{\hat{G}(f)\}$

$$\rightarrow \hat{g}(t) = \sin(2\pi f_c t)$$

$\because F^{-1}\{\frac{1}{2j}[\delta(f-f_c) - \delta(f+f_c)]\} = \sin(2\pi f_c t)$

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* properties of the hilbert transform:

1- $\hat{g}(t)$ and $g(t)$ have the same amplitude spectrum

$$|G(f)| = |\hat{G}(f)| \quad \because \quad |-j\,sgn(f)| = 1 \quad \forall f$$

2- hilbert transform of $\hat{g}(t)$ is $-g(t)$ (provided $G(0)=0$)

- applying the hilbert transform twice to a signal causes a sign reversal.

3- $g(t)$ and $\hat{g}(t)$ (its hilbert transform) are orthogonal

$$\rightarrow \int_{-\infty}^{\infty} g(t)\,\hat{g}(t)\,dt = 0$$

* pre-envelope:

if $g(t)$ is a real-valued signal, then its pre-envelope is:

$$g_+(t) = g(t) + j\,\hat{g}(t)$$

- taking the fourier transform $\rightarrow G_+(f) = G(f) + j[-j\,sgn(f)\cdot G(f)]$

$$\rightarrow G_+(f) = G(f) + sgn(f)\cdot G(f)$$

$$\therefore G_+(f) = \begin{cases} 2G(f), & f>0 \\ G(0), & f=0 \\ 0, & f<0 \end{cases}$$

- hence, $g_+(t)$ can be obtained from:

1- $g_+(t) = g(t) + j\,\hat{g}(t)$ 2- $g_+(t) = 2\int_0^{\infty} G(f)\,e^{j2\pi ft}\,df$

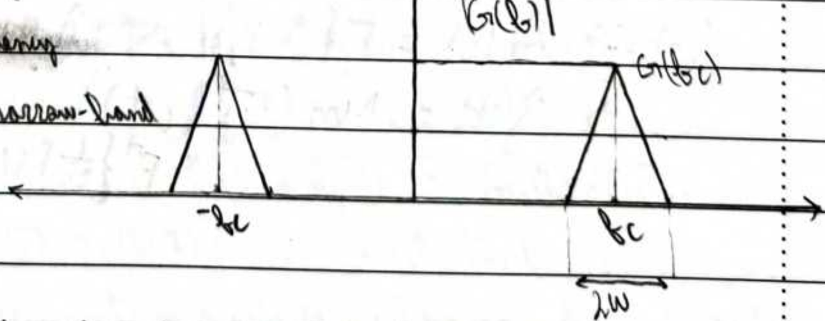
- the "+" subscripts indicate positive frequencies were taken

- the pre-envelope for negative frequencies: $g_-(t) = g(t) - j\,\hat{g}(t)$ indicated by the negative subscript.

$$G_-(f) = \begin{cases} 0, & f>0 \\ G(0), & f=0 \\ 2G(f), & f<0 \end{cases}$$

* Representation of bandpass signals:

- f_c refers to a carrier frequency
- if $f_c >> 2W$, the signal is narrow-band



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- If $g(t)$ is a narrow-band signal with $(G(f))$, then the pre-envelope of $g(t)$ can be expressed as: $g_+(t) = \tilde{g}(t)\, e^{j2\pi f_c t}$

where $\tilde{g}(t)$ is the complex envelope

- Using the frequency shifting property of the fourier transform

→ $G_+(f) = \tilde{G}(f - f_c)$ & $\tilde{g}(t) = g_+(t)\, e^{-j2\pi f_c t}$

- the complex envelope is the shifted version of the pre-envelope
- the complex envelope of a band-pass signal is a low-pass signal

∴ $g(t) = Re\{g_+(t)\} = Re\{\tilde{g}(t)\, e^{j2\pi f_c t}\}$

→ $g(t) = Re\{\tilde{g}(t) \cdot [\cos(2\pi f_c t) + j\sin(2\pi f_c t)]\}$ → euler's formula

If $\tilde{g}(t) = g_I(t) + j\, g_Q(t)$, where $g_I(t)$ & $g_Q(t)$ are real-valued lowpass signals

→ $\boxed{g(t) = g_I(t) \cdot \cos(2\pi f_c t) - g_Q(t) \cdot \sin(2\pi f_c t)}$

- $g_I(t)$ is called the in-phase component
- $g_Q(t)$ is called the quadrature component.

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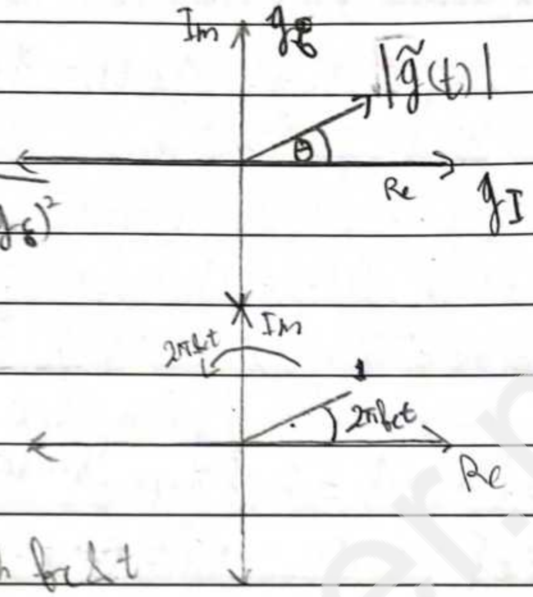
∵ $\tilde{g}(t) = g_I(t) + j\, g_Q(t)$

∴ $\tilde{g}(t)$:



where $|\tilde{g}(t)| = \sqrt{(g_I)^2 + (g_Q)^2}$

and $\theta = \tan^{-1}\left(\frac{g_Q}{g_I}\right)$

∴ $e^{j2\pi f_c t}$

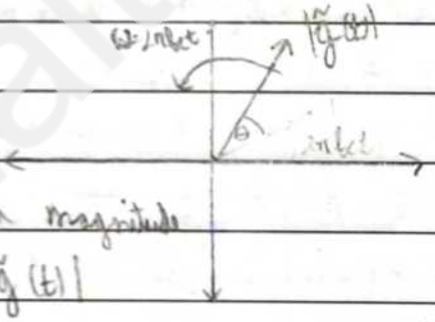
- magnitude : 1
- angle is changing with f_c & t

the line of length (magnitude) one can be assumed to rotate at an angular velocity equal to $2\pi f_c t$

- Since the pre-envelope is equal to the complex envelope multiplied by $e^{j2\pi f_c t}$

→ $g_+(t) = \tilde{g}(t) \cdot e^{j2\pi f_c t}$ ω: angular velocity

then $g_+(t)$:



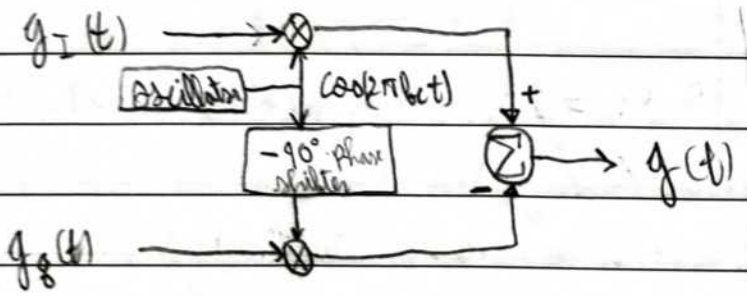
hence, $g_+(t)$ is $e^{j2\pi f_c t}$ with a phase shift of $+\theta$ and a magnitude of $|\tilde{g}(t)|$ & $|g_+(t)| = |\tilde{g}(t)|$

- keep in mind, our signal $g(t)$ is the real part of the pre-envelope

→ $g(t) = Re\{g_+(t)\}$

→ Canonical form of band-pass signal: $g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)$

∴



If the inphase and quadrature components are known, then the band-pass signal can be found.

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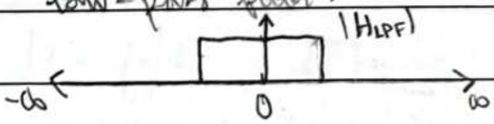
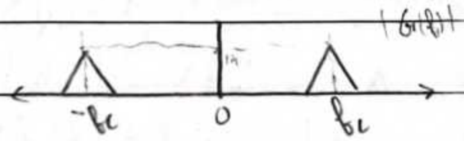
* low-pass signal : a signal whose frequency is close to the zero frequency.

- low-pass filter only passes a signal whose frequency is close to 0

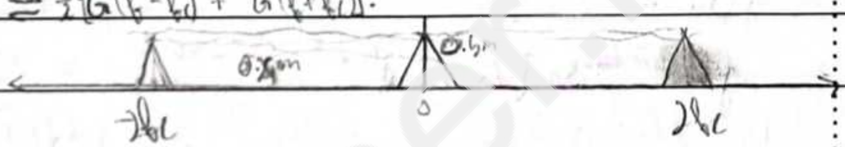
→ low-pass filter:



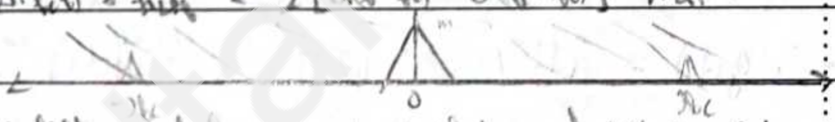
If $g(t) \rightleftharpoons G(f)$:



Then $g(t) \cdot \cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}[G(f-f_c) + G(f+f_c)]$:

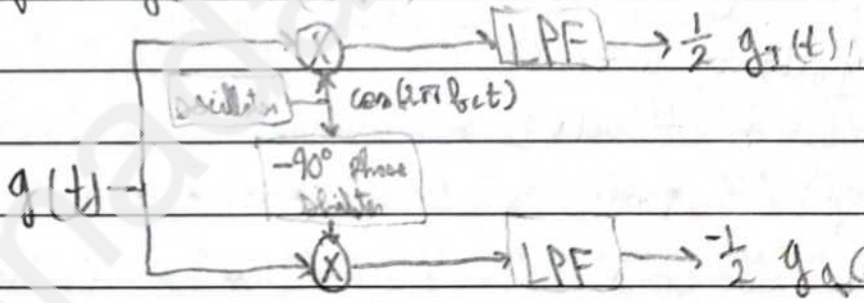


Therefore, $g(t) \cdot \cos(2\pi f_c t) * h_{LPF} \rightleftharpoons \frac{1}{2}[G(f-f_c) + G(f+f_c)] \cdot H_{LPF}$:



hence, the LPF will filter out frequencies $(-2f_c)$ and $(2f_c)$ giving $\frac{1}{2} g_I(t)$ from $g(t) \cdot \cos(2\pi f_c t)$

∴



→ $g(t) \cdot \cos(2\pi f_c t) = \underbrace{\frac{1}{2} g_I(t)}_{LP} + \underbrace{\frac{1}{2} g_I(t) \cdot \cos(4\pi f_c t)}_{HP} - \underbrace{\frac{1}{2} g_Q(t) \sin(4\pi f_c t)}_{HP}$

low-pass filter only passes $\boxed{\frac{g_I(t)}{2}}$

→ $\underbrace{g(t) \cdot \sin(2\pi f_c t)}_{(\cos(2\pi f_c t - 90°))} = \underbrace{\frac{1}{2} g_I(t) \cdot \sin(4\pi f_c t)}_{HP} \underbrace{- \frac{1}{2} g_Q(t)}_{LP} + \underbrace{\frac{1}{2} g_Q(t) \cos(4\pi f_c t)}_{HP}$

low-pass filter only passes $\boxed{-\frac{1}{2} g_Q(t)}$

+ pass-band modulation:

∘∘ complex envelope: $\tilde{g}(t) = g_I(t) + j\, g_Q(t)$ can be represented as a phasor with the same magnitude.

→ $|\tilde{g}(t)| = \sqrt{(g_I(t))^2 + (g_Q(t))^2} = a(t) = |g_+(t)|$

→ $\tan^{-1}[g_Q(t)/g_I(t)] = \phi(t)$ (phase of $g(t)$)

∴ $\tilde{g}(t) = a(t)\angle\phi(t) = \boxed{a(t)\cdot e^{j\phi(t)}}$

& $g_I(t) = a(t)\cdot\cos(\phi(t))$ & $g_Q(t) = a(t)\cdot\sin(\phi(t))$

∘∘ $g(t) = Re\{\tilde{g}(t)\cdot e^{j2\pi f_c t}\}$

→ $g(t) = Re\{a(t)\cdot e^{j\phi(t)}\cdot e^{j2\pi f_c t}\} = \boxed{a(t)\cdot\cos(2\pi f_c t + \phi(t))}$

- $a(t)$ is called the natural envelope of $g(t)$ (or simply the envelope of $g(t)$)
- $\phi(t)$ is the phase of $g(t)$
- $\boxed{g(t) = a(t)\cdot\cos(2\pi f_c t + \phi(t))}$ is called the hybrid form of amplitude and angle modulation

∘∘ pre-envelope $g_+(t) = \tilde{g}(t)\cdot e^{j2\pi f_c t} = a(t)\cdot e^{j\phi(t)}\cdot e^{j2\pi f_c t}$

→ $\boxed{g_+(t) = a(t)\cdot e^{j(2\pi f_c t + \phi(t))}}$

$= a(t)\,[\cos(2\pi f_c t + \phi(t)) + j\sin(2\pi f_c t + \phi(t))]$

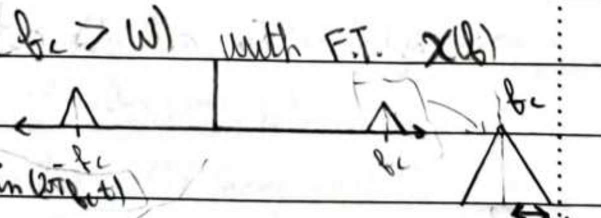
- since the inphase and quadrature components ($g_I(t)$ & $g_Q(t)$) of $\tilde{g}(t)$ contain both amplitude and phase information, therefore both components are required for a unique definition of $\phi(t)$

* band pass system:

- a band pass system's main objective is to attenuate the accumulated noise on a signal

assume $x(t)$ is a narrow band ($f_c > W$) with F.T. $X(f)$

with canonical form:



$x(t) = \underbrace{x_I(t)}_{\text{in phase component}}\cos(2\pi f_c t) + \underbrace{x_Q(t)}_{\text{quadrature component}}\sin(2\pi f_c t)$

signal is between $f_c - W$ and $f_c + W$

∘∘ $\tilde{x}(t) = x_I(t) + j\,x_Q(t)$

& $x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t}$ & $|x_+(t)| = |\tilde{x}(t)|$ & $x(t) = Re\{x_+(t)\}$

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$x(t)$ is then input into an LTI band-pass system $\xrightarrow[X(f)]{x(t)} \boxed{\begin{matrix} h(t) \\ H(f) \end{matrix}} \rightarrow y(t)$

where $H(f)$ is limited to frequencies equal to $\pm B$ multiplied by the carrier frequency where $2B \leq 2W$

$\boxed{\text{canonical representation of } h(t) = h_I(t) \cdot \cos(2\pi f_c t) - h_Q(t) \sin(2\pi f_c t)}$

and the complex impulse response $\tilde{h}(t) = h_I(t) + j \cdot h_Q(t)$

$\rightarrow h(t) = Re\{\tilde{h}(t) \cdot e^{j2\pi f_c t}\}$

pre-envelope: $h_+(t) = h(t) + j\hat{h}(t)$ where $\hat{h}(t)$ is the hilbert transform of $h(t)$

- $h_I(t)$, $h_Q(t)$, and $\tilde{h}(t)$ are all low-pass functions limited to the frequency band $-B \leq f < B$

$\because$ given a function $f_1(t) = Re\{f_2(t)\} \rightarrow f_1(t) = \frac{1}{2}[f_2(t) + f_2^*(t)]$ conjugate

$\wedge$ $h(t) = Re\{\tilde{h}(t) e^{j2\pi f_c t}\}$

$\rightarrow 2h(t) = \tilde{h}(t) e^{j2\pi f_c t} + [\tilde{h}(t) \cdot e^{j2\pi f_c t}]^*$

$\rightarrow \boxed{2h(t) = \tilde{h}(t) \cdot e^{j2\pi f_c t} + \tilde{h}^*(t) \cdot e^{-j2\pi f_c t}}$

$\wedge$ if $F[g(t)] = G(f) \rightarrow g^*(t) \rightleftharpoons G^*(-f)$

$\therefore \boxed{2H(f) = \tilde{H}(f - f_c) + \tilde{H}^*(-f - f_c)}$

$\rightarrow \boxed{\text{if } f > 0, \ 2H(f) = \tilde{H}(f - f_c)}$

- If given $H(f)$, we can find $\tilde{H}(f)$ by taking $H(f)$ for positive frequencies, shifting it to the origin, then multiplying by 2.

* to find output $y(t)$:

1- take the inverse FT of $\tilde{H}(f)$: $\tilde{h}(t) = \int_{-\infty}^{\infty} \tilde{H}(f) \cdot e^{j2\pi f t} \, df$

2- convolve $\tilde{h}(t)$ and $\tilde{x}(t)$: $\tilde{h}(t) * \tilde{x}(t) = 2\tilde{y}(t)$

$\therefore 2H(f) \cdot X(f) = 2Y(f)$

3- half the value and take the real part : $\boxed{y(t) = Re\{\tilde{y}(t) e^{j2\pi f_c t}\}}$

N O T E B O O K

Subject Continuous Wave modulation Date No.

- for a baseband signal ($m(t)$) to travel through a channel, it must be modulated.
- the carrier wave is generally sinusoidal. ($c(t)$)
+ modulating wave: the base band signal
+ modulated wave: the resulting wave after modulation is done

* Amplitude modulation:

$$c(t) = A_c \cdot \cos(2\pi f_c t)$$

where A_c: carrier amplitude, and f_c: carrier frequency

$\rightarrow$ $s(t) = A_c \cdot [1 + k_a\, m(t)] \cdot \cos(2\pi f_c t)$

where $s(t)$ is the modulated wave, k_a: amplitude sensitivity of the modulator, $m(t)$: modulating wave

- A_c and $m(t)$ are typically measured in volts, whereas k_a is measured in V^{-1}

$\because$ $s(t)$ is in canonical form, $\tilde{g}(t) = g_I(t)\cdot\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)$

$\rightarrow$ the quadrature component of $s(t)$ is 0, only an in-phase component exists. in-phase component: $A_c[1 + k_a \cdot m(t)]$

$\rightarrow$ $\tilde{g}(t) = g_I(t) + j\, g_Q(t) = A_c[1 + k_a \cdot m(t)]$

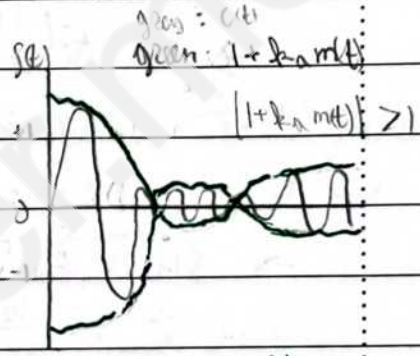
$\rightarrow$ $|\tilde{g}(t)| = |a(t)| = |g_I(t)| = A_c[1 + k_a\, m(t)]$

- Therefore the original signal can be recovered from the modulated signal since the natural envelope ($a(t)$) is equal to the amplitude of the modulated wave
- If the recovered message ($\bar{m}(t)$) is just the baseband signal multiplied by a constant: $\bar{m}(t) = k \cdot m(t)$, $0 < k < 1$
Then "perfect demodulation" takes place, where the carrier wave just needs to be amplified to be recovered.
- perfect demodulation never occurs as there is added noise, time variation and interference from other communication systems using the same frequency

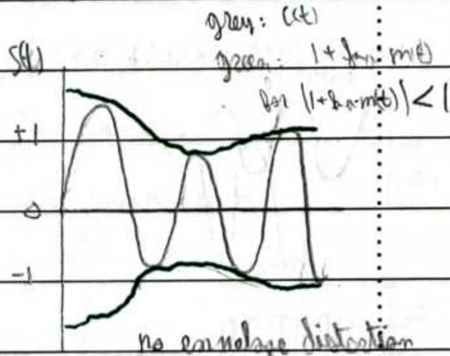
NOTEBOOK

Subject Date No.

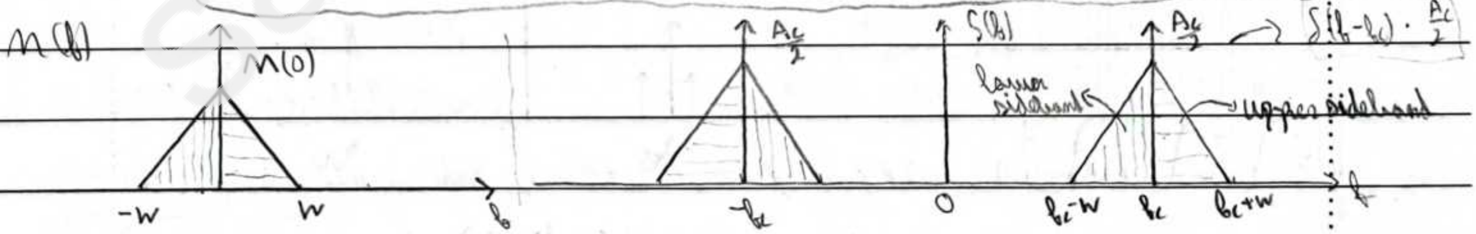
- if $a(t) = A_c[1 + k_a \cdot m(t)]$ | $1 + k_a \cdot m(t) > 0$
  then the envelope of the modulated signal is a scalar multiple of the message signal, plus some dc value (causing a dc offset)
  $\rightarrow a(t) = A_c + A_c k_a \cdot m(t)$.
- hence, the envelope of the modulated wave ($s(t)$) has the same shape of the message signal ($m(t)$) if ① $|k_a \cdot m(t)| < 1 \quad \forall t$
  $\rightarrow 1 + k_a \cdot m(t)$ is always positive
  if $|k_a \cdot m(t)| > 1$, phase reversal occurs.
  $\because -\cos(\theta) = \cos(\theta + \pi)$
  Therefore, envelope distortion occurs.



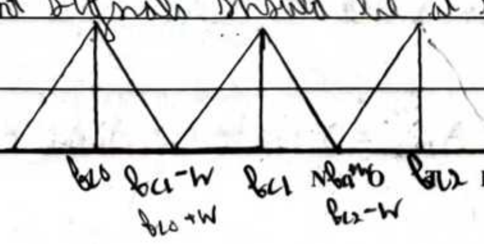
* percentage modulation: $\max[|k_a \cdot m(t)|] \times 100\%$
- the other condition for the envelope to have the same shape as the message signal is the carrier frequency must be larger than the message's bandwidth: $f_c >> W$



$\because s(t) = A_c[1 + k_a \cdot m(t)] \cdot \cos(2\pi f_c t)$
& $\cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\rightarrow \boxed{F[s(t)] = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c \cdot k_a}{2} \cdot [M(f - f_c) + M(f + f_c)]}$



- carrier frequencies for different signals should be at least 2W apart to avoid aliasing



$f_{c0} \leq f_{c1} - 2W$
$f_{c1} \leq f_{c2} - 2W$

NOTEBOOK

- Since three components exist in the frequency spectrum of $S(f)$:
  - upper sideband
  - lower sideband
  - carrier frequency

Therefore, the modulated signal is said to be full AM modulated.

* modulation index : ratio of amplitudes of the carrier wave and the message wave

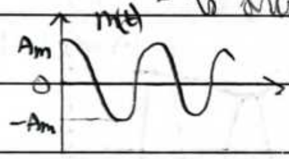
→ modulation index, $m = \dfrac{\max|m(t)|}{A_c} = \dfrac{M}{A}$

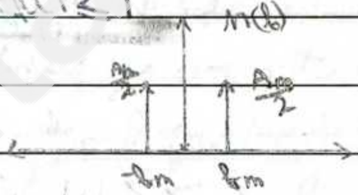
* Single-tone modulated signal : only one frequency :

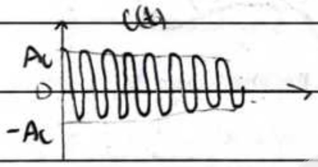
single tone : $m(t) = A_m \cos(2\pi f_m t)$

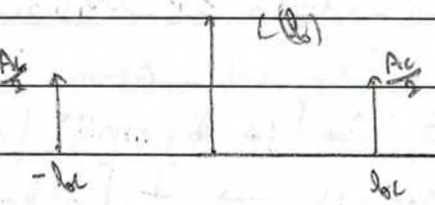
→ $s(t) = A_c[1 + \underbrace{k_a \cdot A_m}_{\mu \text{ : modulation factor}} \cos(2\pi f_m t)] \cos(2\pi f_c t)$

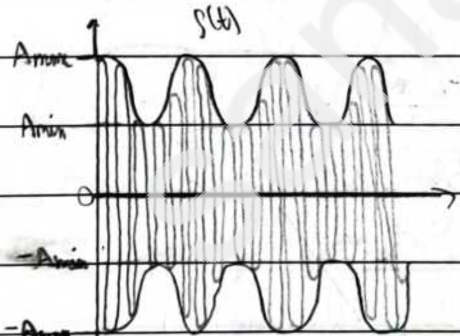
- to avoid envelope distortion, $|\mu| \le 1$



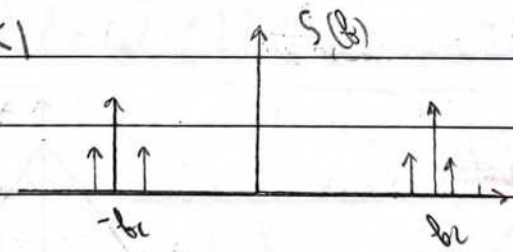








$k_a A_m < 1$



→ modulation index $= \dfrac{A_{max}}{A_{min}} = \dfrac{A_c(1+\mu)}{A_c(1-\mu)}$ → $\boxed{\mu = \dfrac{A_{max} - A_{min}}{A_{max} + A_{min}}}$ modulation index

- $\mu = 1$ if $A_{min} = 0$, trough of wave is at 0 (on the axis).

(from the single-tone example in the previous page):

$\therefore S(t) = A_c \cos(2\pi f_c t) + A_c \mu \cos(2\pi f_m t) \cdot \cos(2\pi f_c t)$

$\rightarrow S(t) = A_c \cos(2\pi f_c t) + \frac{A_c \mu}{2}\left[\cos(2\pi t(f_m - f_c)) + \cos(2\pi t(f_m + f_c))\right]$

$\rightarrow S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A_c \mu}{4}\left[\delta(f - f_m + f_c) + \delta(f + f_m - f_c)\right]$

$+ \frac{A_c \mu}{4}\left[\delta(f - f_m - f_c) + \delta(f_m + f + f_c)\right]$

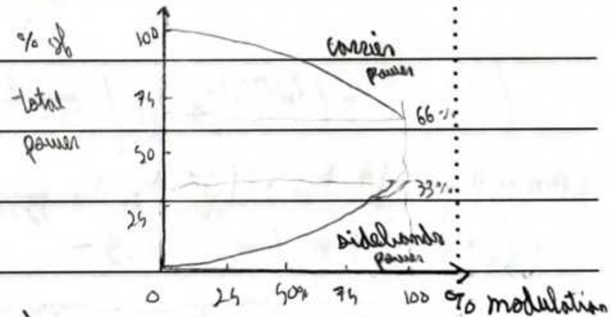
Carrier power $= \frac{A_c^2}{2}$, Upper sideband power $= \frac{A_c^2 \cdot \mu^2}{8} =$ lower sideband power

$\frac{\text{total sideband power}}{\text{total power}} = \frac{(A_c^2 \mu^2)/4}{\frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{4}} = \frac{\mu^2}{2 + \mu^2}$

- If 100% modulation ($\mu = 1$) occurs, then $\frac{\mu^2}{2+\mu^2} = \frac{1}{3}$

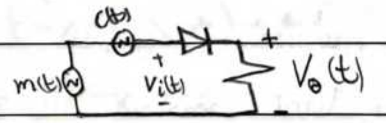
hence, the total power in the two side frequencies of the AM wave is only one third of the power in the modulated wave.

- as the percentage modulation increases, the sidebands's power as a percentage of total power increases:



- If $\mu > 1$, then overmodulation has occurred and the envelope is distorted

& Switching modulator:



1- assume the diode is ideal, $R_d = 0$ in forward bias, $R_d = \infty$ in reverse bias

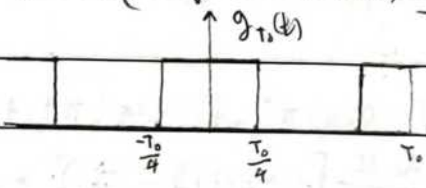
2- $C(t)$ is large in amplitude, $m(t)$ is weak in comparison

$$V_i(t) = A_c \cos(2\pi f_c t) + m(t)$$

$$V_o(t) \simeq \begin{cases} V_i(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$$

→ $V_o(t) \simeq \left[A_c \cos(2\pi f_c t + m(t))\right] \cdot g_{T_0}(t)$, where $g_{T_0}(t)$ is a control signal

$g_{T_0}(t)$:



$T_0 = \frac{1}{f_c}$ fourier series

$$g_{T_0}(t) = \underbrace{\frac{1}{2}}_{\text{dc}} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)]$$

dc shift ↓ average of one period

frequency becomes an integer multiple of f_c

$\therefore\ g_{T_0}(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + \cdots$

& $V_o(t) = V_i(t) \cdot g_{T_0}(t)$

$\rightarrow V_o(t) = \left[A_c \cos(2\pi f_c t) + m(t)\right] \cdot \left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) \cdots \infty\right]$

$\rightarrow V_o(t) = \frac{m(t)}{2} + \frac{A_c}{2}\cos(2\pi f_c t) + \frac{2\,m(t)}{\pi}\cos(2\pi f_c t) + \frac{2A_c}{\pi}\cos^2(2\pi f_c t) \cdots \infty$

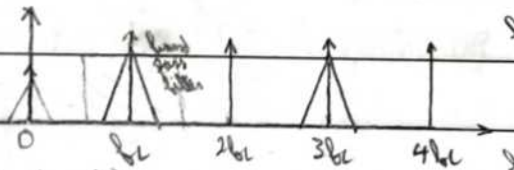
full AM

wanted components: $s(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right] \cdot \cos(2\pi f_c t)$

- by decoupling the dc component and passing through BP filter

Unwanted components: a) dc, $\cos(\pi(2f_c)t)$, $\cos(2\pi(4f_c)t)$, ...

b) $m(t)$, $m(t)\cos(2\pi(3f_c)t)$, $m(t)\cos(2\pi(5f_c)t)$, ...

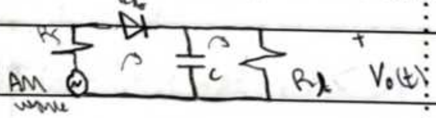


$f_c > 2W$

* envelope detector: (detects the envelope of full AM signals)

R_f : forward resistance of diode

conditions:



1- AM is narrow band, $f_c > W$

R_s : output resistance of amplifier

2- percentage modulation < 100%

3- charging time constant $(R_f + R_s)\cdot C$ must be short relative to the carrier period, $1/f_c$: $(R_f + R_s)\cdot C \ll \frac{1}{f_c}$

4- discharging time constant must be long enough to ensure that the capacitor discharges slowly through the load R_L between positive peaks of the carrier but not so long that the capacitor voltage will not discharge at the max rate of $m(t)$; $\frac{1}{f_c} \ll R_L \cdot C \ll \frac{1}{W}$

MOHAMMAD SANAD ALTAHER 130806

Subject homework #2 Date No.

2.17:

$\because$ time shift : $t - T \rightleftharpoons e^{-j2\pi f T}$

$H_1(f) : X(f) - X(f) \cdot e^{-j2\pi f T} = 1 - e^{-j2\pi f T}$

integration in time domain $\int_{-\infty}^{t} g(\tau)\, d\tau \rightleftharpoons \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2} \delta(f)$

$H_2(f) = \frac{1}{j2\pi f} H_1(f) + \cancel{\frac{H_1(0)}{2} \delta(f)}^{\,0}$ $\quad H_1(0) = X(0) - X(0) \cdot e^{0} \rightarrow H_1(0) = 0$

$H_3 = H_2(f) - H_2(f) \cdot e^{-j2\pi f T}$

$H_4 = \frac{1}{j2\pi f} H_3(f) + \cancel{\frac{H_3(0)}{2} \delta(f)}^{\,0}$ $\quad H_3(0) = H_2(0) - H_2(0) = 0$

$\rightarrow H_4 = \frac{1}{j2\pi f} H_2(f) - \frac{1}{j2\pi f} H_2(f) \cdot e^{-j2\pi f T}$

$\rightarrow H_4 = \frac{1}{j2\pi f} \left[\frac{1}{j2\pi f} \cdot H_1(f) \right] - \frac{1}{j2\pi f} \left[\frac{1}{j2\pi f} H_1(f) \cdot e^{-j2\pi f T} \right]$

$\rightarrow H_4 = \frac{-1}{4\pi^2 f^2} \left[1 - e^{-j2\pi f T} \right] + \frac{1}{4\pi^2 f^2} \left[(1 - e^{-j2\pi f T}) e^{-j2\pi f T} \right]$

$\rightarrow H_4(f) = H(f) = \frac{1}{4\pi^2 f^2} \left[e^{-j2\pi f T} - e^{-j4\pi f T} - 1 + e^{-j2\pi f T} \right]$

$$\therefore \boxed{H(f) = \frac{-1}{4\pi^2 f^2} \cdot \left[e^{-j4\pi f T} + 1 - 2 e^{-j2\pi f T} \right]}$$

NOTEBOOK

Subject Date No.

2.18:

∵ time constant: time for amplitude to drop to 37% of initial value

→ $37\% = e^{-1}$, when $t = T_0 (= RC)$ → $e^{-t/T_0} = e^{-t/RC}$

∵ $t > 0$, fourier pair: $e^{-at} u(t) \rightleftharpoons \frac{1}{a + j2\pi f}$
for $a > 0$, $RC > 0 \ \forall R, C$

→ $F\left[e^{-t/T_0}\right] = \frac{1}{1/T_0 + j2\pi f} = \frac{T_0}{1 + j2\pi f T_0}$

amplitude response = Real part of frequency response.

→ $Re\left\{\frac{T_0}{1+j2\pi f T_0} \cdot \frac{1-j2\pi f T_0}{1-j2\pi f T_0}\right\} = \boxed{\frac{T_0}{1+4\pi^2 f^2 T_0^2}}$

∴ cascading N stages: $\left[\frac{T_0}{1+4\pi^2 f^2 T_0^2}\right]^N$

Subject Date No.

2.23:

$\because g_+(t) = g(t) + j\hat{g}(t)$

a) given the hilbert transform pair: $\operatorname{sinc}(t) \rightleftharpoons \frac{1-\cos(t)}{t}$

$\rightarrow \hat{g}(t) = \frac{1-\cos(t)}{t}$

$\therefore g_+(t) = \frac{\sin(t)}{t} + \frac{j - j\cos(t)}{t}$

$\rightarrow g_+(t) = \frac{-j(\cos(t) + j\sin(t) - 1)}{t}$

$$\therefore \boxed{g_+(t) = j \cdot \left[\frac{e^{j\pi t} - 1}{t}\right]}$$

b) $\because g(t) = \cos(2\pi f_c t) + k\cos(2\pi f_m t)\cdot\cos(f_c t \cdot 2\pi)$

& hilbert transform pair $\cos(t) \xrightleftharpoons{h} \sin(t)$

$\rightarrow \hat{g}(t) = \sin(2\pi f_c t) + \frac{k}{2}\sin(2\pi(f_c - f_m)t) + \frac{k}{2}\sin(2\pi(f_c + f_m)t)$

$\therefore g_+(t) = \cos(2\pi f_c t) + \frac{k}{2}\cos(2\pi(f_c - f_m)t) + \frac{k}{2}\cos(2\pi(f_c + f_m)t)$
$+ j\sin(2\pi f_c t) + j\frac{k}{2}\sin(2\pi(f_c - f_m)t)$
$+ j\frac{k}{2}\sin(2\pi(f_c - f_m)t)$

*using euler's formula:

$\rightarrow g_+(t) = e^{j2\pi f_c t} + \frac{k}{2}e^{j2\pi(f_c - f_m)t} + \frac{k}{2}e^{j2\pi(f_c + f_m)t}$

$= e^{j2\pi f_c t}\left[1 + \frac{k}{2}e^{-j2\pi f_m t} + \frac{k}{2}e^{j2\pi f_m t}\right]$

$$\therefore \boxed{g_+(t) = e^{j2\pi f_c t} \cdot \left[1 + k\cos(2\pi f_m t)\right]}$$

NOTEBOOK

2.31:

$\because$ $$H(f) = \begin{cases} |H(f)|, & f_c - B < f < f_c + B \ \wedge\ -f_c - B < f < -f_c + B \\ 0, & \text{o.w} \end{cases}$$

given $|f_c - f_0| >> 2B$

$\rightarrow f_c - f_0 >> 2B \ \wedge\ f_0 - f_c >> 2B$

hence, f_0 is always out of the bandwidth of the filter and the output is zero.

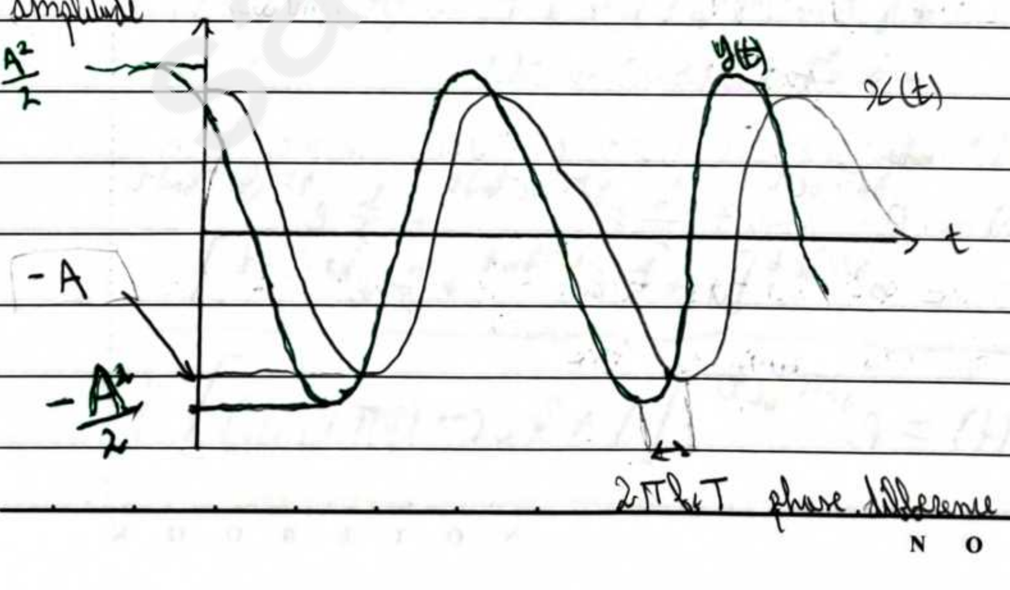
2.32:

$\because$ $$h(t) = \begin{cases} A\cos(2\pi f_c(t - T)), & 0 \le t \le T \\ 0, & \text{o.w} \end{cases}$$ cos is even

$\wedge$ time shifting : $(t - T) = e^{-j2\pi f T}$

$\rightarrow H(f) \cdot X(f) = \frac{A^2}{4}[\delta(f - f_c) + \delta(f + f_c)]^2 e^{-j2\pi f T}$

$\therefore \boxed{y(t) = \frac{A^2}{2} \cdot \cos(2\pi f_c(t - T))} \ \boxed{\forall\ t \le T \wedge t \ge 0}$



same frequency spectrum.
$y(t)$ lags $x(t)$.
$\boxed{|y(t)| = \frac{|x(t)|^2}{2}}$

Subject: full AM (continued) Date 29/3/2021 No.

* limitations of full AM:
  - only a small fraction of the total transmitted power is affected by $m(t)$ hence, inefficient.
  - inefficient use of channel as both sidebands are sent despite only needing one: hence, wasteful of bandwidth
+ modified full AM:
  - double sideband - suppressed carrier (DSB-SC):
    the transmitted wave consists of only the upper and lower sidebands
  - vestigal sideband (VSB):
    one sideband is passed almost completely and just a trace (or vestige) of the other sideband is retained.
    VSB is well suited for the transmission of wideband signals (e.g. television signals) that contain significant components at low frequencies
  - single sideband (SSB):
    only one sideband is transmitted

canonical representation of signal:

$$S(t) = S_I(t) \cdot \cos(2\pi f_c t) - S_Q(t) \cdot \sin(2\pi f_c t)$$

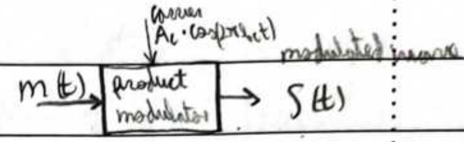
$S_I(t)$: inphase component, $S_Q(t)$: quadrature component

- $S_I(t)$ & $S_Q(t)$ are both low-pass signals that are linearly related to the message signal $m(t)$.
- the single sideband modification is only applicable if there is a gap between the upper and lower bands.
- if there is no gap between the upper and lower bands, then a trace (vestige) of one must be taken

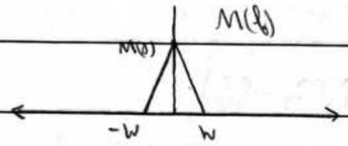
Subject Date No.

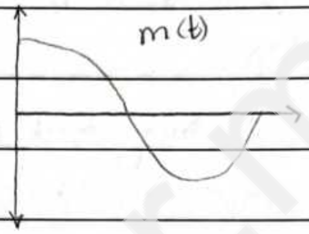
* double sideband - suppressed carrier (DSB-SC):

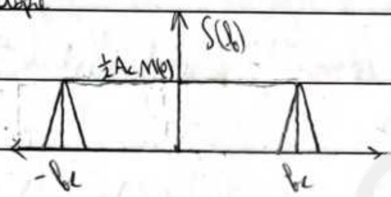
$$s(t) = A_c \cdot m(t) \cdot \cos(2\pi f_c t)$$

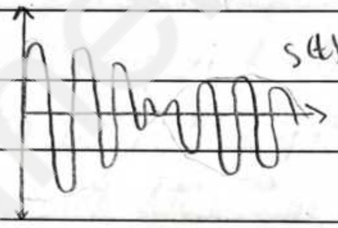


- the modulated wave is just the message signal multiplied by the carrier wave.
- s(t) undergoes a phase reversal whenever m(t) crosses zero causing envelope distortion.







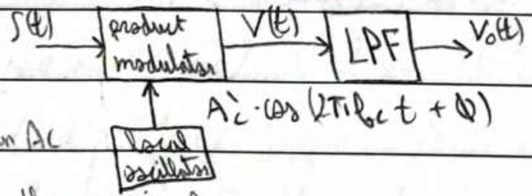


$$S(f) = F\{A_c m(t) \cos(2\pi f_c t)\} = \frac{A_c}{2} \left[M(f-f_c) + M(f+f_c) \right]$$

- modulation translates the spectrum of m(t) by $\pm f$ and scales it $\left(\frac{A_c}{2}\right)$

- since the message signal, m(t), can't be fully recovered using an envelope detector circuit, another circuit must be used.
- as a signal travels through a communication channel, its phase will change

* coherent detection:



where A_c' is a different scalar than A_c

f_c is the same as (coherent with) the carrier frequency

ϕ is the phase difference between the signals (assumed constant)

$$\rightarrow V(t) = s(t) \cdot A_c' \cdot \cos(2\pi f_c t + \phi) = A_c \cdot A_c' \cdot m(t) \cdot \cos(2\pi f_c t) \cdot \cos(2\pi f_c t + \phi)$$

$$\therefore V(t) = \frac{1}{2} \cdot A_c \cdot A_c' \cdot m(t) \cdot \cos(4\pi f_c t + \phi) + \frac{1}{2} \cdot A_c \cdot A_c' \cdot m(t) \cdot \cos(\phi)$$

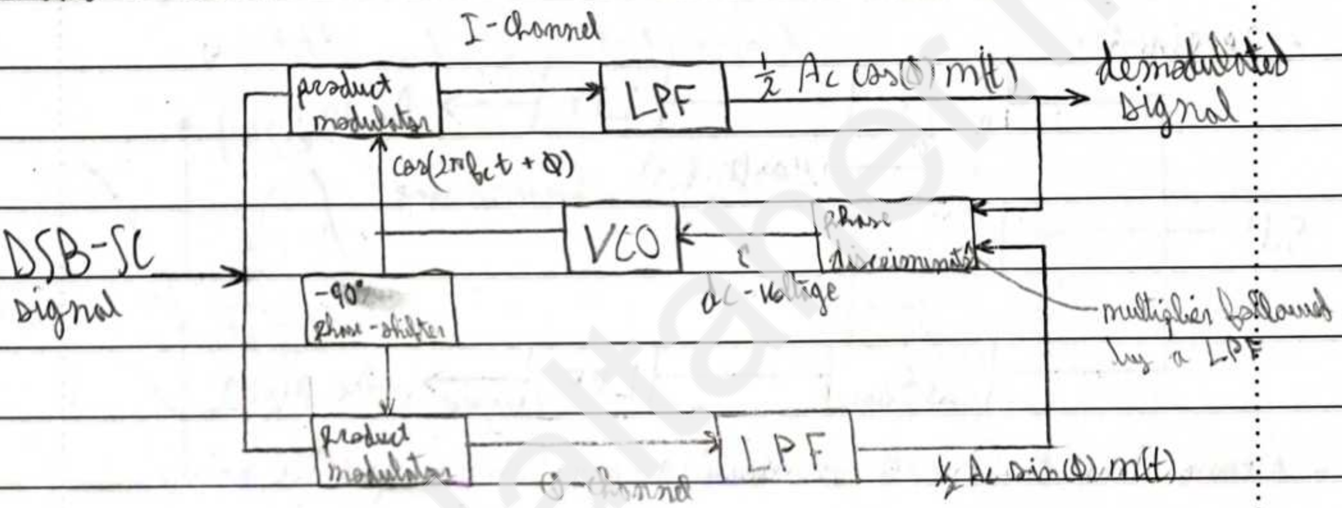
after passing through an LPF, $V_o(t) = \frac{1}{2} \cdot A_c \cdot A_c' \cdot m(t) \cdot \cos(\phi)$

Subject Date No.

- hence $V_o(t)$ has the same shape as the message signal.
- if Φ is constant, $V_o(t) \propto m(t)$

  $V_o(t)$ max at $\Phi = 0$ & min at $\Phi = \pm \frac{\pi}{2}$
- the zero demodulated signal, which occurs at $\Phi = \pm \frac{\pi}{2}$, represents the quadrature null effect of the coherent detector.

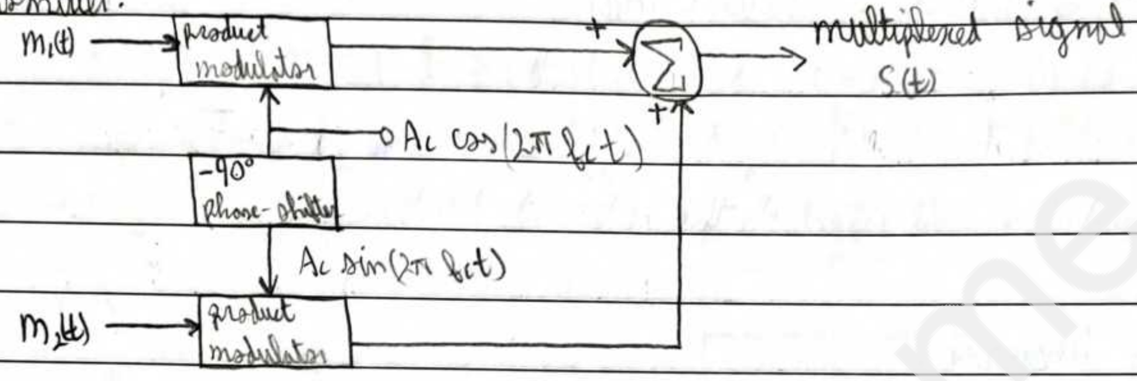
Costas Reciever:



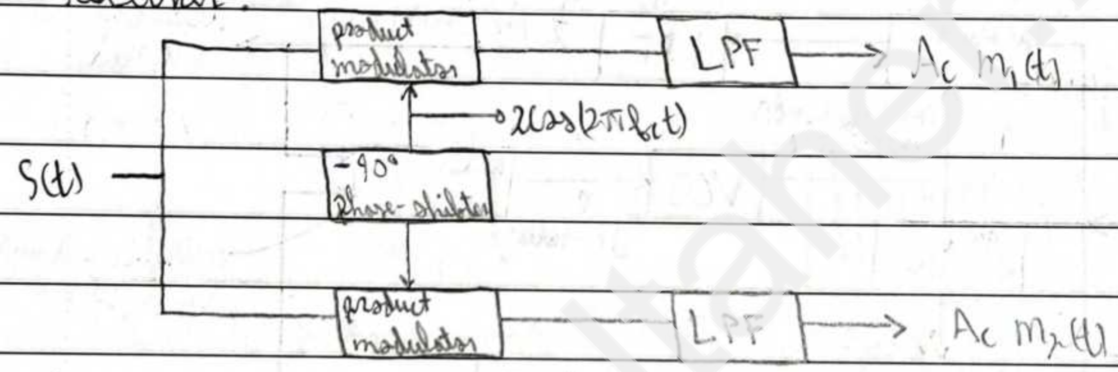
- VCO: Voltage controlled oscillator
- When $\Phi = 0$: I-channel $\propto m(t)$, Q-channel is zero
- When Φ is small value: $\sin\Phi \approx \Phi$, $\cos\Phi \approx 1$
- by creating a feedback loop with $\frac{1}{2} A_c \cos(\Phi) \cdot m(t)$ and $\frac{1}{2} A_c \sin\Phi\, m(t)$ entered into a phase discriminator, we end up with an error signal which is entered into the VCO in order to correct the phase shift
- a sin signal is necessary to identify whether the phase error is positive or negative
- the dc control signal automatically corrects the phase error by offsetting the VCO by an equivalent phase in magnitude but opposite in polarity

Subject part two: continued Date 9/4/2021 No.

* quadrature - carrier multiplexing:
- transmitter:



- receiver:



- allows two DSB-SC modulated waves to occupy the same channel:

$$S(t) = A_c m_1(t) \cdot \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t)$$

* pilot signal: low power sinusoidal tone with frequency and phase related to $c(t)$, often single frequency, sent to control, supervise, equalize, or synchronize a communication channel
- pilot signal may be sent to the receiver alone to maintain synchronization
- the phase difference between the two signals transmitted in $S(t)$ must be maintained at $\frac{\pi}{2}$, hence why synchronization is necessary.

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* Single-Sideband & Vestigial sideband modulation:

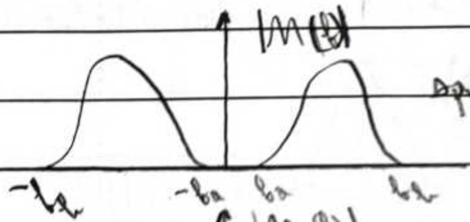
- SSB & VSB signals can be generated using frequency discrimination.

+ the following steps can be used to generate a SSB or VSB signal:

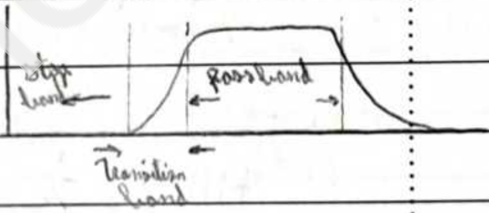
① A product modulator is used to generate a DSB-SC modulated signal.

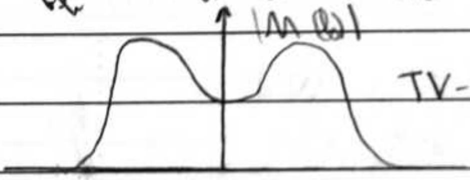
② a) for a SSB signal, a band-pass filter is used to pass one of the sidebands and suppress the other completely

b) for a VSB signal, a band-pass filter is used to pass one sideband completely and a trace of the second.



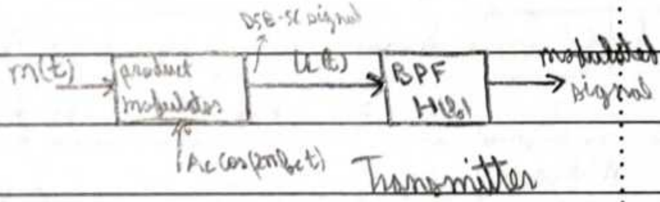
speech signal, can be modulated to SSB





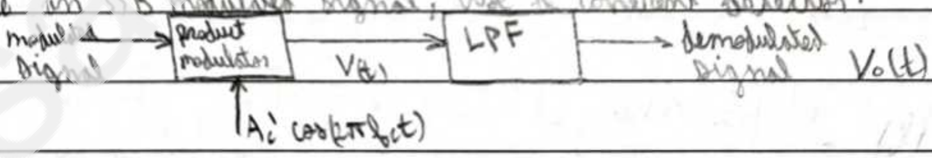
TV-signal contains significant components at low frequency hence cannot be modulated as SSB and must be VSB

- to generate an SSB signal:



Transmitter

- to receive an SSB modulated signal, use a coherent detector:



- mathematical analysis in the frequency domain gives

$$V_o(f) = \frac{A_c \cdot A_c'}{4} \cdot M(f) \cdot [H(f-f_c) + H(f+f_c)]$$

- for a distortionless reproduction of m(t), $V_o(f)$ must be a scaled version of $M(f)$, therefore $H(f)$ should satisfy:

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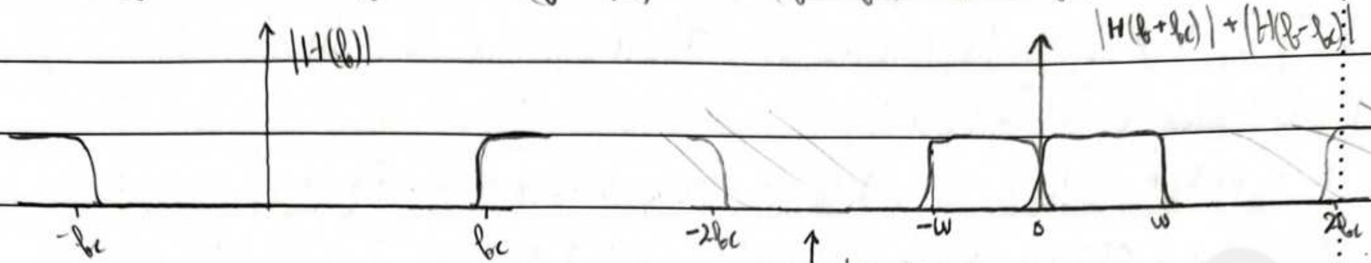
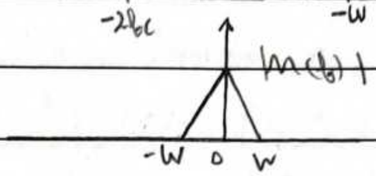
- $H(f)$ should satisfy : $H(f-f_c) + H(f+f_c) = 2H(f_c)$



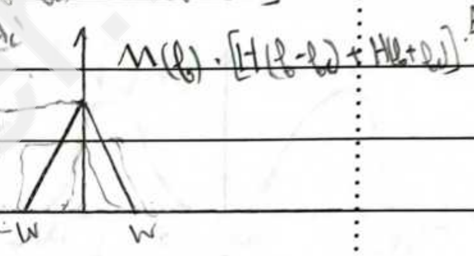
$\because\ m(t) \rightleftharpoons M(f)$



$\therefore$ output $V_o(t) \rightleftharpoons V_o(f) = \frac{A_c A_c'}{4} M(f) \cdot [H(f-f_c) + H(f+f_c)]$

is a scaled version of $M(f)$. scalar $= \frac{A_c A_c'}{4}$

$\because\ M(f) \cdot [H(f-f_c) + H(f+f_c)] = \frac{A_c A_c'}{4} M(f)$



- to simplify the analysis, assume:

$H(f_c) = \frac{1}{2}$, $M(f)$ is confined between $-W$ & W

$H(f-f_c) + H(f+f_c) = 1$, $-W \leq f \leq W$

$V_o(f) = \frac{A_c A_c'}{4} M(f)$

- $s(t)$ is a bandpass signal with canonical form:

$s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t)$

it can be shown that

$$S_I(f) = \begin{cases} S(f-f_c) + S(f+f_c) & , -W \leq f \leq W \\ 0 & , \text{elsewhere} \end{cases}$$

$$\Rightarrow S_I(f) = \begin{cases} \frac{A_c}{2} M(f) \cdot [H(f-f_c) + H(f+f_c)] & , -W \leq f \leq W \\ 0 & , \text{e.w} \end{cases}$$

$\because\ S_I(f) = \frac{A_c}{2} \cdot M(f) \rightarrow s_I(t) = \frac{A_c}{2} m(t)$

$$\& \ S_Q(f) \begin{cases} j \cdot [S(f+f_c) - S(f-f_c)] & , -W \leq f \leq W \\ 0 & , \text{e.w} \end{cases}$$

DSB-SC signal : $A_c\, m(t) \cdot \cos(2\pi f_c t)$ LPF

$A_c m(t) \cdot \cos(2\pi f_c t + \phi) = A_c\, m(t) \frac{1}{2} \left[\cos(\phi) + \cancel{\cos(\ldots t + \phi)} \right]$

$= \frac{1}{2} A_c\, m(t) \cdot \cos\phi$

$V_I \cdot V_Q$

$\frac{1}{4} \cdot A_c^2 \cos\phi \cdot \sin\phi \cdot m^2(t) = \frac{1}{8} A_c^2 \cdot \sin(2\phi) \cdot m^2(t)$

$\cos(\phi) + \cos(\phi - 90°)$

$\frac{1}{2} A_c\, m(t) \left[\cos\phi + \sin\phi \right] = A_c\, m(t) \cdot \left[\cos\left(\frac{2\phi - 90}{2}\right) \cos(45) \right]$

$A_c\, m(t) \cdot \cos(\phi - 45°) \cdot \frac{\sqrt{2}}{2}$

$\because s(t) = A_c\, m_1(t) \cos(2\pi f_c t) + A_c \cdot m_2(t) \sin 2\pi f_c t$

$s(t) \cdot 2\cos(2\pi f_c t) = \frac{1}{2} \cdot 2A_c\, m_1(t) \cdot \left[1 + \cos(4\pi f_c t) \right]$

$+ \frac{1}{2} \cdot 2 A_c\, m_2(t) \cdot \left[\sin(4\pi f_c t) + 0 \right]$

pass through LPF : $s(t) \cdot 2\cos(2\pi f_c t) = A_c \cdot m_1(t)$

$s(t) \cdot 2\sin(2\pi f_c t) = \frac{1}{2} \cdot 2A_c\, m_1(t) \left[\sin(4\pi f_c t) + 0 \right]$

$+ \frac{1}{2} \cdot 2 A_c\, m_2(t) \cdot \left[1 - \cos(4\pi f_c t) \right]$

pass through LPF $\rightarrow A_c\, m_2(t)$

---

in frequency domain:

$\because s(t) = u(t) * h(t) \Rightarrow \frac{A_c}{2} \left[M(f - f_c) + M(f + f_c) \right] \cdot H(f)$

$\because u(t) = m(t) * A_c \cos(2\pi f_c t) = \frac{A_c}{2} M(f) \cdot \left[\delta(f - f_c) + \delta(f + f_c) \right]$

$= \frac{A_c}{2} \cdot \left[M(f - f_c) + M(f + f_c) \right]$

$\rightarrow \boxed{S(f) = \frac{A_c}{2} \cdot \left[M(f - f_c) + M(f + f_c) \right] \cdot H(f)}$ — 1

in demodulation:

$V(t) = s(t)\, A_c' \cos(2\pi f_c t) \rightarrow V(f) = S(f) * \frac{A_c'}{2} \left[\delta(f - f_c) + \delta(f + f_c) \right]$

$\rightarrow \boxed{V(f) = \frac{A_c'}{2} \left[S(f - f_c) + S(f + f_c) \right]}$ — 2

sub. 1 in 2 : $V(f) = \frac{A_c'}{2} \left[\left[\frac{A_c}{2} \left(M(f - 2f_c) + M(f) \right) \right]_{H(f - f_c)} + \left[\frac{A_c}{2} \left[M(f) + M(f + 2f_c) \right] \right]_{H(f + f_c)} \right]$

$\rightarrow V(f) = \frac{A_c' A_c}{4} \left[M(f) \left[H(f - f_c) + H(f + f_c) \right] + M(f - 2f_c) H(f - f_c) + M(f + 2f_c) \cdot H(f + f_c) \right]$

$\therefore V_o(f) = \frac{A_c' A_c}{4} \cdot M(f) \cdot \left[H(f - f_c) + H(f + f_c) \right]$

Removed by LPF

$S_I(t) = S(t) \cdot 2\cos(2\pi f_c t)$ then passed through a LPF

$S_Q(t) = S(t) \cdot 2\sin(2\pi f_c t)$ then passed through a LPF

$$\rightarrow S_I(f) = \begin{cases} S(f-f_c) + S(f+f_c), & -W \le f \le W \\ 0 & e.W \end{cases}$$

$$\wedge\ S(f) = \frac{A_c}{2} \cdot \left[M(f-f_c) + M(f+f_c)\right] \cdot H(f)$$

$$\rightarrow S_I(f) = \frac{A_c}{2}\Big[\cancel{M(f-2f_c) \cdot H(f-f_c)} + M(f) \cdot H(f+f_c) + M(f) \cdot H(f-f_c) + \cancel{M(f+2f_c) \cdot H(f+f_c)}$$

$$= \frac{A_c}{2} \cdot M(f) \cdot \underbrace{\left[H(f-f_c) + H(f+f_c)\right]}_{\simeq 1}$$

$$\rightarrow S_I(f) = \frac{A_c}{2} M(f) \rightarrow S_I(t) = \frac{A_c}{2} m(t)$$

$$\rightarrow S_Q(f) = \begin{cases} j\left[S(f+f_c) - S(f-f_c)\right], & -W \le f \le W \\ 0, & \text{elsewhere} \end{cases}$$

$$S_Q(f) = \frac{jA_c}{2}\Big[M(f)\,H(f+f_c) - \cancel{M(f-2f_c) \cdot H(f-f_c)} + \cancel{M(f+2f_c) \cdot H(f+f_c)} - M(f) \cdot H(f-f_c)\Big] \quad \text{LPF}$$

$$\rightarrow S_Q(f) = \frac{jA_c}{2} \cdot M(f) \cdot \underbrace{\left[H(f+f_c) - H(f-f_c)\right]}_{=1}$$

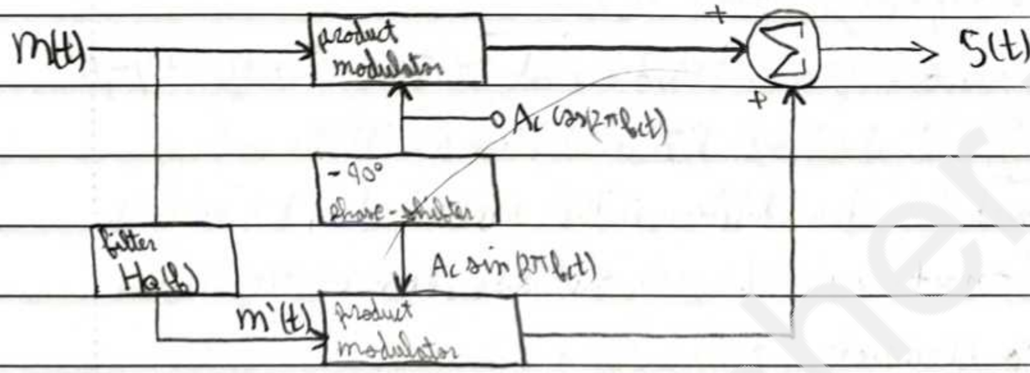
$$\rightarrow S_Q(f) = \qquad S_Q(t) = \frac{A_c}{2} \cdot m'(t)$$

$\rightarrow S_Q(f) = \frac{j}{2} \cdot A_c \cdot M(f) \cdot [H(f+f_c) - H(f-f_c)]$

- $S_Q(t)$ can be generated by passing $m(t)$ through a new filter whose transfer function: $H_Q(f) = j[H(f-f_c) - H(f+f_c)]$, $-W \le f \le W$

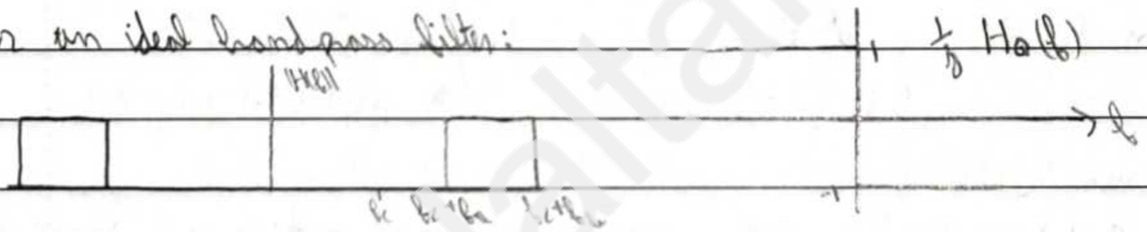
$\rightarrow S_Q(t) = \frac{A_c}{2} \cdot m'(t)$

$\therefore S(t) = \frac{A_c}{2} \cdot m(t) \cdot \cos(2\pi f_c t) - \frac{A_c}{2} \cdot m'(t) \cdot \sin(2\pi f_c t)$



(except for $\frac{1}{2}$ factor)

* for an ideal band pass filter:



- $\frac{1}{j} H_Q(f)$ gives the negative signum function in an ideal BPF

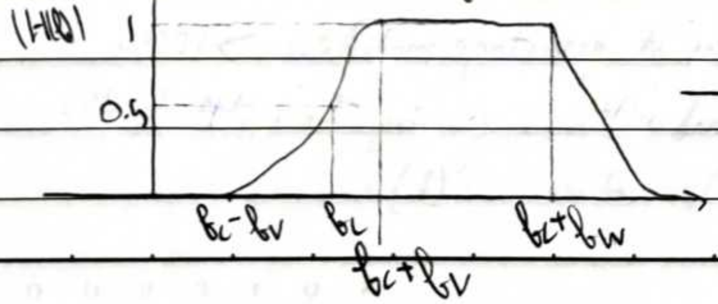
$\therefore H_Q(f) = -j\,\mathrm{sgn}(f)$

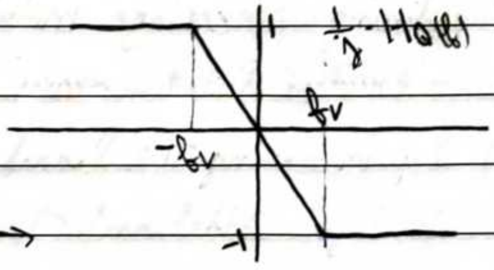
- the quadrature component of the modulated signal is the hilbert transform if the BPF is ideal (SSB signal)

$\rightarrow S(t) = \frac{A_c}{2} \cdot m(t) \cdot \cos(2\pi f_c t) \boxed{-} \frac{A_c}{2} \cdot \hat{m}(t) \cdot \sin(2\pi f_c t)$

where $S(t)$ is the SSB signal. → note 1

* for a non-ideal BPF: Vestigial sideband





Subject Date No.

- note 1: the minus sign is only for the upper sideband. If the lower sideband was passed and the upper sideband rejected, the sign will be a plus (+)

* Television signal:
  - modulated in two parts: audio is FM, whereas video is VSB modulated

modulation of video signals

① video signals have a large bandwidth and contain significant low frequency content, hence VSB modulation should be used

② the circuits used for demodulation in the receiver should be simple and cheap. envelope detection is suitable, however, requires the addition of a carrier.

* wave form distortion in VSB:
  - adding the carrier to $S(t)$ that represent the VSB modulated signal in the canonical form give the new signal as:

$$S(t) = A_c\cdot\left[1 + \tfrac{1}{2} k_a\, m(t)\right]\cdot\cos(2\pi f_c t) - \tfrac{1}{2} k_a A_c\, \hat{m}(t)\sin(2\pi f_c t)$$

the envelope $a(t) = \sqrt{S_I^2(t) + S_Q^2(t)}$

$$= A_c\cdot\left[\left(1+\tfrac{1}{2} k_a\, m(t)\right)^2 + \left(\tfrac{1}{2} k_a\, \hat{m}(t)\right)^2\right]^{1/2}$$

$$\rightarrow a(t) = A_c\cdot\left[1 + \tfrac{1}{2} k_a\, m(t)\right]\cdot\underbrace{\left[1 + \left(\frac{\frac{1}{2} k_a\, \hat{m}(t)}{1+\frac{1}{2} k_a\, m(t)}\right)^2\right]^{1/2}}_{\text{distortion factor}}$$

- $\hat{m}(t)$ contributes to the distortion, distortion can be reduced by:

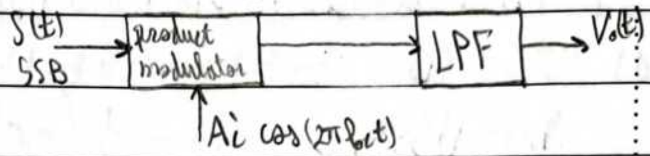
1) reducing percentage modulation to reduce k_a
   - envelope distortion occurs at percentage modulation > 100%

2) by increasing the bandwidth, increasing the width of the vestigal sideband to reduce $\hat{m}(t)$

Subject Date No.

- Reducing percentage modulation increases the carrier's share of the total power, making it less efficient

* demodulation of SSB signal :



$$V_o(t) = \frac{A_c \cdot A_c'}{4} m(t)$$, as long as the local oscillator is synchronized in both frequency and phase with the carrier signal.

If there is a phase difference, ϕ, then it can be shown that:

$$V_o(t) = \frac{1}{4} \cdot A_c \cdot A_c' \cdot [m(t) \cos(\phi) + \underbrace{\hat{m}(t) \sin(\phi)}_{\text{unwanted component}}]$$

$$\rightarrow V_o(f) = \frac{1}{4} A_c \cdot A_c' [M(f) \cos(\phi) + \hat{M}(f) \cdot \sin(\phi)] , \quad \hat{M}(f) = -j \operatorname{sgn}(f) M(f)$$

$$\therefore \quad V_o(f) = \begin{cases} \frac{1}{4} A_c A_c' \cdot M(f) \cdot [\cos(\phi) - j\sin(\phi)] , & f > 0 \\ \frac{1}{4} A_c \cdot A_c' \cdot M(f) \cdot [\cos(\phi) + j\sin(\phi)] , & f < 0 \end{cases}$$

hence, if there is no phase shift, $\phi = 0$ and no distortion occurs

- Phase error in local oscillator results in constant phase distortion
- Phase shift distortion can be tolerated in voice communication since the human ear is relatively insensitive to phase distortion, thus SSB modulation is used almost exclusively for audio signals
- the human eye can sense phase distortion, thus SSB modulation is not used for video.

MOHAMMAD SANAD ALTAHER 130806

Subject homework #3 Date No.

Q1: $m(t) = \cos(2000\pi t) + \cos(4000\pi t)$, $A_m = A_c = 1$

$c(t) = \cos(200\pi \times 10^3 t)$, upper sideband SSB signal

a) $\because$ $S(t) = S_I(t)\cos(2\pi f_c t) - S_Q(t)\sin(2\pi f_c t)$

for USSB, $S_I(t) = \frac{1}{2} m(t)$ & $S_Q(t) = \frac{1}{2}\hat{m}(t)$

$f_c = 1\times 10^5$ Hz, $\hat{m}(t) = \sin(2000\pi t) + \sin(4000\pi t)$

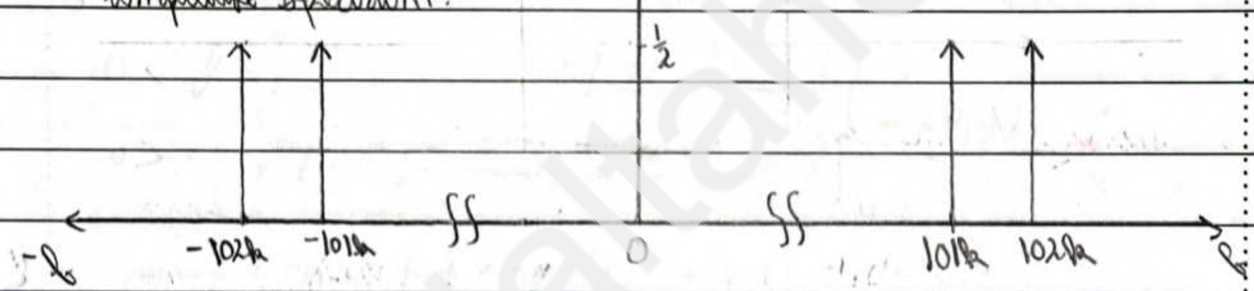
$\rightarrow S(t) = \frac{1}{2}\Big[\cos(2000\pi t)\cdot\cos(200\pi\times 10^3 t) + \cos(4000\pi t)\cdot\cos(200\pi\times 10^3 t)$

$- \sin(2000\pi t)\cdot\sin(200\pi\times 10^3 t) - \sin(4000\pi t)\cdot\sin(200\pi\times 10^3 t)\Big]$

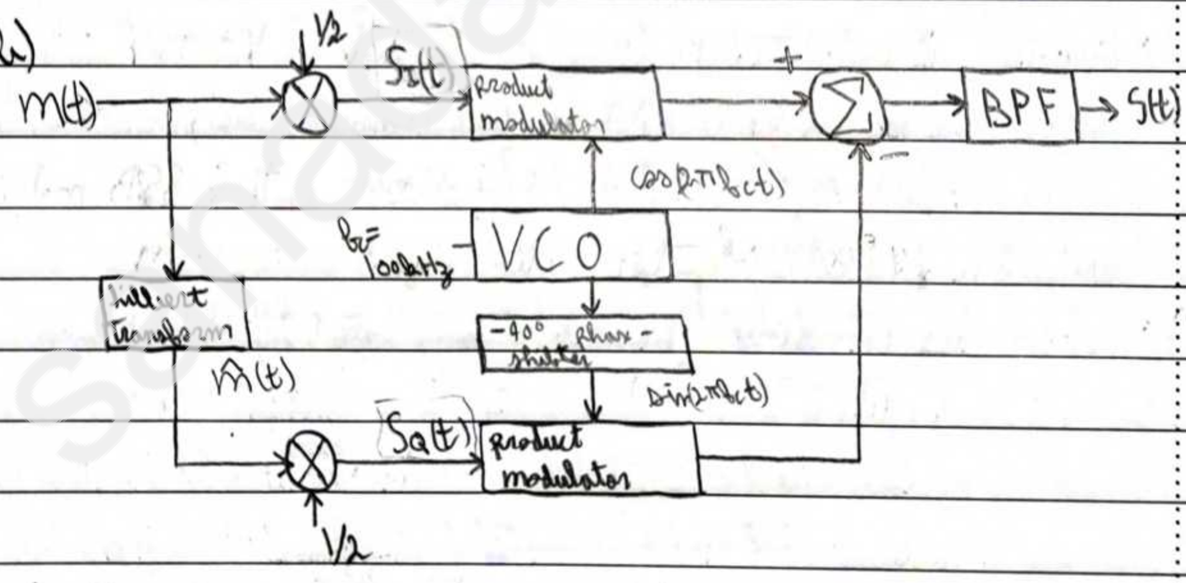
after expanding and canceling equal terms:

$\rightarrow \boxed{S(t) = \frac{1}{2}\left[\cos(202\pi\times 10^3 t) + \cos(204\pi\times 10^3 t)\right]}$

amplitude spectrum:



b)



c) ∵ $P_{avg} = \frac{1}{T}\int_{-T/2}^{T/2} |g(t)|^2 \, dt$

for a sinusoidal signal, $P_{avg} = \frac{(\text{amplitude})^2}{2}$

$P_{carrier} = \frac{1^2}{2} = 0.5$

$P_{sidebands} = \frac{1}{2}\left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right] = \frac{1}{4}$

d) $S(t) \cdot C(t) = \frac{1}{2}\left[\cos(202\pi \times 10^3 t) \cdot \cos(200.04\pi \times 10^3 t) + \cos(204\pi \times 10^3 t) \cdot \cos(200.04\pi \times 10^3 t)\right]$

after expanding and passing through a low-pass filter:

output $= \frac{1}{4}\left[\cos(1960\pi t) + \cos(3960\pi t)\right]$

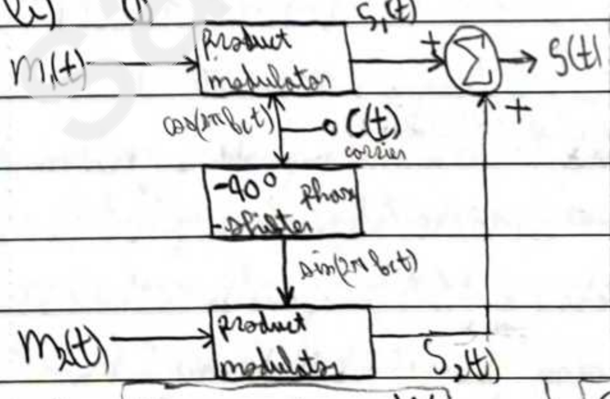
∴ frequency components: $f = 980$ Hz & 1980 Hz

Q2:

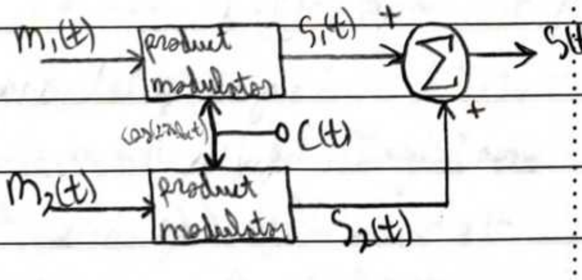
a) ① quadrature-carrier multiplexing allows for two different message signals to modulate the same carrier as long as they remain 90° apart: $S_1(t) = m_1(t) \cdot \cos(2\pi f_c t)$ | $S_2(t) = m_2(t) \cdot \cos(2\pi f_c t - 90°)$

② frequency-division multiplexing allows multiple signals to modulate the same carrier, however, unlike in quadrature carrier multiplexing, the frequencies of the message signal must be different and W apart. $S_1(t) = C(t) \cdot m_1(t)$ | $S_2(t) = C(t) \cdot m_2(t)$

b) ①



②



c) ① $\boxed{S_1 = S_2 = \frac{W}{2}}$ | ② $\boxed{S_1 = S_2 = W}$ bandwidth

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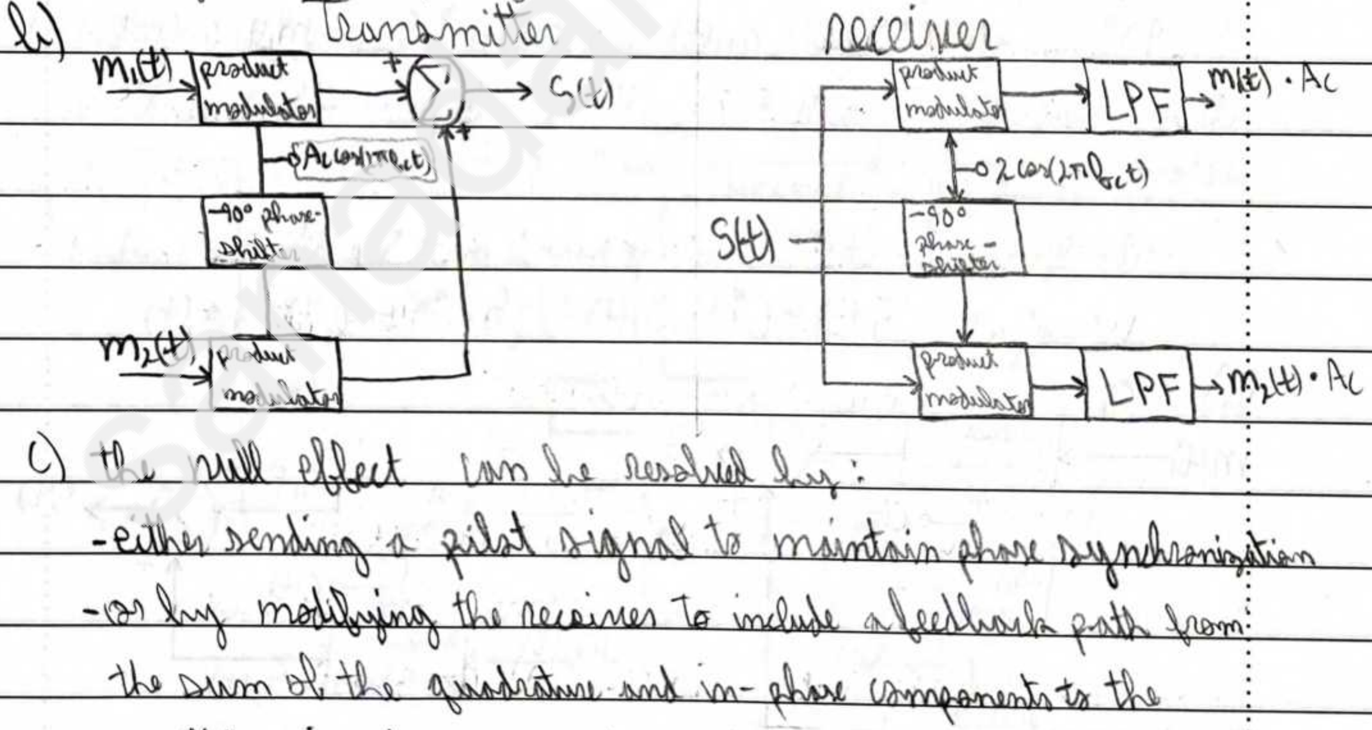
Q3:

a) The quadrature null effect is exploited to send two separate message signals (of the same frequency) on the same carrier by maintaining a 90° phase difference between the parts of the carrier modulated by the message signals (e.g. the carrier component modulated by the first message signal should lead the carrier component modulated by the second signal by exactly 90°).

The modulated signal can then be demodulated by modulating (again) with a sinusoidal signal of the same frequency as the carrier two separate times, once normally, and the second by shifting the signal by (-90°). The two separate signals are now passed through low-pass filters outputting scaled original message signals.

b)



c) the null effect can be resolved by:

- either sending a pilot signal to maintain phase synchronization
- or by modifying the receiver to include a feedback path from the sum of the quadrature and in-phase components to the oscillator in order to correct any phase shift that might have occurred.

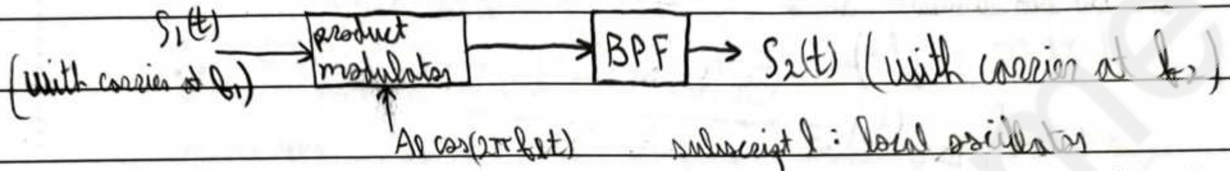
NOTEBOOK

Subject: part 3: frequency modulation   Date 11/4/2021   No.

* frequency Translation:

- mixer: converts the frequency spectrum of a signal either upwards or downwards (to a higher or lower frequency)

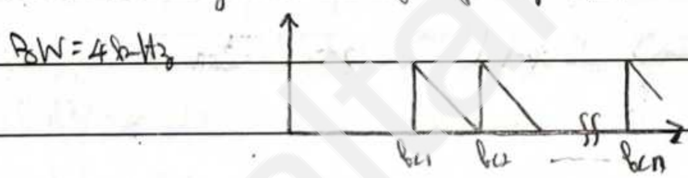
mixer block diagram:

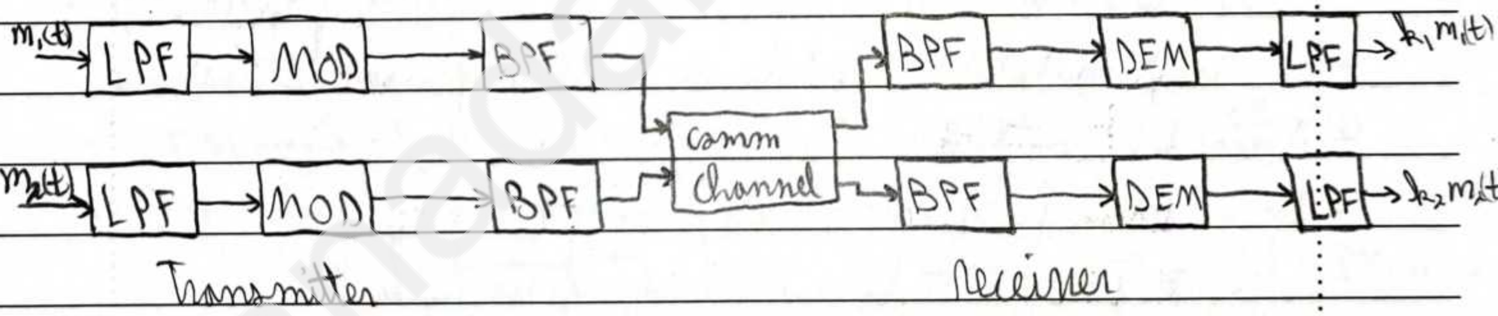


- upward conversion: $f_2 > f_1$: $f_2 = f_1 + f_l$, $f_l = f_2 - f_1$
- downward conversion: $f_2 < f_1$: $f_1 = f_2 + f_l$; $f_l = f_1 - f_2$

* frequency division multiplexing:

Used to separate the signals in frequency after SSB modulation





FDM system

* Angle modulation:

- an advantage of angle modulation (as opposed to amplitude modulation) is that the information is more immune to distortion from noise as the amplitude is mainly affected by it.
- the amplitude of the carrier is held constant so angle modulation can provide better discrimination against noise and interference (compared to AM)

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- Angle modulation needs a bandwidth greater than 2W

+ if $\theta_i(t)$ is the angle of a modulated sinusoidal carrier assumed to be a function of the message signal: $S(t) = A_0 \cos[\theta_i(t)]$

- if $\theta_i(t)$ increases monotonically with time, then the average frequency (in Hz) over an interval t to $t+\Delta t$ is given by:

$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\,\Delta t}$$

hz instead of Rad/s

instantaneous frequency: $f_i(t) = \lim_{\Delta t \to 0} [f_{\Delta t}(t)]$ definition of derivative

$$\rightarrow f_i(t) = \frac{1}{2\pi} \cdot \frac{d\theta_i(t)}{dt}$$

where $W_i(t) = \frac{d\theta_i(t)}{dt}$ angular velocity

- an Angle modulated signal is a Rotating phasor of length A_c and phase $\theta_i(t)$

- for an unmodulated carrier: $\theta_i(t) = 2\pi f_c t + \phi_c$, $\phi_c = \theta_i(0)$

$\rightarrow W_i(t) = 2\pi f_c$ (constant angular velocity)

+ common methods to vary $\theta_i(t)$ according to the message signal:

① phase modulation (PM):

$\theta_i(t) = 2\pi f_c t + k_p \cdot m(t)$, where k_p: phase sensitivity of modulator (in Rad/V)

$\rightarrow S(t) = A_c \cdot \cos[2\pi f_c t + k_p \cdot m(t)]$

② frequency modulation (FM):

the instantaneous frequency varies linearly with $m(t)$:

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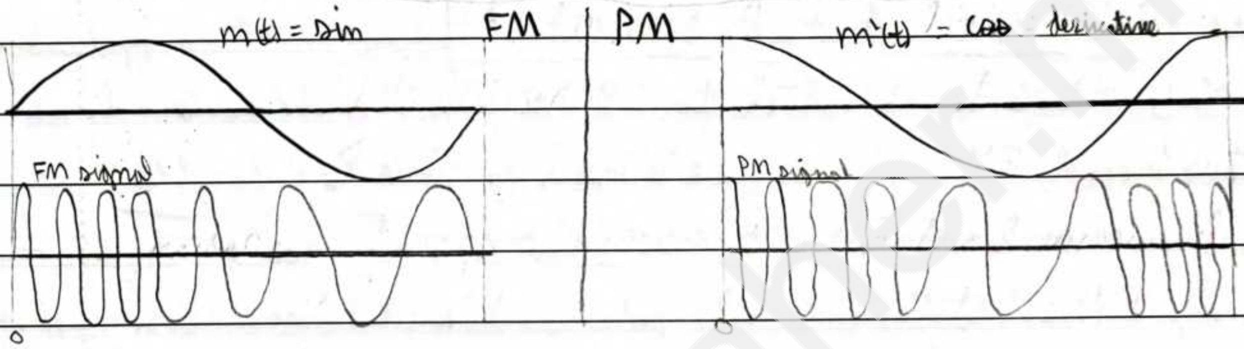
$$f_i(t) = f_c + k_f \cdot m(t) \quad \text{--- ①}$$

where k_f: frequency sensitivity of the modulator (in Hz/V)

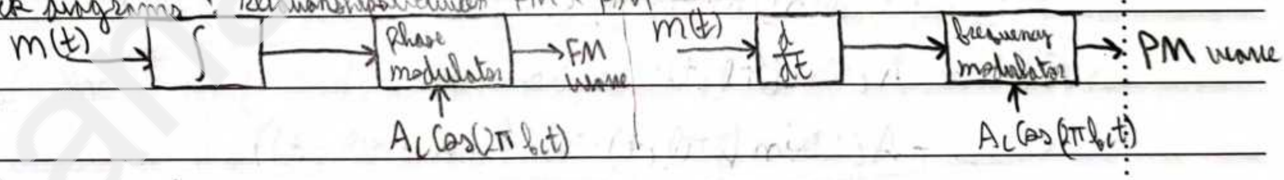
$\because f_i(t) = \frac{1}{2\pi} \cdot \frac{d\theta_i(t)}{dt} \rightarrow \theta_i(t) = 2\pi \int_0^t f_i(\tau)\, d\tau$

sub ①: $\boxed{\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau}$

$$\therefore \boxed{S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \cdot \int_0^t m(\tau)\, d\tau\right]}$$



- note that the frequency is highest when the message signal is max and lowest when the message signal is min.
- $m'(t)$ is the derivative of $m(t)$, starts at maximum while $m(t)$ starts at 0
- FM signal: $f_i(t) = f_c + k_f\, m(t)$ | PM signal: $f_i(t) = f_c + \frac{k_p}{2\pi} m'(t)$
- block diagrams: Relationship between FM & PM



* frequency modulation:

$\because S(t) = A_c \cdot \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$

& $S(t)$ is a nonlinear function of $m(t)$

two cases for $m(t)$:

① simple: single-tone that produces a narrow-band FM signal

② general: single-tone that produces a wide-band FM signal

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if $m(t) = A_m \cos(2\pi f_m t)$

$\rightarrow f_i(t) = f_c + k_f \cdot A_m \cos(2\pi f_m t)$, define frequency deviation, $\Delta f = k_f A_m$

$\rightarrow \theta_i(t) = 2\pi \int_0^t f_i(\tau) d\tau = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$

- frequency deviation, Δf, is measured in Hz since A_m is Volts and k_f is $\frac{Hz}{V}$
- define modulation index, $\beta = \frac{\Delta f}{f_m}$

$$\therefore \boxed{\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)}$$

$$\rightarrow \boxed{S(t) = A_c \cdot \cos[2\pi f_c t + \beta \sin 2\pi f_m t]}$$

+ two types of FM signal can be defined from their β:
  1. narrow-band FM, β is small compared to one radian
  2. Wide-band FM, β is large compared to one radian

& narrow-band frequency modulation:

$\because S(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$

$\wedge \cos(\alpha) \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\wedge \sin(\alpha) \sin(\beta) = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$$\rightarrow S(t) = \frac{A_c}{2} \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] + \frac{A_c}{2} \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] + \frac{A}{2} \cos[2\pi f_c t - \beta \sin(2\pi f_m t)] - \frac{A_c}{2} \cos[2\pi f_c t - \beta \sin(2\pi f_m t)]$$

$$\rightarrow \boxed{S(t) = A_c \cdot \cos(2\pi f_c t) \cdot \cos(\beta \sin(2\pi f_m t)) - A_c \cdot \sin(2\pi f_c t) \cdot \sin(\beta \sin(2\pi f_m t))}$$

$\because x << 1 \rightarrow \cos(x) \cong 1$

$\wedge x << 1 \rightarrow \sin(x) \cong x$

assuming β is smaller compared to 1 (i.e. narrow-band)

$$\rightarrow \boxed{S(t) \cong A_c \cdot \cos(2\pi f_c t) - \beta A_c \cdot \sin(2\pi f_c t) \cdot \sin(2\pi f_m t)}$$

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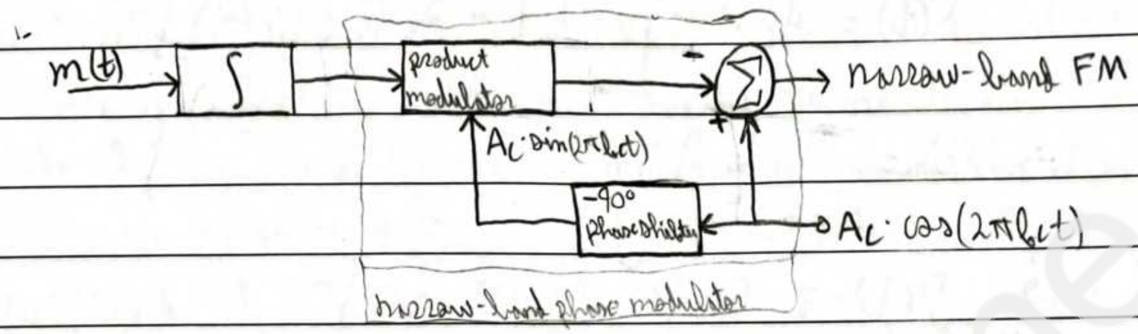
- Block diagram for narrow-band frequency modulator:



- ideally, an FM signal has a constant envelope (constant amplitude). for the case of a sinusoidal modulating signal of frequency f_m, the angle $\theta_i(t)$ is also sinusoidal with frequency (f_m).

$$S(t) = \underbrace{A_c}_{\text{envelope}} \cdot \cos\Big[\overbrace{2\pi f_c t + \beta \sin(2\pi f_m t)}^{\theta_i(t)}\Big]$$

- the generated narrow-band FM using the above simplified block diagram differs from the ideal condition in:
  1. the envelope contains residual amplitude modulation and therefore varies with time.
  2. for a sinusoidal $m(t)$, $\theta_i(t)$ contains harmonic distortions (third and higher odd harmonics)
     - if $\beta < 0.3$ radians, the residual AM and harmonic distortions are at negligible levels.

Subject Wide-band frequency modulation Date 20/4/2021 No.

* β in general:

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$$

$$\rightarrow s(t) = \mathrm{Re}\left\{ A_c e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)} \right\}$$

$$\rightarrow s(t) = \mathrm{Re}\left\{ \tilde{s}(t)\, e^{j2\pi f_c t} \right\}$$

where $\tilde{s}(t)$ is the complex envelope, a periodic function of time with a fundamental frequency of f_m

$$\tilde{s}(t) = A_c e^{j\beta \sin(2\pi f_m t)}$$

given the fourier series: $f(x) = \sum_{n=-\infty}^{\infty} C_n e^{jnx}$

where $C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-jnx} f(x)\, dx$

$$\therefore \tilde{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}$$

$$\rightarrow C_n = f_m \int_{-\frac{f_m}{2}}^{\frac{f_m}{2}} \tilde{s}(t) \cdot e^{-j2\pi n f_m t}\, dt$$

$$\therefore C_n = A_c f_m \int_{-\frac{f_m}{2}}^{\frac{f_m}{2}} e^{[j\beta \sin(2\pi f_m t) - j2\pi n f_m t]}\, dt$$

define $x = 2\pi f_m t$

$$\rightarrow C_n = \frac{A_c f_m}{2\pi f_m} \int_{-\pi}^{\pi} e^{j[\beta \sin(x) - xn]}\, dx$$

$\therefore \boxed{dx = 2\pi f_m\, dt}$

$$\therefore C_n = A_c \cdot \underbrace{\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j[\beta \sin x - nx]}\, dx}_{}$$

n^{th} order bessel function of the first kind of argument of β

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+ notes on the FM signal and its spectrum

1- the spectrum of an fm signal contains a carrier and an infinite number of side frequencies located symmetrically on either side of the carrier frequency with separations of fm, 2fm, 3fm

2- for a small value of β compared to one Radian, only the coefficients of $J_0(\beta)$ and $J_1(\beta)$ have significant values; the FM signal is effectively composed of a carrier and a single pair of side frequencies at $f_c \pm f_m$ (NBFM)

3- the amplitude of the carrier component varies with β according to $J_0(\beta)$

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j[\beta \sin x - nx]}\, dx$$

nth bessel function

$$\rightarrow C_n = A_c J_n(\beta)$$

$\therefore$

$$\tilde{S}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi n f_m t}$$

$$S(t) = A_c \operatorname{Re}\left\{ \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi(f_c + n f_m)t} \right\}$$

$$\rightarrow S(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]$$

$$\rightarrow S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)\right]$$

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+ properties of bessel function:

① if n even : $J_n(\beta) = J_{-n}(\beta)$

n odd : $J_n(\beta) = -J_{-n}(\beta)$

hence $\boxed{J_n(\beta) = (-1)^n J_{-n}(\beta)}$ for all n

② for small values of β : $J_0(\beta) \simeq 1$

$J_1(\beta) \simeq \beta/2$

$J_n(\beta) \simeq 0 \; , n \geq 2$

③ $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

$\therefore$ $S_{FM}(t) = A_c \cdot \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\, dt\right]$

$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[2\pi (f_c + n f_m) t\right]$

$= A_c J_0(\beta) \cos(2\pi f_c t) + A_c \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} J_n(\beta) \cos\left[2\pi (f_c + n f_m) t\right]$

$\Rightarrow \boxed{P_{FM} = \frac{A_c^2}{2}}$ total power

- in FM modulation, the original carrier power (before modulation) is shared by the carrier and the side frequencies.

$$P_{FM} = \frac{1}{2} A_c^2 \underbrace{\sum_{n=-\infty}^{\infty} J_n^2(\beta)}_{1} = \frac{A_c^2}{2}$$

$P_{s(t)} = \frac{A_c^2}{2}$, $P_{\substack{\text{modulated} \\ \text{carrier}}} = \boxed{\frac{[A_c J_0(\beta)]^2}{2}}$, $P_{\text{sidebands}} = P_{s(t)} - P_{\substack{\text{modulated} \\ \text{carrier}}}$

$= \boxed{\frac{(A_c)^2}{2} - \frac{[A_c J_0(\beta)]^2}{2}}$

hence $\boxed{P_{f_c + f_m} = \frac{[A_c J_1(\beta)]^2}{2}}$ because $n = 1$

$\Delta f = k_f \cdot A_m$, $\beta = \frac{\Delta f}{f_m}$

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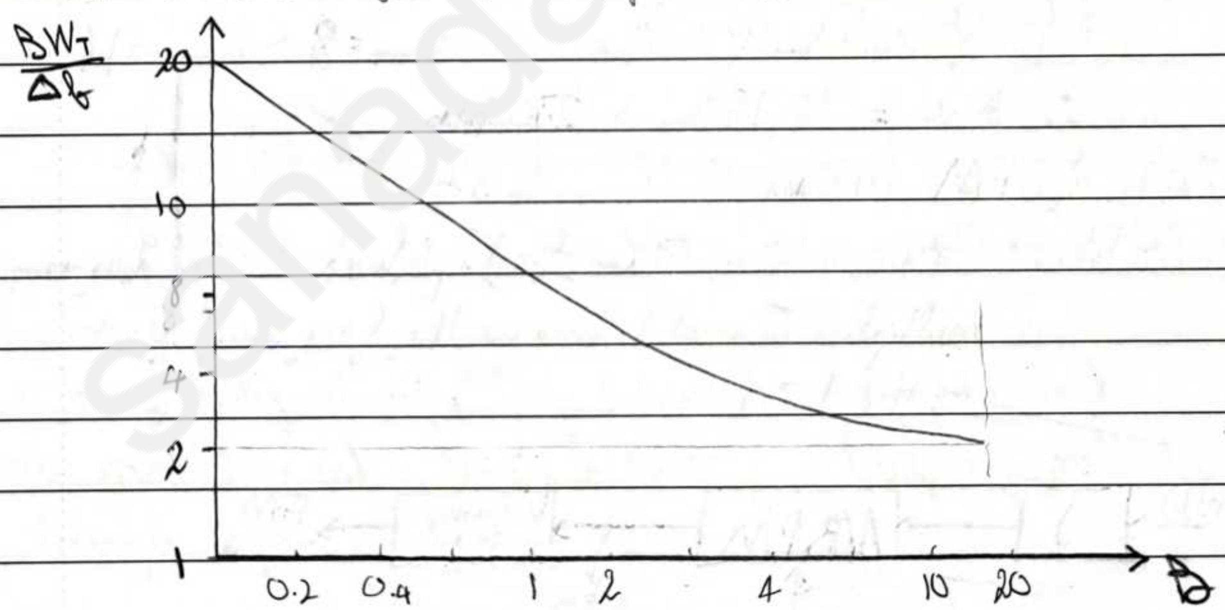
* Transmission bandwidth of FM signal:
  - for a small β, the spectrum is effectively limited to f_c and one pair of side frequencies $f_c \pm f_m$
  - for a large β, the bandwidth is slightly greater than $2\Delta f$

$$BW_T \simeq 2\Delta f + 2f_m, \quad \beta = \frac{\Delta f}{f_m}, \quad \Delta f = A_m k_f$$

$$\simeq 2\Delta f\left(1 + \frac{1}{\beta}\right) \text{ — Carson's Rule}$$

  - the effective bandwidth should include the significant side frequencies whose amplitudes are greater than some selected value (threshold)
  - the threshold is often selected as 1% of the unmodulated carrier
  - $BW_T = 2 n_{max} f_m$, n_{max} is the largest value of the integer n that satisfies $|J_n(\beta)| > 0.01$ (1%)
- Universal curve for evaluating the 1% BW of FM



Subject FM generation and demodulation: Date 20/4/2021 No.

* the more general case of AM:

m(t) has M(f) that vanishes for |f| > W

- frequency deviation ratio: $D = \frac{\Delta f}{W}$, Δf corresponds to max amplitude of m(t)
- to find the bandwidth, we can use carson's rule:

$$BW_T \simeq 2\Delta f + 2W = 2\Delta f(1 + \frac{1}{D})$$

which will yield an underestimated bandwidth. or we can use the universal curve by replacing β with D

- Values of universal curve are found by $BW_T = 2 n_{max} \cdot f_m$

$\rightarrow \frac{BW_T}{\Delta f} = \frac{2 n_{max}}{\beta}$ (or $\frac{2n_{max}}{D}$) take n_{max} where $J_n(\beta) > 0.01$

example: $\Delta f_{max} = 75$ kHz , $f_m = 15$ kHz, find BW_T:

$\therefore D = \frac{\Delta f}{f_m} = \frac{75}{15} = 5$

(1) from carson's rule: $BW_T = 2 \cdot 75k \cdot (1 + \frac{1}{5}) = 180$ kHz

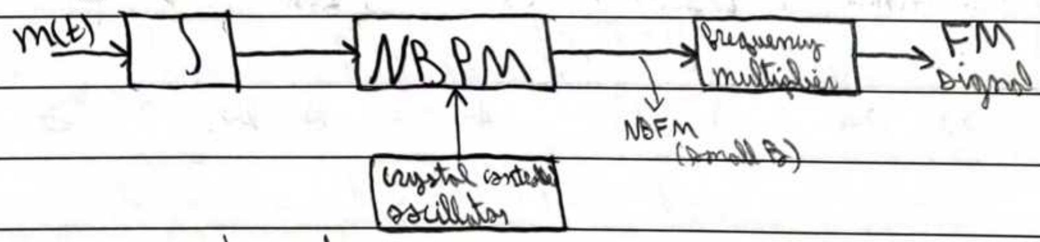
(2) from universal curve: $BW_T = 2n_{max} \cdot f_m$

bessel function table for $\beta = 5 \rightarrow n_{max} = 8 \rightarrow 2n_{max} = 16$

$\therefore BW_T = 16 \cdot 15k = 240$ kHz

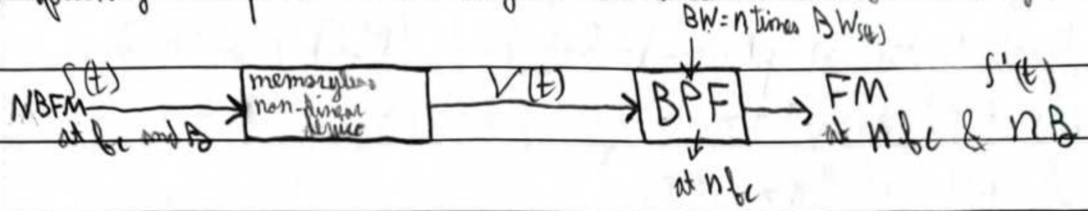
* generation of FM waves:

* indirect FM: a narrowband FM is first produced, then a frequency multiplier is used to increase the frequency deviation to the desired level.



indirect FM generation

* frequency multiplier: a memoryless nonlinear device followed by a BPF



where $V(t) = a_1 S(t) + a_2 S^2(t) + \dots + a_n S^n(t)$

and $a_1, a_2, \dots, a_n$ are coefficients determined by the operating point of the device

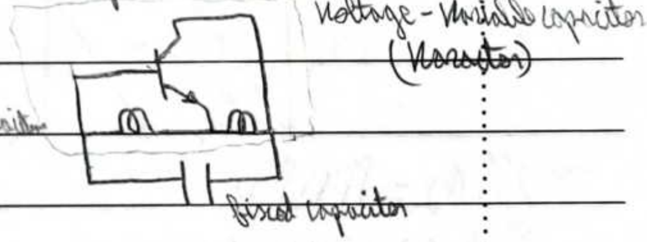
$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\, dt\right]$$

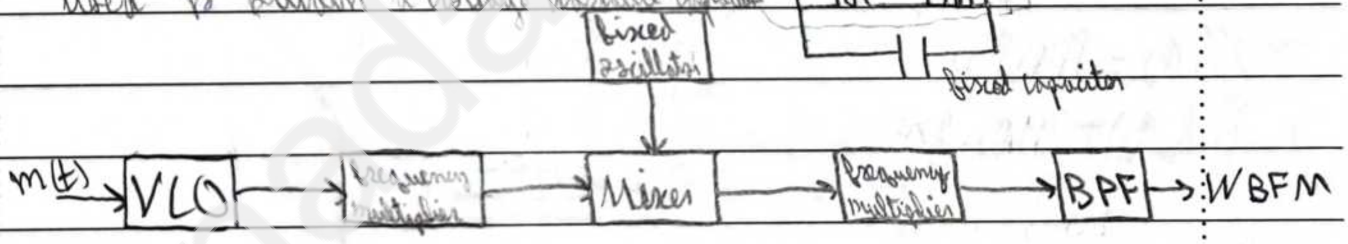
$$S'(t) = A_c' \cos\left[2\pi n f_c t + 2\pi n k_f \int m(t)\, dt\right]$$

$$f_i(t) = n f_c + n k_f m(t)$$

* direct FM: the carrier frequency is directly varied in accordance with m(t)

- the reverse biased P-N junction is used to obtain a voltage variable capacitor





wide band frequency modulation using a voltage controlled oscillator

- a mixer only changes the carrier frequency while keeping the modulation index constant, whereas a frequency multiplexer changes both.

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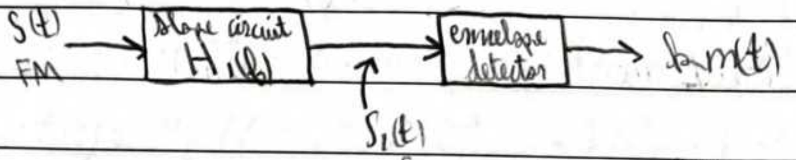
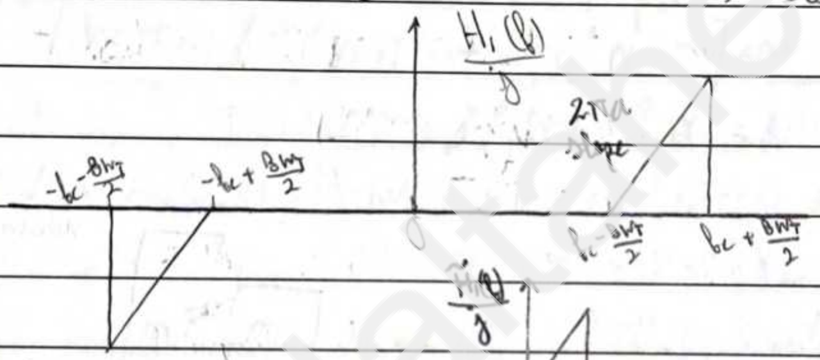
* demodulation of FM:

- frequency discriminator or phase-locked loop (PLL) consists of a slope circuit followed by an envelope detector.

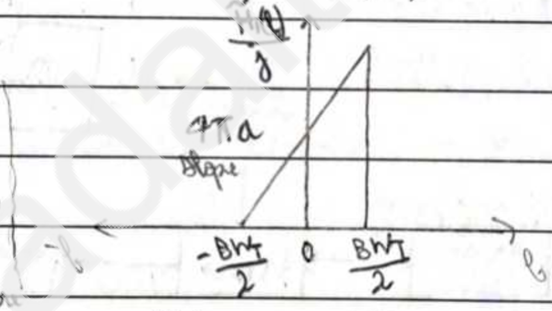


where
$$H_1(f) = \begin{cases} j2\pi a\left(f - f_c + \frac{BW_T}{2}\right), & f_c - \frac{BW_T}{2} \le f \le f_c + \frac{BW_T}{2} \\ j2\pi a\left(f + f_c - \frac{BW_T}{2}\right), & -f_c - \frac{BW_T}{2} \le f \le -f_c + \frac{BW_T}{2} \\ 0, & \text{elsewhere} \end{cases}$$



$\because 2\tilde{S}_1(f) = \tilde{H}_1(f)\tilde{S}(f)$

$\wedge\ \tilde{H}_1(f - f_c) = 2H_1(f),\ f > 0$



$$\tilde{H}_1(f) = \begin{cases} 0, & \text{elsewhere} \\ j4\pi a\left(f + \frac{BW_T}{2}\right), & -\frac{BW_T}{2} \le f \le \frac{BW_T}{2} \end{cases}$$

$\because s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\, dt\right]$

$\wedge\ \tilde{s}(t) = A_c\left[e^{j2\pi k_f \int m(t)\, dt}\right]$

$\rightarrow \tilde{S}_1(f) = \frac{1}{2}\tilde{S}(f)\cdot\tilde{H}_1(f) = \begin{cases} j2\pi a\left(f + \frac{BW_T}{2}\right)\tilde{S}(f), & -\frac{BW_T}{2} \le f \le \frac{BW_T}{2} \\ 0, & \text{elsewhere} \end{cases}$

$\because \frac{dg(t)}{dt} \rightleftharpoons j2\pi f\, G(f)$

$\rightarrow \tilde{s}(t) = a\left[\frac{d\tilde{s}(t)}{dt} + j\pi BW_T \cdot \tilde{s}(t)\right]$

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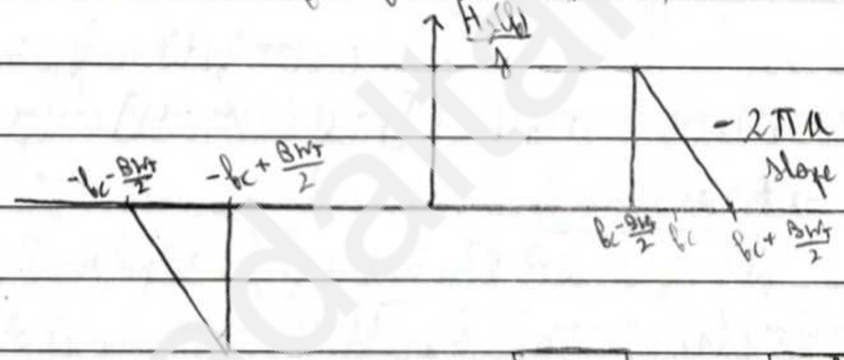
$$\therefore \tilde{S}_1(t) = j\pi BW_T a A_c\left[1+\frac{2k_f}{BW_T}m(t)\right]e^{j2\pi k_f\int m(t)dt}$$

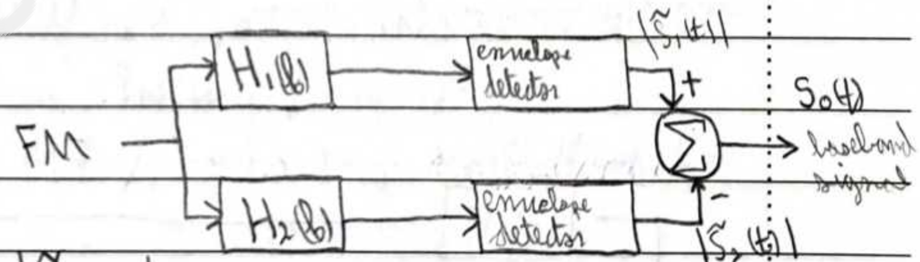
$$\Rightarrow S_1(t) = Re\{\tilde{S}_1(t)e^{j2\pi f_c t}\} = \pi BW_T a A_c\left[1+\frac{2k_f}{BW_T}m(t)\right]\cdot\cos\left(2\pi f_c t + 2\pi k_f\int m(t)dt + \frac{\pi}{2}\right)$$

+ in the condition where $\left|\frac{2k_f}{BW_T}\cdot m(t)\right| < 1$ for all t, the output of the envelope detector:

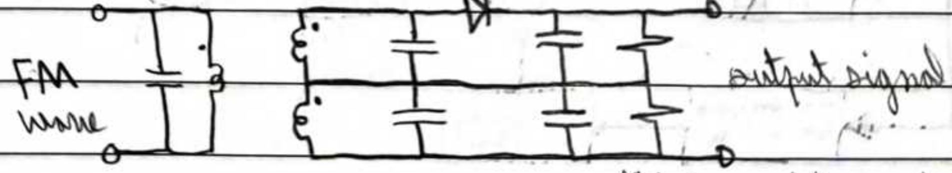
$$|\tilde{S}_1(t)| = \pi BW_T a A_c\left[1+\frac{2k_f}{BW_T}m(t)\right]$$
$$= \underbrace{\pi BW_T a A_c}_{bias} + 2\pi a A_c k_f m(t)$$

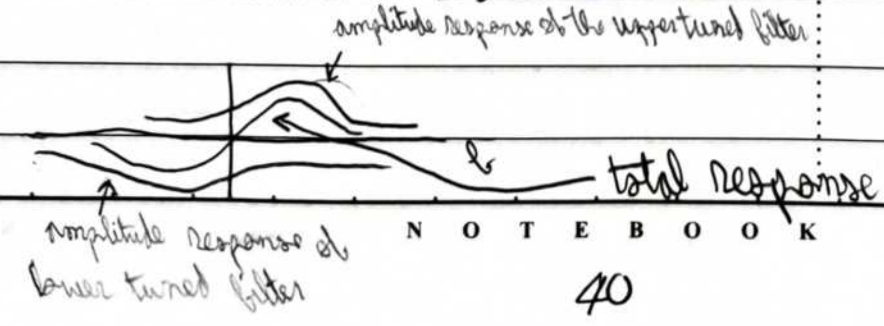
in which the bias can be removed by subtracting from $|S_1(t)|$ the output of a second envelope detector preceded by complementary slope circuit with transfer function $H_2(f)$





$$S_o(t) = |\tilde{S}_1(t)| - |\tilde{S}_2(t)| = 4\pi k_f\, a\, A_c\, m(t)$$





* FM stereo multiplexing:

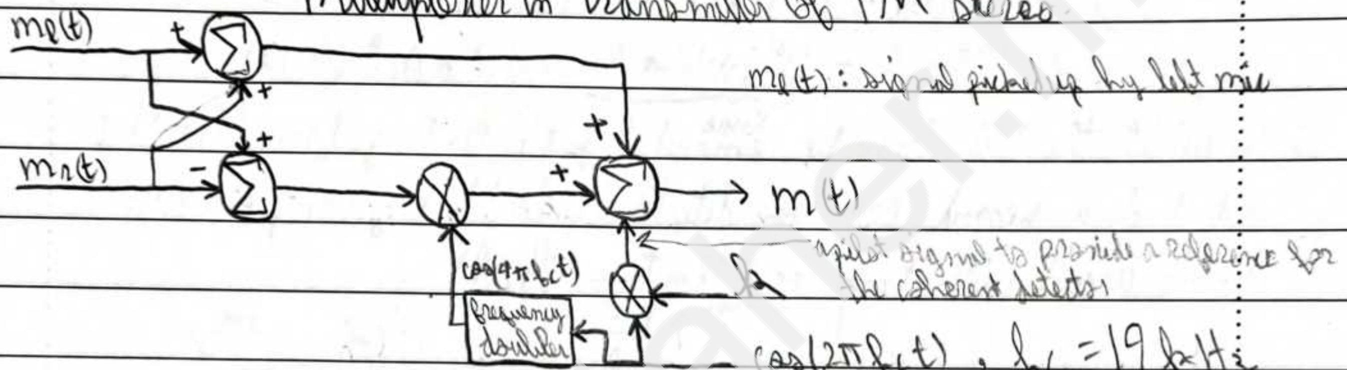
- a form of FDM designed to transmit two separate signals via the same carrier.

+ The standards for FM stereo transmission are influenced by two factors:

(1) The transmission has to operate within the allocated FM broadcast channel

(2) must be compatible with monophonic radio.

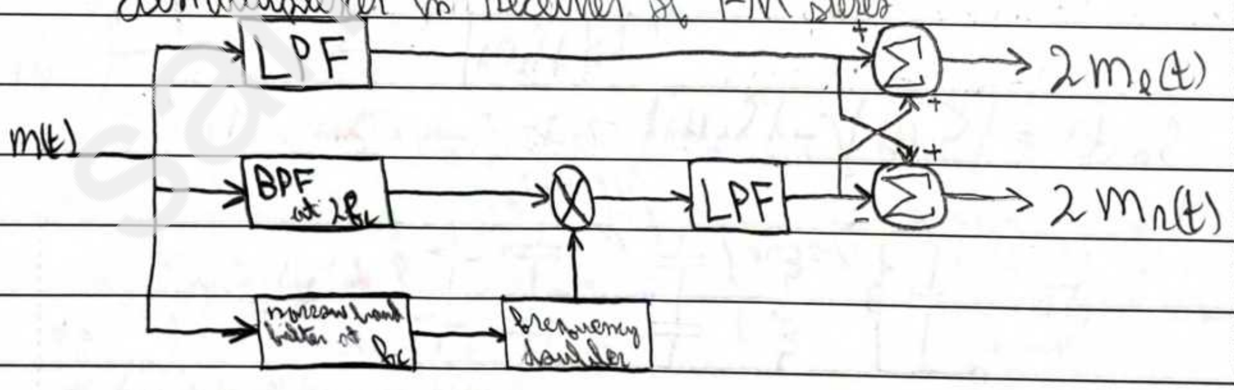
multiplexer in transmitter of FM stereo



$$\rightarrow m(t) = m_l(t) + m_r(t) + [m_l(t) - m_r(t)]\cos(4\pi f_c t) + \underline{k\cos(2\pi f_c t)}$$

pilot signal where k is used to control to power of the pilot signal and keep it in the 8% - 10% range of the peak frequency deviation

demultiplexer in receiver of FM stereo



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MOHAMMAD SANAD ALTAHER 130806

Q1)

$\because S(t) = \cos[3\sin(2000\pi t)]\cos(2\pi f_c t) - \sin[3\sin(2000\pi t)]\sin(2\pi f_c t)$

$\rightarrow S(t) = \frac{1}{2}[\cancel{\cos[2\pi f_c t - 3\sin(2000\pi t)]} + \cos[2\pi f_c t + 3\sin(2000\pi t)]]$

$- \frac{1}{2}[\cancel{\cos[2\pi f_c t - 3\sin(2000\pi t)]} - \cos[2\pi f_c t + 3\sin(2000\pi t)]]$

$\rightarrow \boxed{S(t) = \cos[2\pi f_c t + \underbrace{3\sin(2000\pi t)}_{\theta_i}]} \rightarrow \boxed{\beta = 3 , f_m = 1\,kHz}$

a) $\because S(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \cos[2\pi(f_c + n f_m)t]$

$\rightarrow \boxed{S(t) = \sum_{n=-\infty}^{\infty} J_n(3) \cdot \cos[2\pi(f_c + n \cdot 1000)t]}$

b) $\because$ one percent rule: $BW_T = 2 \cdot n_{max} \cdot f_m$

from bessel tables, $J_7(3) = 0.0025 < 1\%$

$\therefore n_{max} = 6 \rightarrow BW_T = 12 \cdot 1k = \boxed{12\,kHz}$

c) $\because P_{total} = A_c^2/2 = 0.5$

$P_{carrier} = \frac{A_c^2}{2} \cdot [J_0(\beta)]^2 = 0.5 \cdot [-0.2601]^2 = \boxed{0.03383}$

$P_{sidebands} = P_{total} - P_{carrier} = \boxed{0.46617}$

d) $\because$ frequency multiplier changes carrier frequency and β

$\rightarrow \beta_{new} = n \cdot \beta_{in} = 20 \cdot 3 = 60$

$\because \Delta f = f_m \cdot \beta \rightarrow \Delta f = 60 \cdot 1k = \boxed{60\,kHz}$

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e) $\because$ $S(t) = \sin[2\pi f_c t + \phi_1]$ & $r(t) = \cos[2\pi f_c t + \phi_2]$

& $\phi_1 = 3\sin(2000\pi t)$, $\phi_2 = 2\pi k_v \int V(t)\,dt$, $V(t)$: output

for a phase-locked loop , $\phi_1 = \phi_2$

$\rightarrow 2\pi \cdot 10k \int V(t)\,dt = 3\sin(2000\pi t)$

$\rightarrow V(t) = \frac{d}{dt}\left[3\sin(2000\pi t)\cdot \frac{1}{2\pi \cdot 10k}\right] = \frac{3}{20k\pi}\cdot\frac{d}{dt}\left[\sin(2000\pi t)\right]$

$\therefore \boxed{V(t) = \frac{3}{10}\cdot\cos(2000\pi t)}$

f)

$\because V_R = V_{in}\cdot\frac{R}{R + (1/j2\pi f C)} \rightarrow V_R = V_{in}\cdot\frac{R[j2\pi f C]}{1 + R[j2\pi f C]}$

& $1 >> 2\pi f RC \rightarrow 1 + j2\pi f RC \approx 1$

$\therefore V_R = V_{in}\cdot \boxed{j2\pi f RC}$ transfer function of RC

$\rightarrow V_R = S(f)\cdot j2\pi f RC$ $\because V_{in}$ = signal S

$\because \frac{d}{dt}[g(t)] \rightleftharpoons j2\pi f\, G(f)$

$\therefore V_R = RC\cdot\frac{d}{dt}[S(t)]$

$\because S(t) = \cos[2\pi f_c t + 2\pi k_f \int m(t)\,dt]$

$\frac{dS(t)}{dt} = \frac{d}{dt}[2\pi f_c t + 2\pi k_f \int m(t)\,dt]\cdot[-\sin(2\pi f_c t + 2\pi k_f \int m(t)\,dt)]$

$= -[2\pi f_c + 2\pi k_f m(t)]\cdot\sin[2\pi f_c t + 2\pi k_f \int m(t)\,dt]$

$\therefore V_R = -RC\cdot 2\pi f_c\left[1 + \frac{k_f}{f_c}m(t)\right]\sin[2\pi f_c t + 2\pi k_f \int m(t)\,dt]$

- passing through an envelope detector removes the sinusoidal component from V_R, giving:

$$\boxed{V_o(t) = 2\pi\cdot R\cdot C\cdot f_c\left[1 + \frac{k_f}{f_c}\cdot m(t)\right]}$$

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Q2) $\because$ $n_1 = 5$, $n_2 = 15$, amplitudes $= 1$, $\beta_{in} = 0.5$, $\cos(10\pi 10^3 t) = S(t)$

- frequency multipliers change the carrier frequency and modulation index (β)

2.1) - Carson's rule: $BW_T = 2\Delta f(1 + \frac{1}{\beta})$, $\beta = n_1 \beta_{in} = 5 \cdot 0.1 = 0.5$

$\rightarrow BW_T = 2\Delta f(3) = 6 \cdot \Delta f$

$\because \beta = \frac{\Delta f}{f_m} \rightarrow \Delta f = 0.5 \cdot f_m = 0.5 \cdot 5k = 2500$

$\therefore BW_T = 6 \cdot 2500 = \boxed{15\,kHz}$

2.2) $\because P_{total} = 0.5$, $P_{carrier} = 0.5[J_0(0.5)]^2 = 0.4404$

$\rightarrow P_{sidebands} = 0.5 - P_{carrier} = 0.0596$ (in both sidebands)

$\therefore P_{LSB} = P_{SB\,total}/2 = \boxed{0.0298}$

2.3) $\because f_c = 0.2\,MHz \cdot n_1 = 1\,MHz$

$f = 1005\,kHz = f_c + n \cdot f_m \rightarrow n \cdot 5k = 5\,kHz$

$\therefore \boxed{n=1}$, find the nth bessel function's value from the bessel tables $\rightarrow J_1(0.5) = 0.2423$

$P_{1005k} = \frac{1^2}{2} \cdot [J_1(0.5)]^2 = 0.5 \cdot [0.2423]^2 = \boxed{0.0293}$

2.4) $\because \beta = n_1 \cdot \beta_{in}$, $0.5 \leq \beta \leq 4$

for power in modulated carrier to be minimized, $J_0(\beta)$ must be minimal in range $0.5 \leq \beta \leq 4$, $J_0(2.4) = 0.0025$ is the smallest

$\therefore \beta = n_1 \cdot 0.1 = 2.4 \rightarrow \boxed{n_1 = 24}$

2.5) $\because \Delta f = \beta \cdot f_m$, $\beta = \beta_{in} \cdot n_1 \cdot n_2 = 0.1 \cdot 5 \cdot 15 = 7.5$

$\rightarrow \Delta f = 7.5 \cdot 5k = \boxed{37.5\,kHz}$

2.6) $\because$ input is cosine, the output components are follow:

$f = [f_{c2} \pm f_{c1} \cdot n_1] \cdot n_2 \pm n \cdot f_m$

$f = \{199.995, 120.005, 149.995, 150.005\}\,MHz$

$f = 199.995\,MHz \wedge f = 120.005\,MHz$ will be passed only

$\boxed{BW = 10\,kHz}$ $120.005M - 199.995M = 10k$

N O T E B O O K

Subject mid exam practice Date 23/4/2021 No.

mid makeup 2020:

Q1) a) $\because$ $\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$

$\rightarrow A_c \cos[2\pi f_c t + 2\pi k_f \int m(t)\,dt]$

$= A_c \cos(2\pi f_c t)\, S_I(t) - A_c \sin(2\pi f_c t)\, S_Q(t)$

$= A_c \cos(2\pi k_f \int m(t)\,dt)\cos(2\pi f_c t) - A_c \sin(2\pi k_f \int m(t)\,dt)\sin(2\pi f_c t)$

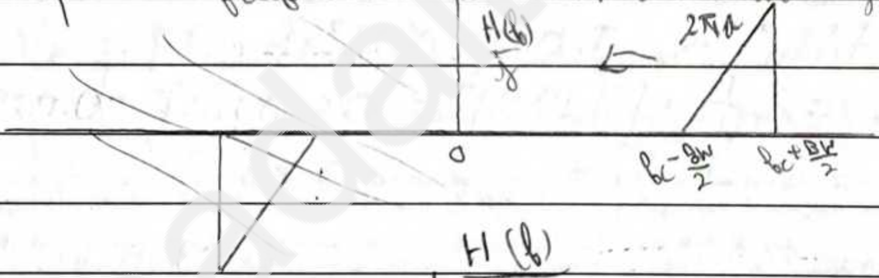
b) $\because$ $a(t) = \sqrt{[g_Q(t)]^2 + [g_I(t)]^2}$

$= \sqrt{A_c^2 \cos^2[2\pi k_f \int m(t)\,dt] + A_c^2 \sin[2\pi k_f \int m(t)\,dt]}$

$= A_c$

c) equivalent low-pass : $\tilde{x}(t) = x_I(t) + j\,x_Q(t)$

for an impulse response : $\tilde{H}(f) = 2H(f)$ corresponding to positive frequencies and shifted to the origin



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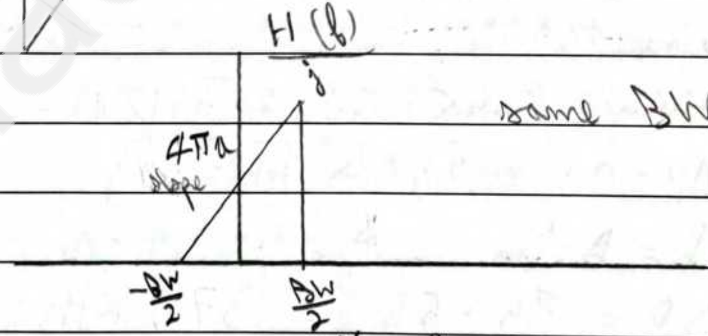


same BW

d) $\because$ $\tilde{y}(t) = F^{-1}\{\tilde{H}(f)\cdot\tilde{G}(f)\} \div 2$

$\tilde{y}(t) = A_c \cos[2\pi k_f \int m(t)\,dt] + j A_c \sin[2\pi k_f \int m(t)\,dt]$

or $\because$ $\tilde{H}(f) = \begin{cases} 4\pi a, & -\frac{BW}{2} < f < \frac{BW}{2} \\ 0, & \text{elsewhere} \end{cases}$

$\rightarrow \frac{1}{2}\tilde{H}(f)\cdot\tilde{G}(f) = \begin{cases} 0, & \text{elsewhere} \\ \frac{1}{2}\cdot j4\pi a\left[f+\frac{BW}{2}\right]\cdot\tilde{G}(f), & -\frac{BW}{2}\leq f\leq\frac{BW}{2}\end{cases}$

$\rightarrow$ when $f = \frac{-BW}{2} \rightarrow \tilde{H}(f)\cdot\tilde{G}(f) = 0$

$\wedge$ when $f = \frac{BW}{2} \rightarrow \tilde{H}(f)\cdot\tilde{G}(f) = \max$, complete overlap

$\rightarrow \frac{1}{2}\tilde{H}(f)\cdot\tilde{G}(f) = j2\pi a f\cdot\tilde{G}(f) + j2\pi a\frac{BW}{2}\cdot\tilde{G}(f)$

$\therefore \frac{d}{dt}g(t) = j2\pi f G(f)$

$\rightarrow F^{-1}\{\frac{1}{2}\tilde{H}(f)\cdot\tilde{G}(f)\} = a\frac{d}{dt}(\tilde{g}(t)) + j2\pi a\frac{BW}{2}\cdot\tilde{g}(t) = \frac{1}{2}\tilde{y}(t)$

$\frac{d}{dt}(\tilde{g}(t)) = -A_c 2\pi k_f m(t)\sin[2\pi k_f\int m(t)dt]$
$+ jA_c 2\pi k_f m(t)\cos[2\pi k_f\int m(t)dt]$

$\therefore \tilde{y}(t) = 2ja\pi\left[A_c\left[k_f m(t)\cos[2\pi k_f\int m(t)dt] + jk_f m(t)\sin[2\pi k_f\int m(t)dt]\right] + \frac{BW}{2}\left[A_c e^{j2\pi k_f\int m(t)dt}\right]\right]$

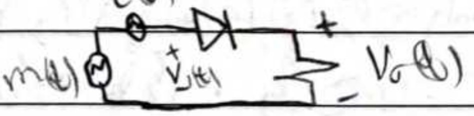
$= 2ja\pi A_c e^{j2\pi k_f\int m(t)dt}\left[k_f m(t) + \frac{BW}{2}\right]$

$= A_c a\pi\cdot BW\cdot\left[1+\frac{k_f}{BW}\cdot 2m(t)\right]e^{j[2\pi k_f\int m(t)dt + \frac{\pi}{2}]}$

e) $y(t) = Re\{\tilde{y}(t)\} = A_c a\pi BW\left[1+\frac{2k_f}{BW}m(t)\right]\cdot\cos(2\pi k_f\int m(t)dt + \frac{\pi}{2})$

$\rightarrow$ output $y(t) = A_c a\pi BW\left[1+\frac{2k_f}{BW}m(t)\right]$

Q2) a)



b) $S^2(t) = 4[1+k_a m(t)]^2\cos^2(2\pi f_c t)$
$= 2[1+k_a m(t)]^2 + 2[1+k_a m(t)]^2\cdot\cos(4\pi f_c t)$

after LPF $= 2[1+k_a m(t)]^2$

$\rightarrow V_3 = \sqrt{2}[1+k_a m(t)]$

Subject Date No.

- conditions: ① BW must be larger than the bandwidth of the message signal but smaller than the carrier frequency minus the ~~signal~~ message bandwidth

square doubles bandwidth

$\rightarrow \quad 2W < BW_f < 2f_c - 2W$

② percentage modulation < 100%

$\rightarrow \quad k_a \cdot m(t) < 1 \quad \therefore |k_a| < \frac{1}{m(t)}$

i) $s(t) = 2\cos(2\pi f_c t) + 2k_a \cdot \frac{1}{2}[\sin(2\pi(f_c+f_m)t) + \sin(2\pi(f_m - f_c)t)]$

$\rightarrow P_{carrier} = \frac{2^2}{2} = 2$

$2P_{USB} = 0.5k_a^2 = 2P_{LSB} \rightarrow P_{USB} = P_{LSB} = \frac{k_a^2}{4}$

ii) $s^2(t)$ passed through LPF $\rightarrow V_2(t) = 2[1 + k_a m(t)]^2$

$= 2 + 4k_a \,\text{sinc}(2Wt) + k_a^2 \,\text{sinc}^2(2Wt)$

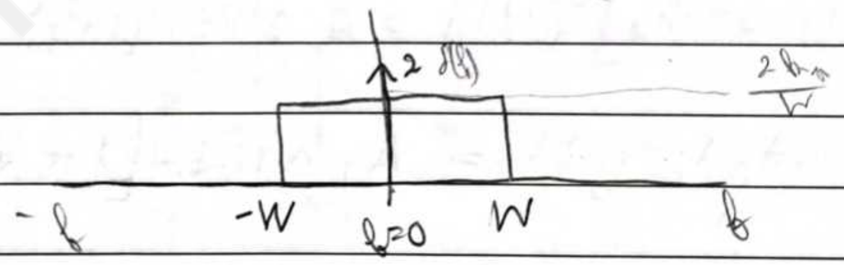
$\text{sinc}(2Wt) = \frac{\sin(2\pi Wt)}{2\pi Wt}$

$\rightarrow 2 + 4k_a \frac{\sin(2\pi Wt)}{2\pi Wt} + k_a^2 \cdot \frac{1}{4\pi W^2 t^2} \cdot \frac{1 - \cos(4\pi Wt)}{2}$

$F\{2\} = 2\delta(f)$

constants removed $\rightarrow 4k_a \frac{\sin(2\pi Wt)}{2\pi Wt} = 4k_a \,\text{sinc}(2Wt)$

fourier trans $\rightarrow 4k_a \cdot \frac{1}{2W} \text{rect}(\frac{f}{2W})$



Q3) a) carson's rule: $BW_T = 2\Delta f(1 + \frac{1}{\beta})$, $\beta = 0.5$

$\Delta f = \beta \cdot f_m = 0.5 \cdot 15k = 7.5k$

$\rightarrow BW_T = 45kHz$

one percent rule: $BW_T = 2n_{max} \cdot f_m$

n_{max} for $\beta = 0.5 = 2 \rightarrow BW_T = 60kHz$

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b) $P_{carrier} = \frac{1}{2}[J_0(0.5)]^2 = 0.44039$

$\rightarrow P_{SB} = 0.5 - 0.44039 = 0.05961$ & $P_{USB} = \frac{0.05961}{2} = 0.0298$

c) $\beta_{out} = 4 \cdot 0.5 = 2$

$\therefore S_{out}(t) = \sum_{n=-\infty}^{\infty} J_n(2) \cdot \cos[2\pi(f_c + f_m \cdot n)t]$

$f_c^{new} = f_c^{old} \cdot n = 10^7 \cdot 4$

$\rightarrow S_{out}(t) = \sum_{n=-\infty}^{\infty} J_n(2) \cdot \cos[2\pi(4\times10^7 + n \cdot 15\times10^3)t]$

---

1) $g(t) = \frac{1}{2}(g_+(t) + g_-(t))$

$\because g_+(t) = g(t) + j\hat{g}(t)$

$\wedge \; g_-(t) = g(t) - j\hat{g}(t)$

3) Hilbert trans. : $\hat{g}(t) = g(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau} d\tau$

$g(t) = \frac{1}{t} \rightarrow \hat{g}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{\tau(t-\tau)} d\tau =$

$\because \delta(t) \overset{H}{\rightleftarrows} \frac{1}{\pi t} \;\; \& \;\; \frac{1}{t} \rightleftarrows -\pi\delta(t)$

1a) $g_+(t) = j\sin(2\pi f_m t) - j[j\cos(2\pi f_m t)]$

$= \cos(2\pi f_m t) + j\sin(2\pi f_m t) = e^{j2\pi f_m t}$

2) $S(t)_{USB} = \frac{A_c}{2} m(t)\cos(2\pi f_c t) - \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$

3) modulation index $= \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

4) $\cos(2\pi f_c t - \theta) = \cos(\theta)\cos(2\pi f_c t) + \sin\theta\sin(2\pi f_c t)$

$(-\sin\theta)$ = quadrature component

Subject Date No.

3) 1) ~~LSM~~ or $\boxed{BBM}$ since mixer designed to take difference

2) $BW = 2n_{max} \cdot f_m \quad \beta_1 = 0.05 \cdot 10 = 0.5$

$\rightarrow n_{max} = 2 \Rightarrow BW = 20kHz$

3) $\because \beta_1 = 0.5 \wedge A_c = 1 \rightarrow P_{carrier} = \frac{1^2}{2}[J_0(0.5)]^2$

$= 0.4404$

4) $P_{st}^f = 0.02935 = 0.5 \cdot [J_n(0.5)]^2$

$\rightarrow J_n(0.5) = 0.24228 = J_1(0.5)$

$\therefore n = 1 \rightarrow f_c + nf_m = 1M + 1 \cdot f_m$

$\& \; 1M + 5k = 1.005 MHz$

5) $\beta = 6 = n_1 \cdot 0.05 \;\&\; n_1 = 120$

6) $\because \Delta f = \beta \cdot f_m \rightarrow f_m = \frac{220}{5.5} = 40kHz$

$\beta = n_1 \cdot n_2 \cdot \beta_{in} = 5.5$

7) $\Delta f = \beta \cdot 5k = 2500$

8) 10 k

9) 5 k since carrier exists

10) $|1M - 9M| = 8\,MHz$

mid - fall 2020

1) $V_i = m(t) + \cos(2\pi f_c t)$, $V_i^2 = m(t)^2 + 2m(t)\cos + \cos^2$

$V_o = V_i + 0.5V_i^2$

$= \frac{\sin 2000\pi t}{2000\pi t} + \cos(2\pi f_c t) + \frac{\sin^2(2000\pi t)}{2 \cdot 2000^2 \pi^2 t^2}$

$+ \underbrace{\frac{\sin(2000\pi t)}{2000\pi t} \cdot \cos(2\pi f_c t)} + \cos^2(2\pi f_c t)$

$\sin(2000t) \cdot \cos(2\pi f_c t) \rightleftharpoons \frac{1}{2000} Rect\left(\frac{f_0}{2w}\right) * \frac{1}{2}\delta(f - f_c) + \delta$

$= \frac{1}{4000}[Rect \ldots$

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2) $P_{carrier}$ in DSB-SC = 0

3) full AM: $A_c[1+k_a m(t)]\cos(2\pi f_c t)$

$\Rightarrow P = \cancel{\frac{1}{2}}$ full AM: $[1+\mathrm{sinc}(2000t)]\cos(2\pi f_c t)$

4) bandwidth of modulated signal is twice message signal

$B = 2k$ due to square $BW = 4k$ after carrier

5) $Y(f) = H(f)\cdot S(f) = j2\pi f\, S(f)$

$\Rightarrow y(t) = \frac{d}{dt} s(t)$

$\Rightarrow y(t) = \frac{-d}{dt}\left[20\pi\times10^6 t + 0.5\sin(3000\pi t)\right]\cancel{\sin} 20\pi \ldots$

$\Rightarrow y(t) = 20\pi\times10^6 t + 1500\pi\cos(3000\pi t)$ envelope det

6) full AM: $A_c[1+k_a\,\mathrm{sinc}(1000t)]\cos(2\pi f_c t)$

$\Rightarrow f_m = 500 \Rightarrow BW = 2f_m = 1000$

7) $m(t)+\cos(2\pi f_c t) + m^2(t) + 2m(t)\cos(2\pi f_c t) + \cos^2(2\pi f_c t)$

full AM: $A_c[1+k_a A_m m(t)]\cos(2\pi f_c t)$

$= [1+\boxed{2}m(t)]\cos(2\pi f_c t)$

k_a

8) Carson's $BW_T = 2\Delta f(1+\frac{1}{\beta})$, $\beta = 1 \Rightarrow BW_T = 2\Delta f(2)$ $f_m = 15k$

$\Delta f = \beta\cdot f_m = f_m \Rightarrow \boxed{BW_T = 60\,kHz}$

1%: $BW_T = 2n_{max} f_m$

$= 6\cdot f_m = 90\,kHz$

9) at integer multiples of π $\frac{\sin(2000\pi t)}{2000\pi t}$

never

10) $\tilde{x}(t) = \mathrm{Rect}\left[\frac{t-\frac{T}{2}}{T}\right]$

NOTEBOOK

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11) $\therefore$ $V_o(t) =$ Cannot be recovered

$\frac{-2}{3\pi} \cdot \cos(2\pi 3 f_c t) \cdot [m(t) + A_c \cancel{\cos(2\pi f_c t)}]$

= DSB-SC

- Use coherent detector to detect DSB-SC signal

12) $s(t) = \cos(4000\pi t + 10\pi \times 10^3 t)$

$= \cos[\pi t (14000)] \rightarrow f_m = 7\ kHz$

13) $\therefore$ $P_{total} = 0.5$, $P_{carrier} = 0.29296$

$\rightarrow P_{SB_1} = 0.20704 \rightarrow P_{USB} = 0.10352$

14) $\cos[80\pi \times 10^6 t + 2\sin(30000\pi t)]$

$$= \sum_{m=-\infty}^{\infty} J_m(2) \cdot \cos[2\pi(4\times10^7 + 15000\, m)t]$$

15)

Subject Mid exam Date 24/4/2021 No.

Q1)

Q2) $S(t) = \cos(2\pi(f_1+f_2)t)$

$f_1 + f_2 =$

Q3) duality $u(t-t_0) \rightleftharpoons \left[\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}\right]e^{-j2\pi f t_0}$

$\rightarrow u(f-2) \rightleftharpoons G(-t)$

$\rightarrow \frac{1}{2}\left[\delta(-t) + \frac{-1}{j2\pi t}\right] \cdot e^{j2\pi f t_0}$

$= \frac{1}{2}\left[\delta(t) + \frac{j}{2\pi t}\right] e^{j4\pi f t}$

Q4) $6\left[m(t) + 8\cos(2\pi f_c t)\right] + 2\left[m^2(t) + 16m(t)\cos(2\pi f_c t) + 64\cos^2(2\pi f_c t)\right]$

Q5) $A \cos(2\pi(f_1+f_2)t)$ $\quad 6\cdot 8 \quad \frac{6.3^2}{4} = 9.92$

Q6) $m(t) + \cos(2\pi f_c t) + A\left[m^2(t) + 2m(t)\cos(2\pi f_c t) + \cos^2 2\pi f_c t\right]$

$\rightarrow S(t) = \left[1 + 2A\, m(t)\right]\cos(2\pi f_c t)$

$\rightarrow 2A \cdot m(t) < 1$

$\rightarrow m(t) < \frac{1}{2A} \rightarrow m(t) < 0.125$

Subject Date No.

Q7) $x(t) = Re\{\tilde{x}(t) \cdot e^{j2\pi f_c t}\}$

$\rightarrow x(t) = \tilde{x}(t) \cos(2\pi f_c t)$

$\rightarrow \tilde{x}(t) = \tan(2\pi f_c t)$

$\tilde{x}(t) = x_+(t) \cdot e^{-j2\pi f_c t}$

$[\sin(2\pi f_c t) + j \cos(2\pi f_c t)] e^{-j2\pi f_c t}$

$\rightarrow \tilde{x}(t) = \sin(2\pi f_c t) \cdot \cos(2\pi f_c t) - j\sin^2(2\pi f_c t) \sin$

$- j \cos^2 - \sin \cos 2\pi f_c t$

$\rightarrow \tilde{x}(t) = \sin 0 - j[1]$

$= -j$

Q8)

$y(f) = H(f) \cdot S(f) = \frac{jf}{1000} \cdot S(f)$

$\rightarrow 1000 \cdot 2\pi \cdot Y(f) = j2\pi f \; S(f)$

$\rightarrow 1000 \cdot 2\pi \cdot y(t) = \frac{d}{dt} s(t)$

$\rightarrow -[2\pi f_1 + 2\pi f_0 \cos(2\pi f_2 t)] \sin[2\pi f_1 t + \ldots]$

$\rightarrow 2\pi\, 112 + 2\pi \cdot 3$

$= 224\pi + 6\pi \cos = 1000\, 2\pi\, y(t)$

$\rightarrow y(t) = \frac{224\pi}{1k \cdot 2\pi}$

$= 112 + 3\cos$

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9) $6[m(t) + 9\cos(2\pi f_c t)] + 4[m^2(t) + 2m(t)\cdot 9\cos 2\pi f_c$

$\rightarrow A_c [1 + k_a m(t)] \cos(2\pi f_c t)$

$\rightarrow A_c = 9$

$6 \times 9 [1 + \frac{8}{6}$

10) $S(t) =$

$\frac{(4.1)^2}{2} [J_0(0.5)] =$

11) $x(t) * h(t) \rightleftharpoons X(f)\cdot H(f)$

$= [x(t) + x(t-1)]\, u(t)$

$= X(f) + X(f)\cdot e^{-j2\pi f}$

$\rightarrow X(f)\cdot H(f) \rightarrow H(f) = 1 + e^{-j2\pi f_c}$

13) $BW_T = 2 n_{max} f_m = 2 n_{max} \cdot 5k$

$n_{max}(0.5) = 2 \rightarrow 4 \cdot 5k = 20$

14) $\beta = 0.5 \cdot 4 = 2$

$\rightarrow P_{carrier} = \frac{3.2^2}{2} [J_0(2)]^2 -$

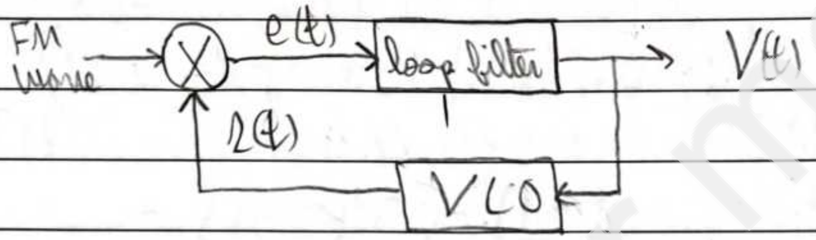
15)

N O T E B O O K

Subject frequency modulation (continued) Date 1/5/2021 No.

* phase-locked loop:

+ a negative feedback system that can be used for:
  - synchronization - frequency division/multiplication
  - indirect frequency demodulation



- the VCO is initially adjusted to satisfy two conditions for when the voltage is zero:
  (1) the frequency of the VCO is exactly equal to the unmodulated carrier frequency
  (2) the VCO output has a 90° phase shift with respect to the unmodulated carrier wave.
- if $s(t)$ is an FM wave applied to the input terminal of the PLL, where $s(t) = A_c \sin(2\pi f_c t + \phi_1(t))$
  & $\phi_1(t) = 2\pi k_f \int m(t)\, dt$
  then $r(t) = A_v \cos(2\pi f_c t + \phi_2(t))$
  & $\phi_2(t) = 2\pi k_v \int v(t)\, dt$, k_v: frequency sensitivity of VCO
- the objective of the PLL is to generate a VCO output ($r(t)$) with a phase angle equal to the input FM signal
  → $\phi_1(t) = \phi_2(t)$

* nonlinear model of PLL:

+ $e(t)$ has two components:
  - high frequency: $k_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]$

- low frequency: $k_m A_c A_v \cdot \sin(\phi_1(t) - \phi_2(t))$
  where k_m is the multiplier gain
- the loop filter is a low-pass filter, hence the high frequency components are negligible.

$\Rightarrow e(t) = k_m A_c A_v \sin[\phi_e(t)]$, $\phi_e(t) = \phi_1(t) - \phi_2(t)$

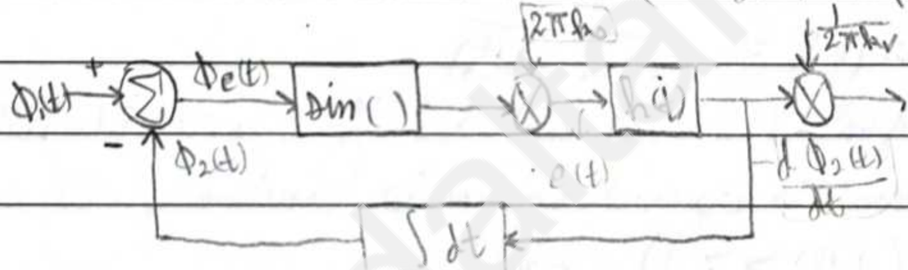
$\Rightarrow \phi_e(t) = \phi_1(t) - 2\pi k_v \int V(t)\, dt$

$\because V(t) = e(t) * h(t)$, $h(t)$: impulse response of loop filter

$= \int e(\tau)\, h(t-\tau)\, d\tau$

$\Rightarrow \frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_o \int \sin[\phi_e(t)]\, h(t-\tau)\, d\tau$

where $k_o = k_m k_v A_v A_c$ (in Hz), loop gain parameter

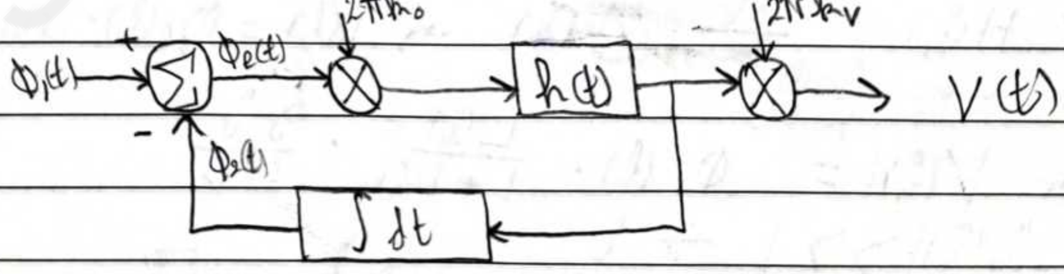


nonlinear model
(nonlinearity caused by sin)

& linear model of PLL:

- the main simplification of the linear PLL is that does not require the sine component as it is assumed that the $\phi_e(t)$ is smaller than 1 (rad), since $\sin(x) \approx x \ \forall\ x \ll 1$

$\therefore \sin(\phi_e(t)) \approx \phi_e(t)$



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from the linearized model of the PLL:

$$\frac{d\phi_e(t)}{dt} + 2\pi k_0 \int \phi_e(\tau) h(t-\tau) d\tau = \frac{d\phi_1}{dt}$$

fourier transform: $j2\pi f \, \phi_e(f) = j2\pi f \, \phi_1(f) - 2\pi k_0 \, \phi_e(f) \cdot H(f)$

rearrange:

$$\phi_e(f) = \frac{\phi_1(f)}{1 + \frac{k_0}{jf} \cdot H(f)}$$

define: $L(f) = \frac{k_0}{jf} \cdot H(f)$, open loop transfer function of PLL

$$\Rightarrow \phi_e(f) = \frac{\phi_1(f)}{1 + L(f)}$$

- to achieve the goal of a PLL and lock the phases (s.t $\phi_e(t) = 0$) we can increase the open loop transfer function for all frequencies ($L(f) >> 1$), hence:

$$\lim_{\substack{L(f) \to \infty \\ \forall f}} \phi_e(f) = 0 \quad \rightarrow \text{synchronized}$$

$\because V(f) = \phi_e(f) \cdot 2\pi k_0 \cdot H(f) \cdot \frac{1}{2\pi k_v}$

$\Rightarrow V(f) = \phi_e(f) \cdot H(f) \cdot \frac{k_0}{k_v}$

$\because H(f) = \frac{jf}{k_0} \cdot L(f) \Rightarrow V(f) = \phi_e(f) \cdot L(f) \cdot \frac{jf}{k_v}$

, $\phi_e(f) = \frac{\phi_1(f)}{1 + L(f)}$

$$\therefore V(f) = \phi_1(f) \cdot \frac{L(f)}{1 + L(f)} \cdot \frac{jf}{k_v}$$

if $|L(f)| >> 1 \quad \forall f$:

$$\therefore V(f) \approx \phi_1(f) \cdot \frac{jf}{k_v}$$

- performing an inverse fourier transform gives:

$$V(t) = \frac{1}{2\pi k_v} \cdot \frac{d\,\phi_1(t)}{dt} \quad \& \quad \phi_1(t) = 2\pi k_f \int m(t)\,dt$$

$$\therefore \quad V(t) = \frac{k_f}{k_v} \cdot m(t)$$

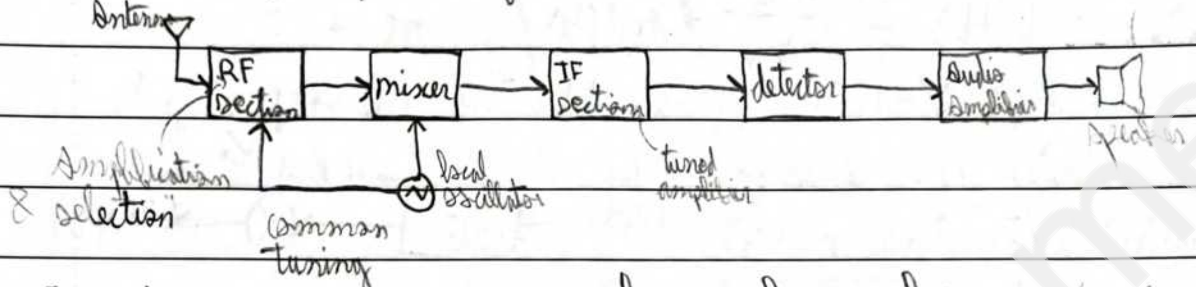
- hence, the output $V(t)$ is approximately the message signal ($m(t)$) scaled.

simplified linear PLL model:
(assuming $|L(f)| \gg 1$)

$$\phi_1(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow \bigotimes_{\;1/2\pi k_v} \rightarrow V(t)$$

Subject Superheterodyne receiver Date 13/5/2021 No.

* The superheterodyne receiver has the following tasks:
  - demodulation - filtering - amplification
  - carrier-frequency tuning



* IF : intermediate frequency , $f_{IF} = f_{RF} - f_{LO}$ (LO: local oscillator)

- RF section is composed of a band pass filter and an amplifier (IF section, too)

\+ Normal frequencies:

	AM radio	FM radio
RF carrier range	0.530 - 1.605 MHz	88 - 108 MHz
IF frequency	0.455 MHz	10.7 MHz
IF bandwidth	10 kHz	200 kHz

- multiple amplifiers can be used to avoid operating in nonlinear regions of amplifiers, keep the costs low, and reduce noise.
- detector stage can be an envelope detector, coherent detector, or other depending on the type of modulation used.

Subject: introduction to noise analysis   Date: 13/5/2021   No.

* noise: unwanted signal that tends to distort the transmission and processing of signals in communication systems. noise signals are incompletely controlled.

+ Sources of noise:
  - internal: shot noise and thermal noise (e.g., due to the heating of a copper conductor and the increased speed of electrons)
  - external: atmospheric and man-made.

* White noise: ideal noise whose power spectral density is independent of the operating frequency. (PSD = const $\forall f$)

$\therefore \; S_W(f) = \frac{N_0}{2}$ (N_0 in W/Hz)

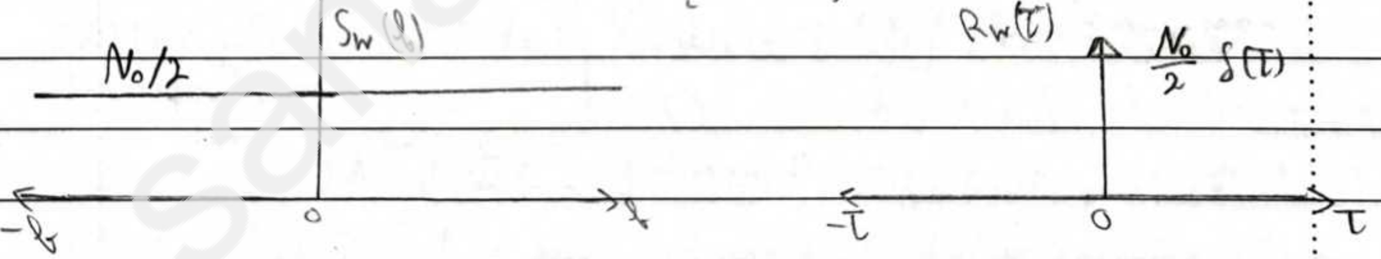
- assuming thermal noise: $N_0 = k\,T_e$, k: boltzmann constant, T_e: equivalent noise temperature (in K)

$\because S_W(f)$ is constant, the autocorrelation function ($R_W(\tau)$):

$\rightarrow R_W(\tau) = \frac{N_0}{2}\,\delta(\tau)$

Recall: power spectral density is equal to the fourier transform of the autocorrelation function.

$$S_{xx}(f) = F\{R_{xx}(\tau)\}$$



- the noise signal is random (indeterministic), hence it is a power signal
- any two different samples of white noise (regardless how close they are) are uncorrelated.
- If the white noise is also gaussian, then two samples are statistically independent.

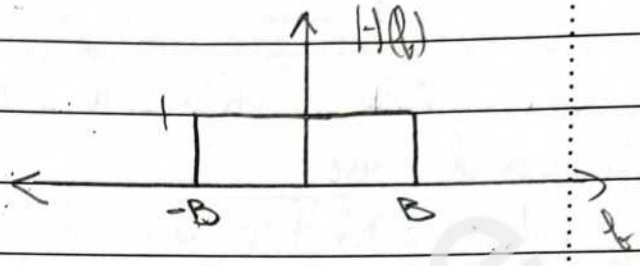
NOTEBOOK

- If the bandwidth of a noise process at the input of a signal is significantly larger than that of the system, then it can be modeled as white noise.

Subject: noise analysis continued: Date: 16/5/2021 No.

* ideal low-pass filtered white noise

$W(t)$ white gaussian noise of zero mean and $S_W(f) = \frac{N_0}{2}$ → Ideal LPF $H(f)$ → $n(t)$ band limited gaussian noise with zero mean.

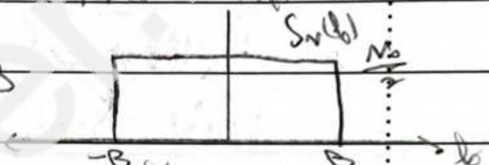


∵ $n(t) = h(t) * w(t) \rightarrow N(f) = H(f) \cdot W(f)$

- $W(f)$ cannot be obtained since $w(t)$ is random.

$S_N(f) = |H(f)|^2 \cdot S_W(f)$, where $|H(f)|^2 = H(f) \cdot H^*(f)$

$$\rightarrow S_N(f) = \begin{cases} \frac{N_0}{2} , & -B < f < B \\ 0 , & \text{elsewhere } (|f| > B) \end{cases}$$



∵ $S_N(f)$ is the power spectral density of the output, therefore the autocorrelation of the output is $F^{-1}\{S_N(f)\} = N_0 B \operatorname{sinc}(2B\tau)$

- if $n(t)$ is sampled at a rate of $2B$ per second, then the resulting noise samples are uncorrelated. Since the noise samples are gaussian, hence they are statistically independent.
- each sample has a zero mean and a variance of $N_0 B$.

∵ $\sigma_x^2 = E[(x(t) - \mu_x)^2] = E[x^2(t)] = R_x(0) = N_0 B$

* noise equivalent bandwidth:

∵ $S_N(f) = S_W(f) \cdot |H(f)|^2 = \frac{N_0}{2} \cdot |H(f)|^2$ $W(t)$ → LPF $H(f)$ → $N(t)$

→ the average output noise power: $N_{out} = \int_{-\infty}^{\infty} S_N(f)\, df$

∵ $|H(f)|^2$ is an even function → $\int_{-\infty}^{\infty} |H(f)|^2 df = 2\int_0^{\infty} |H(f)|^2 df$

∴ $N_{out} = N_0 \cdot \int_0^{\infty} |H(f)|^2 df$ —— (1)

→ for an ideal LPF: $N_{out} = N_0 \cdot H^2(0) \cdot B$ —— (2)

∴ $B = \dfrac{\int_0^{\infty} |H(f)|^2 df}{H^2(0)}$ | - $N_0 \cdot B$ is the area under $S_N(f)$

* narrow-band noise:

gaussian zero mean and unity PSD $W(t) \rightarrow$ [narrow band filter] $\rightarrow n(t)$ narrow band noise

$$S_N(f) = |H(f)|^2 \underbrace{S_W(f)}_{=1} = |H(f)|^2$$

- assuming $n(t)$ has a power spectral density centered about f_c:

$$n_+(t) = n(t) + j\hat{n}(t) \quad \& \quad \tilde{n} = n_+(t)\, e^{-j2\pi f_c t}$$
$$= n_I(t) + j\, n_Q(t)$$

$$\rightarrow n_I(t) = n(t)\cos(2\pi f_c t) + \hat{n}(t)\sin(2\pi f_c t)$$
$$\& \quad n_Q(t) = \hat{n}\cos(2\pi f_c t) - n(t)\sin(2\pi f_c t)$$
$$\therefore n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

* properties of $n_I(t)$ & $n_Q(t)$ of a narrow-band noise:

① $n_I(t)$ & $n_Q(t)$ have a zero mean

② if $n(t)$ is gaussian, then $n_I(t)$ & $n_Q(t)$ are jointly gaussian

③ if $n(t)$ is wide-sense stationary, then $n_I(t)$ & $n_Q(t)$ are jointly wide-sense stationary

$\rightarrow$ constant mean & $R_x(t, t+\tau) = R_x(\tau)$

④ $$S_{N_I}(f) = S_{N_Q}(f) = \begin{cases} S_N(f-f_c) + S_N(f+f_c); & -B \le f \le B \\ 0 & , \text{elsewhere} \end{cases}$$

⑤ $n_I(t)$ & $n_Q(t)$ have the same variance as $n(t)$

⑥ the cross-spectral density of $n_I(t)$ & $n_Q(t)$ is purely imaginary

$$S_{N_I N_Q}(f) = -S_{N_Q N_I}(f)$$
$$= \begin{cases} j[S_N(f+f_c) - S_N(f-f_c)], & -B \le f \le B \\ 0 & , \text{otherwise} \end{cases}$$

⑦ if $n(t)$ is gaussian with zero mean and $S_N(f)$ is locally symmetric about the mid-band frequency, f_c, then $n_I(t)$ & $n_Q(t)$ are statistically independent.

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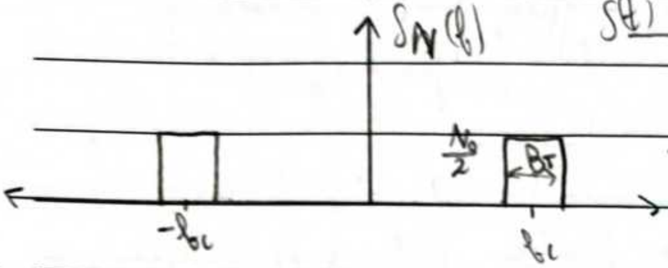
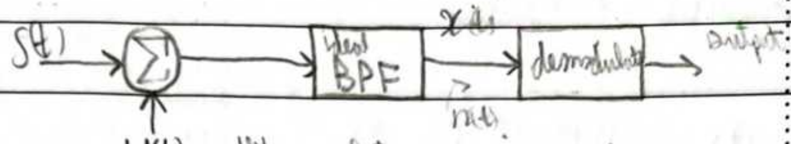
* receiver model:





additive white gaussian noise

$S_W(f) = \frac{N_0}{2}$

- assuming an ideal BPF, $f_c > B_T$, and $n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$

- $x(t) = s(t) + n(t)$

→ the average noise power at the demodulator input: $P_N = 2\left(\frac{N_0}{2}\right)B_T = N_0 B_T$

* $(SNR)_I$ = input signal-to-noise ratio, which is the ratio of the average power of $s(t)$ to the average power of the filtered noise $n(t)$.

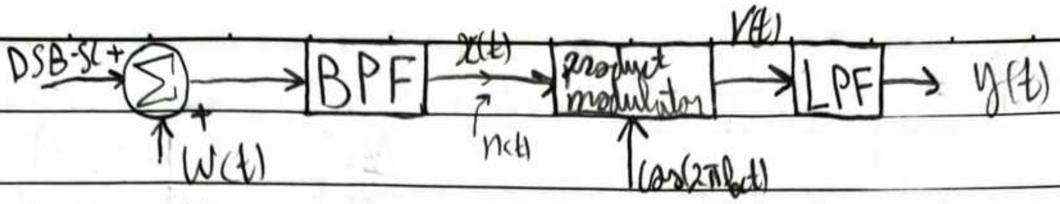
* $(SNR)_O$: output signal-to-noise ratio, which is the ratio of average power of demodulated message signal to the average power of noise, both measured at the receiver output

- to compare two modulation schemes:
  1) $s(t)$ of both schemes should have the same power.
  2) $W(t)$ has the same average power in the message bandwidth

*$(SNR)_C$: channel signal-to-noise ratio, which is the ratio of the average power of the modulated signal to the average power of the noise in the message bandwidth, both measured at the receiver input.

- figure of merit: $\frac{(SNR)_O}{(SNR)_C}$

Subject: Noise in DSB-SC Receiver   Date: 17/5/2021   No.



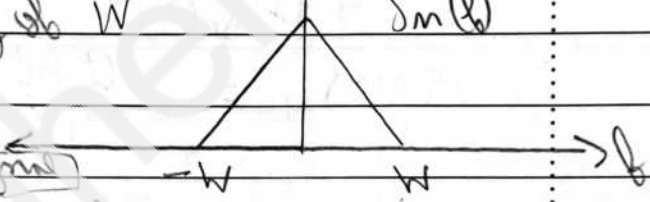
- The DSB-SC component in $x(t)$ is:

$$s(t) = C A_c \cos(2\pi f_c t) \cdot m(t)$$

where C is a constant that ensures $s(t)$ is measured in the same units as $n(t)$

- assuming that $m(t)$ is a sample function of a stationary random process with zero mean, its power spectral density $S_m(f)$ is limited to a maximum frequency of W

$$P = \int_{-W}^{W} S_m(f)\, df$$



P: average power in the message signal $m(t)$

- the carrier is statistically independent of $m(t)$:

$$s(t) = C \cdot A_c \cos(2\pi f_c t + \Theta) \cdot m(t)$$ — Random signal

where Θ is a uniformly distributed random variable in the interval $[0, 2\pi]$

- The goal is to obtain the figure of merit, we start by obtaining the power of $s(t)$. first, we find the autocorrelation function of $s(t)$:

$$R_s(\tau) = E[s(t+\tau) \cdot s(t)]$$ — autocorrelation function

$$\boxed{R_s(\tau) = \frac{1}{2} C^2 A_c^2 R_m(\tau) \cos(2\pi f_c \tau)}$$

where $R_m(\tau) = E[m(t) \cdot m(t+\tau)]$, which was separated because it is statistically independent of the carrier.

$$\therefore S_s(f) = \frac{1}{4} C^2 A_c^2 [S_m(f - f_c) + S_m(f + f_c)]$$

$$\rightarrow P_s = \frac{1}{4} C^2 A_c^2 \cdot (2P) = C^2 A_c^2 \cdot P/2$$

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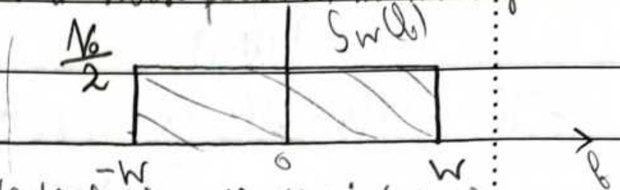
$\therefore \ (SNR)_c = \dfrac{C^2 A_c^2 P/2}{W \cdot N_0}$, WN_0 is the noise power in the message bandwidth



$\because \ x(t) = s(t) + n(t)$

$= CA_c \cos(2\pi f_c t)\, m(t) + n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)$

$\wedge \ v(t) = x(t) \cdot \cos(2\pi f_c t)$

- passing $v(t)$ through an LPF:

$$y(t) = \underbrace{\tfrac{1}{2} A_c C\, m(t)}_{\substack{\text{average power:} \\ (\frac{1}{2}A_c C)^2 \cdot P}} + \underbrace{\tfrac{1}{2} n_I(t)}_{\substack{\text{average power:} \\ (\frac{1}{2})^2 P_{n_I(t)} = \frac{1}{4} \cdot 2WN_0}}$$

$\therefore \ (SNR)_o = \dfrac{C^2 A_c^2 \cdot P/4}{WN_0/2} = \boxed{\dfrac{C^2 A_c^2 P}{2WN_0}}$

$\therefore$ the figure of merit: $\dfrac{(SNR)_o}{(SNR)_c} = 1$

MOHAMMAD SANAD ALTAHER 130806

Subject homework #1 Date No.

problem 2.1 :

a) $\because \; g(t) = A \cdot \cos(2\pi f_c t) \cdot \text{Rect}\left(\frac{t}{T}\right) \;,\; f_c = \frac{1}{2T}$

$$\rightarrow g(t) = A \cdot \frac{1}{2} \cdot \left[e^{j2\pi f_c t} + e^{-j2\pi f_c t} \right] \cdot \text{Rect}\left(\frac{t}{T}\right)$$

$$\because \; G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt$$

$$\rightarrow \frac{2}{A} G(f) = \int_{-T/2}^{T/2} e^{j2\pi t (f_c - f)}\, dt + \int_{-T/2}^{T/2} e^{-j2\pi t (f_c + f)}\, dt$$

$$\rightarrow \frac{2}{A} G(f) = \frac{1}{j2\pi(f_c - f)} \left[e^{j2\pi t (f_c - f)} \right]_{-T/2}^{T/2} + \frac{-1}{j2\pi(f_c + f)} \left[e^{-j2\pi t(f_c + f)} \right]_{-T/2}^{T/2}$$

(expanded using euler's formula)

$$\rightarrow \frac{2}{A} G(f) = \frac{1}{j2\pi(f_c - f)} \left[2j \sin(\pi T (f_c - f)) \right] + \frac{-1}{j2\pi(f_c + f)} \left[-2j \sin(\pi T (f + f_c)) \right]$$

$$\rightarrow G(f) = \frac{A}{2} \left[\frac{\sin\left(\frac{\pi}{2} - \pi f T\right)}{\pi/2T - \pi f} + \frac{\sin\left(\frac{\pi}{2} + \pi f T\right)}{\pi/2T + f\pi} \right]$$

$$\therefore \boxed{G(f) = \frac{AT}{2} \cdot \left[\text{sinc}\left(\tfrac{1}{2} - fT\right) + \text{sinc}\left(\tfrac{1}{2} + fT\right) \right]}$$

$\because$ sinc is an even function,

$$\text{sinc}\left(\tfrac{1}{2} - fT\right) = \text{sinc}\left(fT - \tfrac{1}{2}\right)$$

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problem 2.1:

(b) $\because \; G(f) = \frac{AT}{2} \cdot \left[\operatorname{sinc}(fT - \tfrac{1}{2}) + \operatorname{sinc}(fT + \tfrac{1}{2}) \right]$

$$\rightarrow G(f) = \frac{AT}{2} \cdot \left[\frac{e^{j(\pi fT - \frac{\pi}{2})} - e^{-j(\pi fT - \frac{\pi}{2})}}{j2\pi fT - j\pi} + \frac{e^{j(\pi fT + \frac{\pi}{2})} - e^{-j(\pi fT + \frac{\pi}{2})}}{j2\pi fT + j\pi} \right]$$

$\wedge$ time shifting $\rightarrow \quad g(t - t_0) \rightleftharpoons G(f) \cdot e^{-j2\pi f t_0}$

$t_0 = T/2$

$\therefore \; G(f) \cdot e^{-j\pi fT}$, after expanding with euler's formula:

$$\frac{AT}{2} \cdot \left[\frac{-j - \sin(2\pi fT) - j\cos 2\pi fT}{2j\pi fT - j\pi} + \frac{j + \sin(2\pi fT) + j\cos(2\pi fT)}{2j\pi fT + j\pi} \right]$$

then cross multiply and simplify:

$$\rightarrow G(f) \cdot e^{-j\pi fT} = \left[\frac{2\pi - 2j\pi \sin(2\pi fT) + 2\pi\cos(2\pi fT)}{\pi^2 - 4\pi^2 f^2 T^2} \right] \frac{AT}{2}$$

$$\therefore \; \boxed{G(f) \cdot e^{-j\pi fT} = \frac{AT}{2} \cdot \frac{2 + 2e^{-j2\pi fT}}{\pi(1 - 4f^2 T^2)}}$$

where $G(f) \cdot e^{-j\pi fT} \rightleftharpoons g(t - \frac{T}{2})$

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problem 2.1

e) $\because$ $g(t) = A \cdot \sin(2\pi f_c t) \cdot \text{rect}\left(\frac{t}{2T}\right)$ & $f_c = \frac{1}{2T}$

$\because$ $g_1(t) \cdot g_2(t) \rightleftharpoons G_1(f) * G_2(f)$

& $A \cdot \sin\left(\frac{\pi t}{T}\right) \rightleftharpoons 0.5 \cdot j \cdot \left[\delta\left(f + \frac{1}{2T}\right) - \delta\left(f - \frac{1}{2T}\right)\right]$

& $\text{rect}\left(\frac{t}{2T}\right) \rightleftharpoons 2T \cdot \text{sinc}(2fT)$

$\rightarrow G(f) = \frac{2TAj}{2} \cdot \text{sinc}(2fT) * \left[\delta\left(f + \frac{1}{2T}\right) - \delta\left(f - \frac{1}{2T}\right)\right]$

$\rightarrow G(f) = jAT\left[\int_{-\infty}^{\infty} \text{sinc}(2fT) \cdot \delta\left(f + \frac{1}{2T}\right) dt - \int_{-\infty}^{\infty} \text{sinc}(2fT) \cdot \delta\left(f - \frac{1}{2T}\right) dt\right]$

$\therefore G(f) = j \cdot A \cdot T \left[\text{sinc}(2fT + 1) - \text{sinc}(2fT - 1)\right]$

If π is included inside sinc:

$$\boxed{G(f) = j \cdot A \cdot T \cdot \left[\text{sinc}(2\pi fT + \pi) - \text{sinc}(2\pi fT - \pi)\right]}$$

problem 2.2:

$\because$ fourier transform pair : $e^{-at} \cdot \sin(2\pi f_c t) \cdot u(t) \rightleftharpoons \frac{2\pi f_c}{(a + j2\pi f)^2 + 4\pi^2 f_c^2}$

for $a > 1$

& $g(t) = e^{-(1)t} \cdot \sin(2\pi f_c t) \cdot u(t)$

$$\boxed{\rightarrow G(f) = \frac{2\pi f_c}{(1 + j2\pi f)^2 + 4\pi^2 f_c^2}}$$

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problem 2.4:

$\because$ $|G(f)| \begin{cases} 1, & -W<f<W \\ 0, & \text{o.w} \end{cases}$ & $\arg(G(f)) \begin{cases} \frac{\pi}{2}, & -W<f<0 \\ -\frac{\pi}{2}, & 0<f<W \end{cases}$

$\rightarrow G(f) \begin{cases} 1 \cdot e^{j\frac{\pi}{2}}, & -W<f<0 \\ 1 \cdot e^{-j\frac{\pi}{2}}, & 0<f<W \end{cases}$ & inverse fourier transform: $\int_{-\infty}^{\infty} G(f) e^{j2\pi f t} \, df$

$\rightarrow g(t) = e^{j\frac{\pi}{2}} \cdot \int_{-W}^{0} e^{j2\pi f t} \, df + e^{-j\frac{\pi}{2}} \int_{0}^{W} e^{j2\pi f t} \, df$

$\rightarrow g(t) = \left[\frac{e^{j\frac{\pi}{2}}}{j2\pi t} - \frac{e^{-j(2\pi W t - \frac{\pi}{2})}}{j2\pi t} \right] + \left[\frac{e^{j(2\pi W t - \frac{\pi}{2})}}{j2\pi t} - \frac{e^{-j\frac{\pi}{2}}}{j2\pi t} \right]$

$\because$ $e^{j\frac{\pi}{2}} = j$

$\rightarrow g(t) = \frac{1}{2\pi t} - \frac{2j \sin(2\pi W t - \frac{\pi}{2})}{j \cdot 2\pi \cdot t} + \frac{1}{2\pi t}$

$\therefore$ $\boxed{g(t) = \frac{1}{\pi t} + \frac{\cos(2\pi W t)}{\pi t}}$

problem 2.10:

$\because$ $x(t) \rightleftharpoons X(f)$ & $X(f) \begin{cases} |X(f)| \cdot \arg(X(f)) & -W \le f \le W \\ 0 & \text{o.w} \end{cases}$

$\rightarrow x(t) \rightleftharpoons X(f) \cdot \text{Rect}\left(\frac{f+W}{2W}\right)$

$\because$ $y(t) = x^2(t) \rightarrow Y(f) = X(f) * X(f)$

$\rightarrow Y(f) = \int_{-\infty}^{\infty} X(f) \cdot X(f) \cdot \text{Rect}\left(\frac{f+W}{2W}\right) \cdot \text{Rect}\left(\frac{f+W}{2W}\right) df$

$\rightarrow Y(f) = X^2(f) \left[\int_{-W-W}^{W} \text{Rect}\left(\frac{f}{2W}\right) df + \int_{W}^{W+W} \text{Rect}\left(\frac{f}{2W}\right) df \right]$

$\rightarrow Y(f) := X^2(f) \cdot \int_{-2W}^{2W} \text{Rect}\left(\frac{f}{2W}\right) df$

$\therefore$ $\boxed{Y(f) = X^2(f) \cdot \text{Rect}\left(\frac{f+2W}{4W}\right) \rightarrow y(t) \begin{cases} x^2(t), & -2W \le f \le 2W \\ 0, & \text{o.w} \end{cases}}$

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problem 2.12:

$$\because G(f) = \begin{cases} 1, & f > 0 \\ \frac{1}{2}, & f = 0 \\ 0, & f < 0 \end{cases}$$

for a signum function, $sgn(f) = \begin{cases} 1, & f > 0 \\ 0, & f = 0 \\ -1, & f < 0 \end{cases}$

$$\therefore \boxed{G(f) = \frac{1}{2} + \frac{1}{2} sgn(f)}$$

$\because sgn(t) \rightleftharpoons \frac{1}{j\pi f}$ & duality: $G(t) \rightleftharpoons g(-f)$

$\rightarrow \frac{1}{2} sgn(f) \rightleftharpoons \frac{1}{2} \cdot \frac{1}{j\pi(-t)}$ & $\frac{1}{2} \rightleftharpoons \frac{1}{2}\delta(t)$

$$\therefore \boxed{g(t) = \frac{1}{2}\delta(t) + \frac{j}{2\pi t}}$$

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Subject: noise analysis (continued) Date: 21/5/2021 No.

* noise in SSB Receivers:

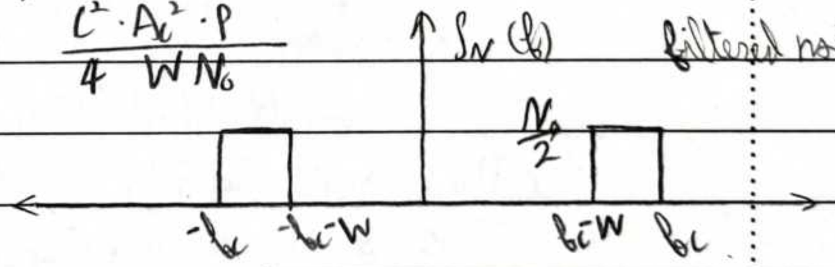
$$s(t) = \underbrace{\tfrac{1}{2} C A_c \cos(2\pi f_c t) \cdot m(t)}_{s_1(t)} + \underbrace{\tfrac{1}{2} C A_c \sin(2\pi f_c t)\, \hat{m}(t)}_{s_2(t)}$$

- The plus sign indicates that we are transmitting the lower side-band.
- the power spectral densities of $m(t)$ & $\hat{m}(t)$ are additive.
- $H(f) = -j\,\mathrm{sgn}(f) \rightarrow |H(f)|^2 = 1 \;\forall f$
  $\Rightarrow m(t)$ & $\hat{m}(t)$ have the same power spectral densities.
- to arrive at the figure of merit, start by finding the channel's signal to noise ratio.
  assuming the power in the message signal is P:

$$P_{s_1(t)} = P_{s_2(t)} = \tfrac{1}{2} C^2 A_c^2 \left(\tfrac{1}{2}\right)^2 \cdot P = C^2 A_c^2 \cdot \tfrac{P}{8}$$

$$\therefore\ P_S = 2 \cdot \tfrac{1}{2} C^2 A_c^2 \left(\tfrac{1}{2}\right)^2 \cdot P = C^2 A_c^2 \cdot P/4$$

$$\rightarrow (SNR)_c = \frac{C^2 \cdot A_c^2 \cdot P}{4\, W N_0}$$



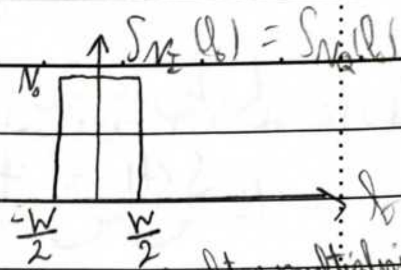
$$\rightarrow n(t) = n_I(t) \cos\left[2\pi \left(f_c - \tfrac{W}{2}\right)t\right] - n_Q(t) \sin\left[2\pi \left(f_c - \tfrac{W}{2}\right)t\right]$$

- centered at f_c

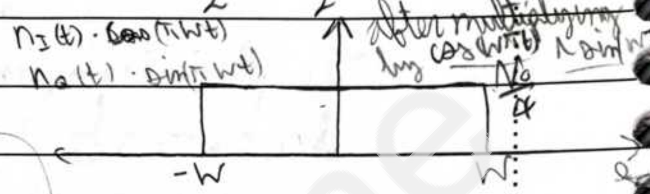
- pass the filtered noise plus the signal through a coherent detector then a low-pass filter.

$$\rightarrow y(t) = \underbrace{\tfrac{1}{4} C A_c\, m(t)}_{P_1 = \frac{C^2 A_c^2 P}{16}} + \underbrace{\tfrac{1}{2} n_I(t) \cos(\pi W t)}_{P_2 = \frac{1}{4} \cdot \frac{N_0 W}{2}} + \underbrace{\tfrac{1}{2} n_Q(t) \sin(\pi W t)}_{P_3 = \frac{1}{4} \cdot \frac{N_0 W}{2}}$$

- the power in the noise is shifted to the center and the I & Q components are added.



$\rightarrow (SNR)_o = \dfrac{C^2 A_c^2 P/16}{N_0 W/4}$



$\therefore (SNR)_o / (SNR)_c = 1$

* (full) noise in AM receiver:

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$
$$= A_c\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t)$$

→ the average power in the carrier component is $A_c^2/2$

→ average power in the information bearing component is $\dfrac{A_c^2 k_a^2 P}{2}$

$\therefore$ the average power in $s(t)$ is $\dfrac{A_c^2}{2}[1 + k_a^2 P]$

- given the average power in the filtered noise equal to WN_0:

$$(SNR)_c = \frac{A_c^2[1 + k_a^2 P]}{2WN_0}$$

$\therefore x(t) = n(t) + s(t)$

$$= A_c[1 + k_a m(t)]\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$\therefore x(t) = [A_c + A_c k_a m(t) + n_I(t)]\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

- envelope detector will square the inphase and quadrature components, ~~add them~~ then output the square-root.

$$\rightarrow y(t) = \sqrt{[A_c + A_c k_a m(t) + n_I(t)]^2 + [n_Q(t)]^2}$$

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- assuming that the power in the carrier is large compared to the noise power:

$$\rightarrow y(t) = \sqrt{[A_c + A_c k_a m(t) + n_I(t)]^2 + n_Q^2(t)}$$

$$\approx A_c + \underbrace{A_c k_a m(t)}_{A_c^2 k_a^2 P} + \underbrace{n_I(t)}_{2N_0W}$$

average power: $A_c^2 k_a^2 P$ $2N_0W$

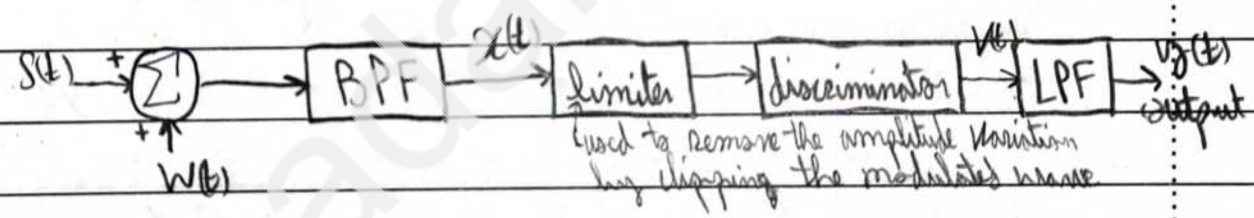
$$\rightarrow (SNR)_o \simeq \frac{A_c^2 k_a^2 P}{2N_0W}$$

if ① noise power << carrier power
② for given percentage modulation $\leq 100\%$

$$\therefore \frac{(SNR)_o}{(SNR)_c} \simeq \frac{k_a^2 P}{1 + k_a^2 P} < 1$$

- Due to the wasted power in transmitting the carrier, the noise performance of DSB-SC is better than that of AM

* noise in FM Receiver:



+ discriminator is:
  - slope network or differentiator
  - envelope detector.

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$n(t) = r(t)\cos[2\pi f_c t - \psi(t)]$$

where $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$ & $\psi(t) = \tan^{-1}\left[\frac{n_Q(t)}{n_I(t)}\right]$

- If $W(t)$ is gaussian with zero mean, then $n(t)$ is also gaussian with zero mean $\rightarrow$ $n_I(t)$ & $n_Q(t)$ are jointly gaussian and statistically independent

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+ Using Random Variable transformation:

- $r(t)$ is Rayleigh distributed, $f_R(r) \begin{cases} \frac{r}{\sigma^2} e^{-\left(\frac{r^2}{2\sigma^2}\right)} & , r \geq 0 \\ 0 & , \text{elsewhere} \end{cases}$
- ψ is uniformly distributed.

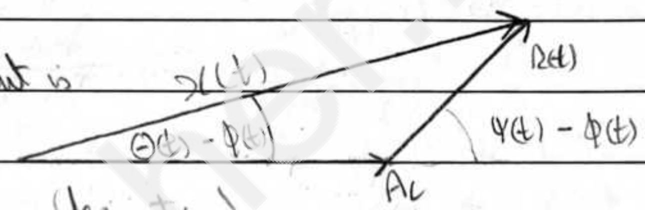
$\because$ $s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\, dt\right]$

$\wedge$ $\phi(t) = 2\pi k_f \int m(t)\, dt \rightarrow s(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right]$

$$\rightarrow \underbrace{x(t)}_{\substack{\text{phasor with} \\ \text{angle } \theta(t)}} = \underbrace{A_c \cos\left[2\pi f_c t + \phi(t)\right]}_{\text{reference}} + \underbrace{r(t) \cos\left[\pi f_c t + \psi(t)\right]}_{\text{phasor with angle } \psi(t)}$$

- an ideal discriminator's output is proportional to $\frac{\theta'(t)}{2\pi}$

where $\theta'(t) = \frac{d\theta(t)}{dt}$ (derivative)



$$\rightarrow \theta(t) - \phi(t) = \tan^{-1}\left[\underbrace{\frac{r(t) \sin\left[\psi(t) - \phi(t)\right]}{A_c + r(t) \cos\left[\psi(t) - \phi(t)\right]}}_{y}\right]$$

$\because$ Taylor series expansion:

$$\tan^{-1}(y) = y - \frac{y^3}{3} + \frac{y^5}{5} - \frac{y^7}{7} + \cdots$$

- if we assume that the carrier-to-noise ratio is large, then $A_c + r(t) \cos\left[\psi(t) - \phi(t)\right] \approx A_c$

$$\rightarrow \theta(t) \simeq \phi(t) + \frac{r(t)}{A_c} \sin\left[\psi(t) - \phi(t)\right]$$

$$\rightarrow \theta(t) \simeq 2\pi k_f \int m(t)\, dt + \frac{r(t)}{A_c} \sin\left[\psi(t) - \phi(t)\right]$$

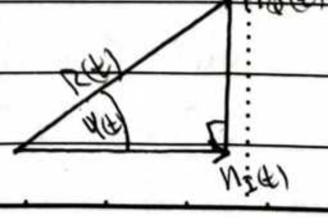
$\because$ $v(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = k_f\, m(t) + n_d(t)$

s.t. $n_d(t) = \frac{1}{2\pi A_c} \cdot \frac{d}{dt}\left[r(t) \sin\left(\psi(t) - \phi(t)\right)\right]$

- assume that $\psi(t) - \phi(t)$ is also uniformly distributed, then $n_d(t)$ is independent of $m(t)$

$\therefore$ $n_d(t) \simeq \frac{1}{2\pi A_c} \frac{d}{dt}\left[r(t) \sin\left[\psi(t)\right]\right]$

$\because$ $n_Q(t) = r(t) \sin\left[\psi(t)\right]$



$\therefore$ $n_d(t) \simeq \frac{1}{2\pi A_c} \cdot \frac{d}{dt}\left[n_Q(t)\right]$

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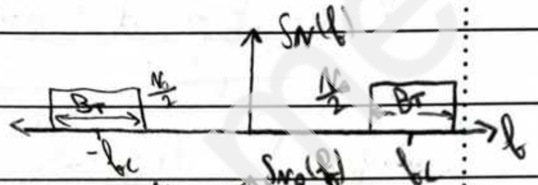
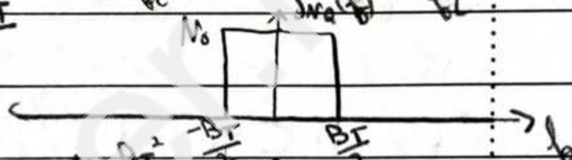
- $n_d(t) \simeq \frac{1}{2\pi A_c} \cdot \frac{d}{dt}[n_Q(t)]$. In the frequency domain, this equation can be seen as passing $n_Q(t)$ through a system with transfer function $H(f) = \frac{jf}{A_c}$

$n_Q(t),\ S_{N_Q}(f) \rightarrow \boxed{H(f)} \rightarrow n_d(t),\ S_{N_d}(f)$

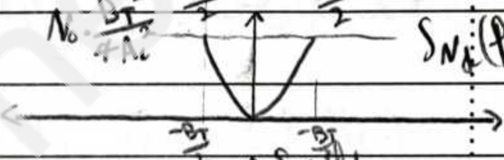
$\rightarrow S_{N_d}(f) = |H(f)|^2 \cdot S_{N_Q}(f)$

$= \frac{f^2}{A_c^2} \cdot S_{N_Q}(f)$

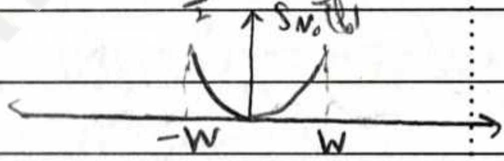




$$\rightarrow S_{N_d}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| \leq \frac{B_T}{2} \\ 0, & \text{otherwise} \end{cases}$$



$n_d(t),\ S_{N_d}(f) \rightarrow \boxed{\text{LPF, BW} = W} \rightarrow n_o(t),\ S_{N_o}(f)$



$$\rightarrow S_{N_o}(f) \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| \leq W \\ 0, & \text{otherwise} \end{cases}$$

- W is usually smaller than $\frac{B_T}{2}$
- power in the filtered noise:

$$P_{n_o(t)} = \int_{-W}^{W} S_{N_o}(f)\, df = \frac{N_0}{A_c^2} \int_{-W}^{W} f^2\, df$$

$$\rightarrow P_{n_o(t)} = \frac{2 N_0 W^3}{3 A_c^2}$$

$\therefore\ v(t) \simeq \underbrace{k_f\, m(t)}_{k_f^2 \cdot P} + \underbrace{n_d(t)}_{\int_{-B_T/2}^{B_T/2} S_{N_d}(f)\, df}$

power:

$$y(t) \simeq \underbrace{k_f\, m(t)}_{k_f^2 P} + \underbrace{n_o(t)}_{\frac{2 N_0 W^3}{3 A_c^2}}$$

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$$\therefore (SNR)_{o,FM} = \frac{3A_c^2 k_f^2 P}{2N_0 W^3}$$

$$\& \ (SNR)_{c,FM} = \frac{P_{s(t)}}{P_{\text{noise in } W}} = \frac{A_c^2/2}{N_0 W}$$

$$\therefore \text{figure of merit: } \frac{(SNR)_o}{(SNR)_c} = \frac{3k_f^2 P}{W^2}$$

- $\Delta f = k_f \cdot A_m \rightarrow \Delta f \propto k_f$
- $D = \frac{\Delta f}{W} \longrightarrow D \propto \Delta f$, $P = \frac{A_m^2}{2} \rightarrow A_m \propto \sqrt{P}$

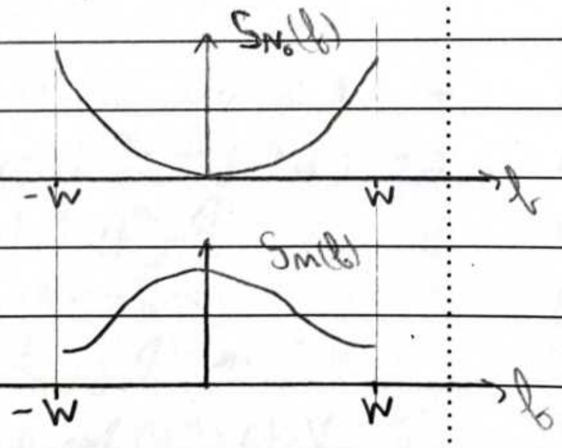
$\therefore D \propto (k_f \sqrt{P})/W$

$$\rightarrow \frac{(SNR)_o}{(SNR)_c} = \frac{3k_f^2 P}{W^2} \propto D^2$$

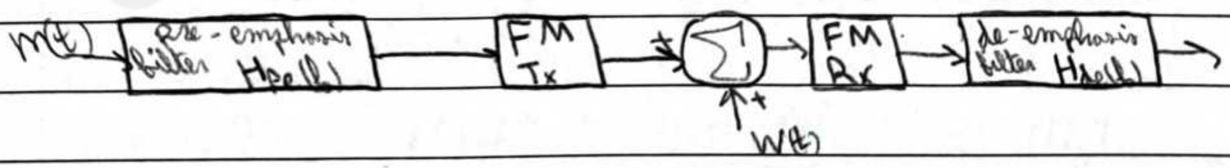
- in FM: $B_T \propto D$, D: frequency deviation

$\rightarrow$ increasing B_T improves the noise performance.

* pre-emphasis and de-emphasis:



- typically the power spectral density of a message signal decreases as the frequency increases. the converse is true for the noise. hence the noise's effect is worse at higher frequencies.



- effectively, the $(SNR)_o$ is being increased.

$$H_{de}(f) = \frac{1}{H_{pe}(f)}$$

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$$S_{N_d}(f) = \begin{cases} \dfrac{N_0 f^2}{A_c^2} &, |f| \leq \dfrac{B_T}{2} \\ 0 &, \text{otherwise} \end{cases}$$

$S_{N_d}(f)$ is the power spectral density of the noise at the output of the discriminator

- PSD with de-emphasis : $S'_{N_d}(f) = |H_{de}(f)|^2 S_{N_d}(f)$

$$I = \frac{P_{noise} \text{ without pre-emphasis \& de-emphasis}}{P_{noise} \text{ with pre-emphasis \& de-emphasis}}$$

at the output of the LPF :

$$I = \frac{2N_0W^3/3A_c^2}{\frac{N_0}{A_c^2}\int_{-W}^{W} f^2 |H_{de}(f)|^2 df} = \frac{2W^3}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 df}$$

- typical value of $I \simeq 21 \simeq 3\,dB$ (larger = better)
- for $H_{pe}(f) = 1 + \frac{jf}{f_0} \Rightarrow H_{de}(f) = \frac{1}{H_{pe}(f)} = \frac{f_0}{f_0 + jf}$

$W = 15\,kHz$ & $f_0 = 2.1\,kHz$

Subject pulse modulation Date 23/5/2021 No.

- Carrier is a pulse train in pulse modulation
+ types of pulse modulation:
  a) pulse amplitude modulation (PAM)
  b) pulse duration modulation (PDM) } analog
  c) pulse position modulation (PPM)
  d) pulse code modulation (PCM) } digital

* the sampling process:
  - given that the fourier transform of a periodic signal:

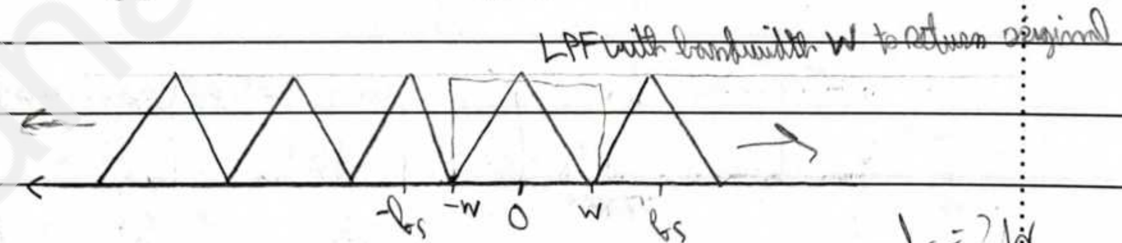
$$\sum_{m=-\infty}^{\infty} g(t - mT_0) \rightleftharpoons f_0 \sum_{n=-\infty}^{\infty} G(nf_0)\,\delta(f - f_0)$$

and our signal is $g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s)$

where T_s is the sampling interval, f_s: sampling frequency.

$\therefore$ by using the duality property:

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$$



$f_s = 2W$

* pulse amplitude modulation: (PAM)
  - the message signal is multiplied by a periodic train of rectangular pulses.
  + the generation of PAM signals is done by:
    - instantaneous sampling of $m(t)$ every T_s

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- lengthening the duration of each sample to be T, inorder to reduce the bandwidth of our sampled signal ($s(t)$)

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, h(t-nT_s)$$

$$\text{s.t. } h(t) = \begin{cases} 1, & 0<t<T \\ 0.5, & t=0 \text{ \& } t=T \\ 0, & \text{otherwise} \end{cases}$$

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(t-nT_s)$$ message samples

$$s(t) = m_\delta(t) * h(t) = \int_{-\infty}^{\infty} m_\delta(\tau)\, h(t-\tau)\, d\tau$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s) \cdot \underbrace{\int_{-\infty}^{\infty} \delta(\tau-nT_s) h(t-\tau)\, d\tau}_{\text{becomes discrete}}$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s) \cdot h(t-nT_s)$$

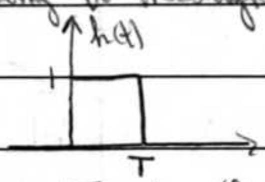
& $S(f) = M_\delta(f) \cdot H(f)$

where $M_\delta(f) = f_s \sum_{k=-\infty}^{\infty} M(f-kf_s)$

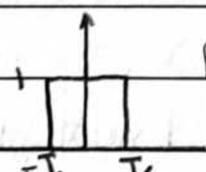
$\rightarrow S(f) = f_s \sum_{k=-\infty}^{\infty} M(f-kf_s) \cdot H(f)$

- $s(t)$ can be used to recover the message signal ($m(t)$) by passing it through a LPF

$\because$ $h(t)$:



shifted version of



$\text{Rect}\left(\frac{t}{T}\right)$

$\rightarrow H(f) = T \,\text{sinc}(fT)\, e^{-j\pi f T}$

- $M(f) \cdot H(f)$ is equivalent to passing the message signal through a LPF with $H(f)$
- Using flat-top samples (sample & hold) causes amplitude

N O T E B O O K

distortion and a delay of $T/2$. this distortion is the "aperature effect".

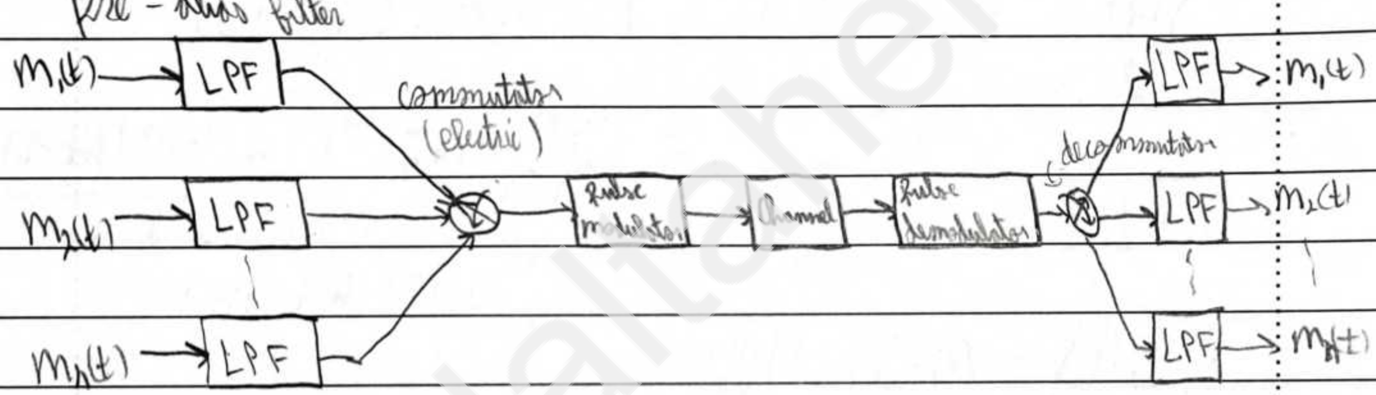
- an equalizer can be used to minimize the distortions:

PAM → [reconstruction filter] → [equalizer $H_{eq}(f)$] → $m(t)$

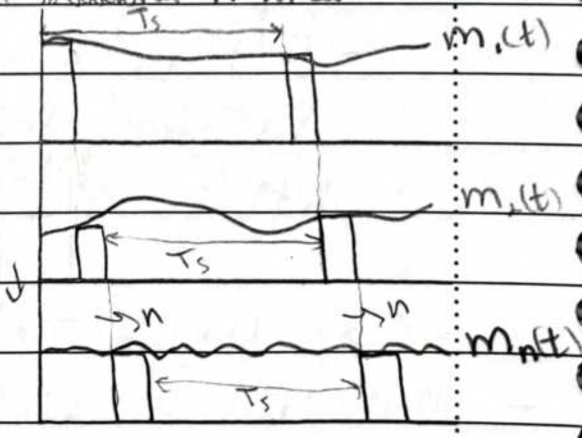
$$\text{s.t.}: |H_{eq}(f)| = \frac{1}{|H(f)|} = \frac{1}{|T \cdot \text{sinc}(fT)|}$$

- if $\frac{T}{T_s} \leq 0.1$, the amplitude distortion is less than 0.5% hence, the equalizer is not needed.

* Time - division multiplexing: (TDM)



- PAM is used for message processing in TDM.
- in TDM, the sampling interval (T_s) is used to sample different message signals. hence, the commutator switches n times during T_s
- as soon as the first message signal is sampled, the commutator flip to sample the next message signal and so on.



* pulse - position modulation:

- PPM is done by varying the time of occurance of the leading edge, trailing edge, or both edges of the pulse

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- an intermediate step of PPM is pulse-width modulation (PWM), which is done by varying the width of the pulse.
- PWM is not used solely as long pulses expend considerable power making PWM inefficient.

Subject Pulse modulation continued   Date 24/5/2021   No.

* PPM :

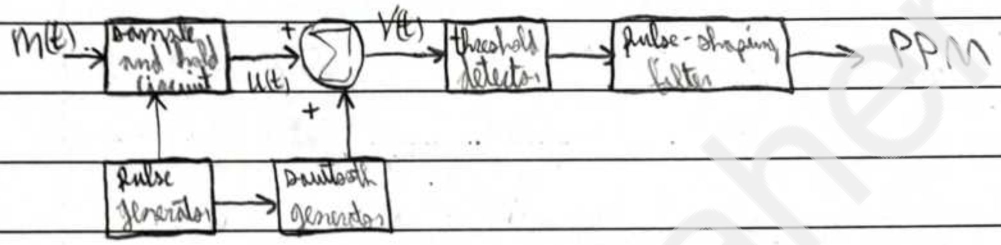
$$s(t) = \sum_{n=-\infty}^{\infty} g(t - nT_s - k_p\, m(nT_s))$$

where k_p is the sensitivity of the pulse-position modulator.

- to avoid overlapping between pulses, the following condition should be maintained :

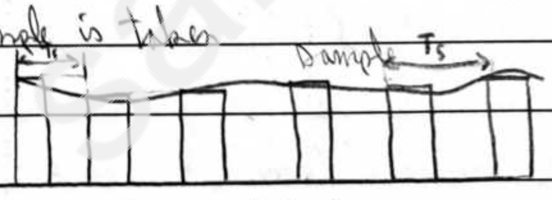
$$k_p \cdot |m(t)|_{max} < \frac{T_s}{2}$$

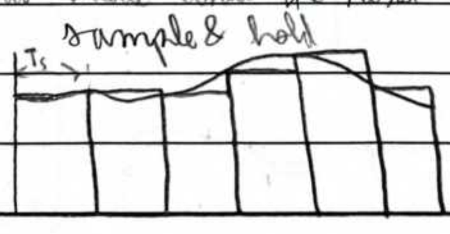
- the following block diagram is used for PPM wave generation:



+ to detect a PPM wave:
1- convert the received wave into PDM.
2- integrate the PDM using a device with a finite integration time
- this step is done to compute the area under each pulse of PDM.
3- sample the output of the integrator at a uniform rate to produce a PAM which can be used to recover m(t) by filtering.

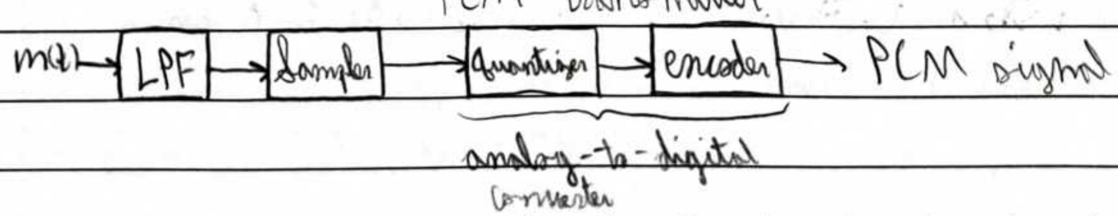
- sample and hold takes a sample and holds its value until the next sample is taken





* Pulse code modulation:

PCM Transmitter



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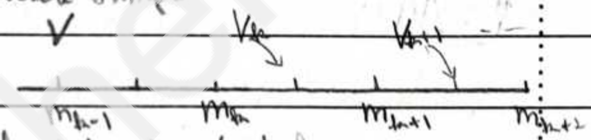
PCM receiver

received PM signal → [regeneration circuit] → [decoder] → [reconstruction LPF] → $m(t)$

* Quantization:

- amplitude quantization is the process of transforming the sample amplitude $m(nT_s)$ of $m(t)$ into discrete amplitude $V(nT_s)$ taken from a finite set of possible amplitudes.

$m(t)$ → [sampler] → [quantizer $g(\cdot)$] → discrete sample



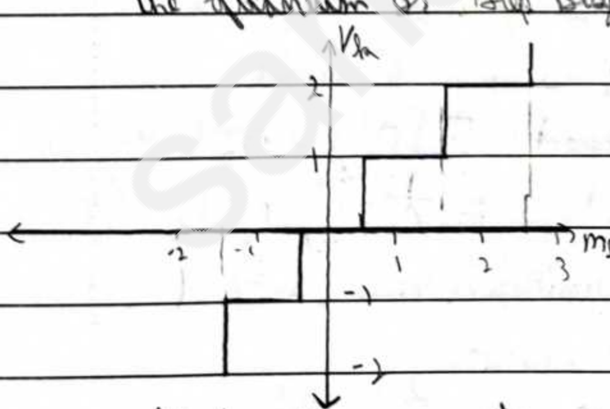
- the signal amplitude is specified by the index k if it lies inside the interval: s.t. L: total number of amplitude levels.

$$y_k : \{m_k < m \leq m_{k+1}\}, \ k = 1, 2, \dots, L$$

where m_k, $k = 1, 2, 3, \dots, L$ are the decision levels

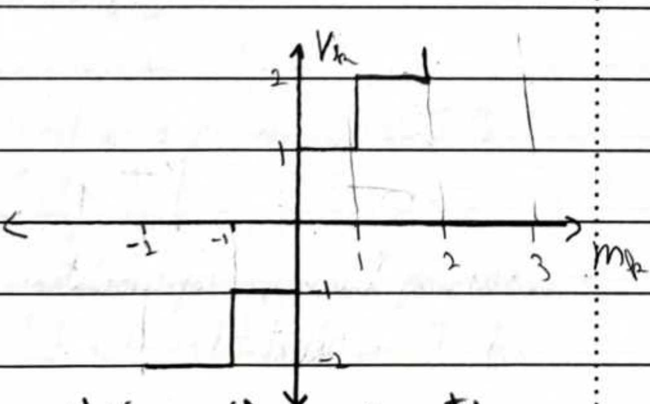
and V_k, $k = 1, 2, \dots, L$ are the representation levels.

- the spacing between two adjacent representation levels is called the quantum or step size.



midtread uniform quantizer

zero is a representation level



midrise uniform quantizer

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* Quantization noise
- If our message signal ($m(t)$) is a sample function of a zero-mean random process and m is the sample value of $m(t)$, then

$$V = g(m) \text{, where } g \text{ is the quantizer}$$

such that $g(\cdot)$ maps the input random variable M of continuous amplitudes into discrete random variable V:
where the quantization error (q) is given by: $q = m - v$

$$\text{(or) } Q = M - V$$

- If M has zero mean, $g(\cdot)$ is a uniform symmetric quantizer, then Q & V will also have zero mean.
- m is between $(-m_{max})$ & (m_{max}). assuming uniform midrise quantizer, then $\Delta = \frac{2\, m_{max}}{L}$.

$$\rightarrow \frac{-\Delta}{2} \leq q \leq \frac{\Delta}{2}$$

- If Δ is sufficiently small (L is large), then we can assume that Q is a uniformly distributed random variable.

$\therefore$ the probability density function of Q:

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & \frac{-\Delta}{2} < q \leq \frac{\Delta}{2} \\ 0, & \text{otherwise} \end{cases}$$

$\because$ zero-mean $\rightarrow$ second-order moment: $E[Q^2] = \sigma_Q^2$ (variance)

$$\therefore \sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} q^2 f_Q(q)\, dq = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2\, dq = \frac{\Delta^2}{12}$$

- assuming binary representation $\rightarrow$ number of levels, $L = 2$ $(0, 1)$ and R is the total number of bits used, hence:

$$\Delta = \frac{2\, m_{max}}{2^R}$$

$$\therefore \sigma_Q^2 = \frac{1}{3}\, m_{max}^2 \cdot 2^{-2R}$$

- σ_Q^2 is the average noise power

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- if P is the average power in $m(t)$, then $(SNR)_o = \frac{P}{\sigma_Q^2} = \left(\frac{3P}{m_{max}^2}\right) \cdot 2^{2R}$

$\therefore \quad (SNR)_o \big|_{PCM} \propto 2^{2R}$

- hence, increasing R will give a higher bit rate, which will increase the bandwidth.
- since the quantization noise is the dominant noise, all other types of noise were ignored in the above calculations.
- for a sinusoidal modulating signal/full-load message signal whose amplitude is A_m and utilizes all the representation levels.

$\rightarrow P = A_m^2/2$, average power in the message signal.

$\lambda \quad m_{max} = A_m \quad \& \quad \sigma_Q^2 = \frac{1}{3} A_m^2 \cdot 2^{-2R}$

$\therefore \quad (SNR)_o = \frac{A_m^2/2}{\frac{1}{3} A_m^2 \cdot 2^{-2R}} = \frac{3}{2} \cdot 2^{2R}$

$\rightarrow 10 \log_{10}[(SNR)_o] = 1.76 + 6R \text{ dB}$

$\frac{1}{\Delta} \cdot \frac{1}{3} \left[q^3 \right]_{-\Delta/2}^{\Delta/2} = \frac{1}{3\Delta \cdot 8} \cdot 2\Delta^3 = \frac{\Delta^3}{12\Delta}$

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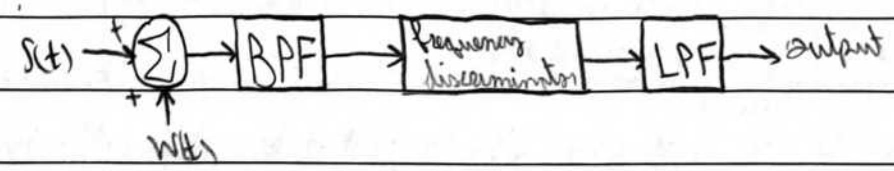
MOHAMMAD SANAD ALTAHER 130806

Q13 → Q17 :



$$S(t) = 4\cos\left(200\pi \times 10^6 t + 2\pi k_f \int m(t)\,dt\right)$$

$$\rightarrow f_c = 100\ \text{MHz} \quad \& \quad A_c = 4$$

Q13) $\because (SNR)_c = \dfrac{P_{\text{modulated signal}}}{P_{\text{noise in message BW}}}$, $P_{\text{noise in message BW}} = \dfrac{N_0}{2} \cdot 2 \cdot 4k$

$$\rightarrow (SNR)_c = \frac{(4)^2/2}{\frac{2N_0}{2} \cdot 4k} = \boxed{\frac{1}{500 N_0}}$$

Q14) BPF bandwidth : 200k → $P_{\text{noise BPF}} = \dfrac{N_0}{2} \cdot 2 \cdot 200k$
$= \boxed{200k \cdot N_0}$

Q15) $\because S_{N_Q}(f) = \begin{cases} N_0, & |f| \leq \frac{200k}{2} \\ 0, & \text{elsewhere} \end{cases}$

$$\rightarrow P_{N_Q} = \int_{-100k}^{100k} N_0 \cdot df$$

$$= \boxed{200k \cdot N_0}$$

Q16) $\because$ bandwidth of quadrature component $= \frac{1}{2}$ BPF bandwidth
& sampling rate must be double the bandwidth
→ sampling rate $= \frac{1}{2} \cdot 200k \cdot 2 = \boxed{200k \text{ samples/s}}$

Q17)

$$\because P_{N_o} = \int_{-\infty}^{\infty} S_{N_o}(f)\,df = \int_{-W}^{W} \frac{N_0 f^2}{A_c^2}\,df$$

$$\rightarrow P_{N_o} = \frac{N_0}{(4)^2 \cdot 3} \cdot \left[f^3\right]_{-W}^{W}, \quad W = 4k$$

$$\therefore \boxed{P_{N_o} = 2.67 \times 10^{9} \cdot N_0}$$

Q18) the center frequency of an SSB receiver's BPF $= f_c - \dfrac{W}{2}$
assuming $f_c = 100\ \text{MHz}$ & $W = 4k$
→ center frequency : 100M − 2k

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Q19) $P_{N_2} = \int_{-\frac{W}{2}}^{W/2} S_{N_2}(f)\, df = N_0 \cdot [f]_{-W/2}^{W/2} = N_0 W$

If $W = 4k$ → $\boxed{P_{N_2} = N_0 \cdot 4k}$

Q20) ∵ $\frac{T}{T_s} \leq 0.1$ for amplitude distortion to be less than 0.5%

→ $T \leq \frac{0.1}{f_s}$ ∧ $f_s = 8\, kHz$

∴ $T \leq 12.5\, \mu s$

Q23)

∵ $|q_{max}| = \frac{\Delta}{2}$ ∧ $\Delta = \frac{2 m_{max}}{L}$, $L = 2^2$

∧ $m_{max} = 2$ → $\boxed{q_{max} = 0.5}$

Q24) The quantization error is mainly affected by the message amplitude and the number of representation levels. Hence, I would increase the number of bits to increase the number of representation levels and decrease the quantization error.

$q \propto \frac{1}{2^R}$, s.t. R: number of bits.

Subject final exam practice Date 7/6/2024 No. δT $(f-\frac{1}{2T})T$

- negative frequencies are not included in bandwidth
- baseband signal has BW = W → carrier signal is bandpass with BW = 2W

HW 1:

a) half cosine : $\cos(2\pi \frac{1}{2T} t) \cdot Rect(\frac{t}{T})$

$\rightarrow A\cos(\frac{\pi t}{T}) \cdot Rect(\frac{t}{T})$

multiplication in time → convolution in frequency

$\cos(\frac{\pi t}{T}) \rightleftharpoons \frac{A}{2}[\delta(f-\frac{1}{2T}) + \delta(f+\frac{1}{2T})]$

$\wedge \quad Rect(t/T) \rightleftharpoons T\,sinc(fT)$

$\therefore \quad \frac{AT}{2}\left[[\delta(f-\frac{1}{2T}) + \delta(f+\frac{1}{2T})] * sinc(fT)\right]$

$\rightarrow \frac{AT}{2}\left[sinc(fT-\frac{1}{2}) + sinc(fT+\frac{1}{2})\right]$

b) $\frac{AT}{2} \cdot e^{-j2\pi \frac{1}{2T} \cdot \frac{T}{2}} \rightarrow \frac{AT}{2} e^{-\frac{\pi}{2}}\left[sinc(fT-\frac{1}{2}) + sinc(fT+\frac{1}{2})\right]$

or $\frac{AT}{2} e^{-\pi \cdot fT}\left[sinc(fT-\frac{1}{2}) + sinc(fT+\frac{1}{2})\right]$

c) $A\cos(2\pi \frac{1}{2T} \cdot t) \cdot Rect(\frac{t}{2T})$ not cos

$\frac{A}{2}[\delta(f-\frac{1}{2T}) + \delta(f+\frac{1}{2T})] * 2T\,sinc(2fT)$

$\rightarrow AT\left[sinc(2fT-1) + sinc(2fT+1)\right]$

~~Take phase shift~~

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$$A \sin\left(2\pi \frac{1}{2T} t\right) \cdot \mathrm{Rect}\left(\frac{t}{2T}\right) \rightarrow 2T \,\mathrm{sinc}(2fT)$$

$$\frac{A}{2j}\left[\delta\left(f-\frac{1}{2T}\right) - \delta\left(f+\frac{1}{2T}\right)\right]$$

$$\rightarrow \frac{AT}{j}\left[\mathrm{sinc}(2fT-1) - \mathrm{sinc}(2fT+1)\right]$$

$$\rightarrow jAT\left[\mathrm{sinc}(2fT+1) - \mathrm{sinc}(2fT-1)\right]$$

2.2:

$$g(t) = e^{-t} \sin(2\pi f_c t)\, u(t)$$

$$\rightarrow \frac{2\pi f_c}{(1+j2\pi f)^2 + 4\pi^2 f_c^2}$$

2.4:

$$|G(f)| = \begin{cases} 1 & , -W < f < W \\ 0 & , \text{o.w} \end{cases}$$

$$\arg G(f) = \begin{cases} \pi/2 & , -W < f < 0 \\ -\pi/2 & , 0 < f < W \\ 0 & , \text{o.w} \end{cases}$$

$$\rightarrow G(f) = \begin{cases} 1 \cdot e^{j\pi/2} & , -W < f < 0 \\ 1 \cdot e^{-j\pi/2} & , 0 < f < W \\ 0 & , \text{otherwise} \end{cases}$$

use inverse fourier equation

$$\int_{-W}^{0} e^{j\pi/2}\, e^{j2\pi f t}\, df + \int_{0}^{W} e^{-j\frac{\pi}{2}}\, e^{j2\pi f t}\, df$$

$$= \frac{-j}{2\pi t}\left[e^{j(2\pi f t + \frac{\pi}{2})}\right]_{-W}^{0} - \frac{j}{2\pi t}\left[e^{j(2\pi f t - \frac{\pi}{2})}\right]_{0}^{W}$$

$$\rightarrow \frac{-j}{2\pi t}\left[e^{j\frac{\pi}{2}} - e^{j(-2\pi W t + \frac{\pi}{2})} - e^{-j\frac{\pi}{2}} + e^{j(2\pi W t - \frac{\pi}{2})}\right]$$

$$\rightarrow \frac{-j}{2\pi t}\left[2j\sin\left(\frac{\pi}{2}\right) + 2j\sin\left(2\pi t W - \frac{\pi}{2}\right)\right]$$

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→ $\frac{1}{\pi t}\left[\sin(\frac{\pi}{2}) + \sin(2\pi W t - \frac{\pi}{2})\right]$

$= \frac{1}{\pi t}\left[1 - \cos(2\pi W t)\right] = \frac{2}{\pi t}\sin^2(\pi W t)$

2.10: $y(t) = x(t) \cdot x(t)$

→ $Y(f) = X(f) * X(f)$

→ $Y(f) = \int_{-\infty}^{\infty} X(\lambda) \cdot X(f-\lambda)\, d\lambda$

→ $Y(f) = \int_{-W}^{W} X(\lambda) \cdot X(f-\lambda)\, d\lambda$

∵ $X(f-\lambda)$ is limited to $|W|$

→ $X(f-\lambda)$ for $-W < f_y - \lambda < W$

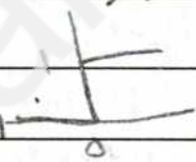
λ $X(\lambda)$ → $-W < \lambda < W$

if $\lambda = W$ → $0 < f_y < 2W$

if $\lambda = -W$ → $-2W < f_y < 0$

∴ $-2W < f < 2W$

2.11

→ $G(f)$ is half signum function $+ \frac{1}{2}$  → $G(f) = \frac{1}{2} + \frac{1}{2}\mathrm{sgn}(f)$

→ $\frac{1}{2}\delta(t) + \frac{1}{2}\frac{1}{j\pi t} = \frac{1}{j2\pi t} + \frac{1}{2}\delta(t)$

⇒ $\frac{-1}{j2\pi t} + \frac{1}{2}\delta(t)$

Or simply $G(f) = u(f)$

$u(t) \rightleftharpoons \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$

take duality → $G(f)$ → $g(t) = \frac{1}{2}\delta(-t) - \frac{1}{j2\pi t}$

→ $\boxed{\frac{1}{2}\delta(t) - \frac{1}{j2\pi t}}$

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$1 - 2e^{-j f_c 2\pi T} - e^{-j4\pi f_c T}$

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$e^{-j2\pi f_c T}\left[e^{j2\pi f_c T} - 2 - e^{-j2\pi f_c T}\right]$

$\rightarrow e^{-j2\pi f_c T}\left[e^{j\pi f_c T} - e^{-j\pi f_c T}\right]^2$

HW2:

① $x(t) - x(t-T)$

② $\int_{-\infty}^{t} x(\tau) - x(\tau - T)\, d\tau \rightarrow g(t)$

③ $g(t) - g(t-T)$

④ $y(t) = \int_{-\infty}^{t} g(\tau) - g(t-T)\, d\tau$

a) in frequency domain

① $X(f)\left[1 - e^{-j2\pi f_c T}\right]$

② $\frac{1}{j2\pi f} X(f)\left[1 - e^{-jf_c 2\pi T}\right] + 0 \quad \because X(0) = 0$

③ $\frac{1}{j2\pi f} X(f)\left[1 - e^{-jf_c 2\pi T}\right]\left[1 - e^{-jf_c 2\pi T}\right]$

$\rightarrow \frac{1}{j2\pi f} X(f)\left[1 - e^{-jf_c 2\pi T}\right]^2$

④ $\frac{1}{(j2\pi f)^2} \cdot X(f)\left[1 - e^{-jf_c 2\pi T}\right]^2$

$\rightarrow X(f) \cdot e^{-j2\pi f_c T}\left[\frac{e^{j\pi f_c T} - e^{-j\pi f_c T}}{j2\pi f}\right]^2$

$\rightarrow X(f) \cdot e^{-j2\pi f_c T}\left[\frac{j2\sin(\pi f_c T)}{j2\pi f}\right]^2$

for the transfer function remove $X(f)$

$\rightarrow e^{-j2\pi f_c T} \cdot T^2 \cdot \left[\frac{\sin(\pi f_c T)}{\pi f_c T}\right]^2$

$\rightarrow e^{-j2\pi f_c T} \cdot T^2 \cdot \mathrm{sinc}^2(fT)$

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2.18:

$$V_c = V_0 \cdot \frac{Z_c}{Z_c + R} \quad \wedge \quad Z_c = \frac{1}{j2\pi f_0 C}$$

$$\rightarrow V_c = V_0 \cdot \frac{1}{1 + j2\pi f_0 CR} \quad , \; RC = T_0$$

$$V_{c2} = V_{c1} \cdot \frac{1}{1 + j2\pi f_0 T_0} \quad \therefore N = \frac{1}{(1 + j2\pi f_0 T_0)^N}$$

amplitude response = Re{frequency response}

$$Re\left\{\frac{1}{(1 + j2\pi f_0 T_0)^N}\right\} \qquad (1 + j2\pi f_0 T_0)^2 = 1 + j4\pi f_0 T_0 - (2\pi f_0 T_0)^2$$

$$\left[(1 + j2\pi f_0 T_0)^2\right]^{N/2}$$

$$\left(1^2 + j4\pi f_0 T_0 - (2\pi f_0 T_0)^2\right)^{N/2}$$

$$\frac{1}{[1 + (2\pi f_0 T_0)^2]^{N/2}}$$

23: a) $g_+(t) = g(t) + j\hat{g}(t)$

$$\because g(t) = \text{sinc}(t) \rightarrow \hat{g}(t) = \frac{1 - \cos(\pi t)}{\pi t}$$

$$\rightarrow g_+(t) = \frac{\sin(\pi t)}{\pi t} + j\,\frac{1 - \cos(\pi t)}{\pi t}$$

$$\rightarrow g_+(t) = \frac{j + \sin(\pi t) - j\cos(\pi t)}{\pi t}$$

$$\therefore g_+(t) = \frac{j}{\pi t}\left[1 - \cos(\pi t) - j\sin(\pi t)\right]$$

$$= \frac{j}{\pi t}\left[1 - e^{j\pi t}\right]$$

b) $\because g(t) = [1 + k\cos(2\pi f_m t)]\cos(2\pi f_c t)$

$\rightarrow g(t) = \cos(2\pi f_c t) + \frac{k}{2}[\cos(2\pi(f_m + f_c)t) + \cos(2\pi(f_c - f_m)t)]$

$\therefore \hat{g}(t) = \sin(2\pi f_c t) + \frac{k}{2}[\sin(2\pi(f_m + f_c)t) + \sin(2\pi(f_c - f_m)t)]$

$\rightarrow g_+(t) = [1 + k\cos(2\pi f_m t)]\cos(2\pi f_c t) + j\sin(2\pi f_c t) + \frac{jk}{2}[\sin(2\pi(f_m + f_c)t) + \sin(2\pi(f_c - f_m)t)]$

$\rightarrow g_+(t) = e^{j2\pi f_c t} + \frac{k}{2}[\cos(2\pi(f_m + f_c)t) + j\sin(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t) + j\sin(2\pi(f_c - f_m)t)]$

$\rightarrow g_+(t) = e^{j2\pi f_c t} + \frac{k}{2}[e^{j2\pi(f_m + f_c)t} + e^{j2\pi(f_c - f_m)t}]$

$\rightarrow g_+(t) = e^{j2\pi f_c t}[1 + \underbrace{\frac{k}{2}e^{j2\pi f_m t} + \frac{k}{2}e^{-j2\pi f_m t}}]$

$e^{j2\pi f_c t}[1 + \frac{k}{2}2\cos(2\pi f_m t)]$

$\therefore g_+(t) = e^{j2\pi f_c t}[1 + k\cos(2\pi f_m t)]$

31: find complex envelope transfer function

$\rightarrow \tilde{H}(f) = 2\,\mathrm{Rect}(\frac{f}{2B}) \cdot e^{-2\pi f_c t_0}$

$\rightarrow \tilde{h}(t) = 4B\,\mathrm{sinc}(2B(t - t_0))$

$\because x(t) = \mathrm{Re}\{\tilde{x}(t)\,e^{j2\pi f_c t}\} = A\cos(2\pi f_c t)$

$\rightarrow \tilde{x}(t) = A\,u(t)$

$\tilde{y}(t) = \frac{1}{2}\tilde{x}(t) * \tilde{h}(t) = \frac{4AB}{2}\int_0^\infty \mathrm{sinc}(2B(t - \tau - t_0))\,d\tau$

$\rightarrow \tilde{y}(t) = 2AB\int_0^\infty \mathrm{sinc}(2B(t - \tau - t_0))\,d\tau$

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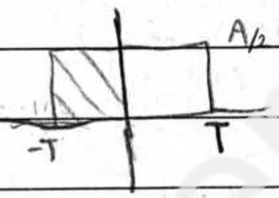
assume $\lambda = 2B(t-T-t_0)\pi$

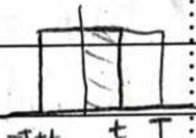
Q2: $\tilde{x}(t) = \begin{cases} A & , 0 \le t \le T \\ 0 & , o.w \end{cases}$

$h(t) = x(-(t-T))$ $\quad \tilde{h}(t) = \begin{cases} A & , 0 \le t \le T \\ 0 & , o.w \end{cases}$

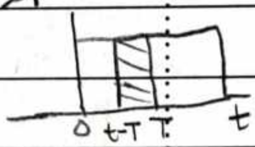
$\rightarrow \tilde{y}(t) = \frac{1}{2}\tilde{x}(t) * \tilde{h}(t)$ $\qquad A^2 \int_0^{t} dt$



shaded area $= (t-0) \cdot A^2/2$



$\rightarrow \tilde{y}(t) = \begin{cases} 0 , & 2T > t < 0 \\ \frac{A^2}{2} t, & 0 \le t \le T \\ \frac{A^2}{2}(T-(t-T)) = \frac{A^2}{2}(2T-t), & T \le t \le 2T \end{cases}$



$\therefore y(t) = \begin{cases} Re\{\frac{A^2}{2} t \, e^{j2\pi f_c t}\} , & 0 \le t \le T \\ Re\{A^2(T-\frac{t}{2}) e^{j2\pi f_c t}\}, & T \le t \le 2T \\ 0 & , e.w \end{cases}$

$\rightarrow y(t) = \begin{cases} \frac{A^2}{2} t \cos(2\pi f_c t) & , 0 \le t \le T \\ A^2(T-\frac{t}{2}) \cos(2\pi f_c t) & , T \le t \le 2T \\ 0 & , e.w \end{cases}$

HW3:

Q1) $m(t) = \cos(2000\pi t) + \cos(4000\pi t)$, $c(t) = \cos(200\pi 10^3 t)$

$c(t) \cdot m(t) =$

$\frac{1}{2}\left[\cos(202 \times 10^3 \pi t) + \cos(204 \times 10^3 \pi t)\right]$

$\therefore$ upper SSB: $\frac{1}{2}\left[A_c m(t) \cos(2\pi f_c t) - A_c \hat{m}(t) \sin(2\pi f_c t)\right]$

$A_c = 1$ $\quad \hat{m}(t) = \sin(2000\pi t) + \sin(4000\pi t)$

$\rightarrow \frac{1}{2}\left[\frac{1}{2}\cos(202 \times 10^3 \pi t) + \frac{1}{2}\cos(204 \times 10^3 \pi t) + \frac{1}{2}\cos(198 \times 10^3 \pi t) + \frac{1}{2}\cos(196\pi t \times 10^3)\right.$

$\left. - \frac{1}{2}\cos(198 \times 10^3 \pi t) + \frac{1}{2}\cos(202 \times 10^3 \pi t) + \frac{1}{2}\cos(204 \times 10^3 \pi t) - \frac{1}{2}\cos(196\pi t \times 10^3)\right]$

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$\rightarrow \frac{1}{2}\left[\cos(202\times10^3\pi t)+\cos(204\times10^3\pi t)\right]$

c) P in carrier $= 0 = P_{LSB}$, $P_{USB} = \frac{(\frac{1}{2})^2}{2} + \frac{(\frac{1}{2})^2}{2} = \frac{1}{4}$

HW4:

a) $s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cdot\cos(2\pi(f_c+nf_m)t)$

$s(t) = \cos(2\pi f_c t + 3\sin(2000\pi t))$ $\rightarrow$ 3 is modulation index

$\rightarrow s(t) = \sum_{n=-\infty}^{\infty} J_n(3)\cos(2\pi(f_c+n1000)t)$

b) $BW = 2n_{max}f_m$ $|J_n(\beta)| > 0.01$

$n_{max}|_3 = 6 \rightarrow BW = 12\cdot1000 = 12\text{kHz}$

c) ~~$A_c = 1$, $P_{carrier} = \frac{A_c^2}{2} = 0.5W$~~

total: $P_{s(t)} = \frac{A_c^2}{2} = 0.5\,W$

$P_{\text{modulated carrier}} = \frac{A_c^2}{2}\left[J_0(\beta)\right]^2 = 0.5\cdot[-0.2601]^2$

$= 0.03383\,W$

$P_{sidebands} = P_{total} - P_{carrier} = 0.466194\,W$

d) $\because$ for a new f_m $\beta = 3\cdot20$

, $\beta = \frac{\Delta f}{f_m} = 60\cdot1\text{kHz} = 60\text{kHz}$

e) $V(t) = \frac{k_f}{k_v}m(t)$, $k_v = 10\text{kHz}/V$

$\because \beta = \frac{\Delta f}{f_m} = 3 \rightarrow \Delta f = 3k$, $\Delta f = k_f A_m$

$\rightarrow k_f = 3\text{kHz}/V \rightarrow V(t) = 0.3\cdot m(t)$

or $V(t) = \frac{1}{2\pi k_v}\frac{d\phi_1(t)}{dt}$, $\phi_1(t) = 2\pi k_f\int m(t)\,dt$

$\phi_1(t) = 3\sin(2000\pi t)$

$\rightarrow V(t) = \frac{1}{2\pi k_v}\frac{d[3\sin(2000\pi t)]}{dt}$

$\rightarrow V(t) = \frac{1}{2\pi k_v}\cdot3\cos(2000\pi t)\cdot2000\pi = \frac{3000}{10000}\cos$

$0.3\cos(2000\pi t)$

NOTEBOOK

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b) $V_R = V_{in} \cdot \frac{R}{R + \frac{1}{j2\pi f_c C}} = V_{in} \frac{j2\pi f_c CR}{1 + j2\pi f_c CR}$

$\because 2\pi f_c CR << 1 \rightarrow V_R \simeq V_{in} \cdot j2\pi f_c RC$

$\rightarrow V_R = S(t) \cdot j2\pi f_c RC \quad \because j2\pi f\, G(f) = \frac{d}{dt} g(t)$

$\Rightarrow V_R = RC \cdot \frac{d}{dt}[S(t)]$

$\because S(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right)$

$\rightarrow \frac{dS(t)}{dt} = A_c \frac{d}{dt}\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right] \cdot \sin\left(2\pi f_c t + 2\pi k_f \int m(t)\,dt\right)$

$\rightarrow \frac{dS(t)}{dt} = A_c\left[2\pi f_c + 2\pi k_f m(t)\right] \cdot \sin$ —

envelope detector removes sin

$\rightarrow \text{output} = A_c\left[2\pi f_c + 2\pi k_f m(t)\right] \cdot RC$

$\rightarrow \text{output} = A_c 2\pi f_c \cdot RC\left[1 + \frac{k_f}{f_c} \cdot m(t)\right]$

2.1: $BW_T = 2\Delta f + 2W$

$\because \beta = 0.1 \rightarrow \Delta f = 0.1 \cdot 5000 = 500\ Hz$

$BW_T = 2\Delta f\left(1 + \frac{1}{\beta}\right) = 2\Delta f + 2f_m$

$\rightarrow BW_T = 1000(1 + 10) = 11\ kHz$

$\beta = 0.5 \rightarrow \Delta f = 2500 \rightarrow 5000(3) = 15\ kHz$

2.2: $\frac{0.5 - 0.5 \cdot [J_0(0.5)]^2}{2} = 0.0298\ W$

2.3: $f = f_c + n f_m \qquad f_c = 5 \cdot 0.2M = 1000\ kHz$

$\rightarrow 1005\ kHz = 1000\ kHz + n \cdot 5000\ Hz$

$\rightarrow n = 1 \ \wedge\ \beta = 0.5$

$\therefore P = 0.5 \cdot [J_1(0.5)]^2 = 0.02935$

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2.4: power in modulated carrier $= \frac{A_c^2 \left[J_0(\beta)\right]^2}{2}$ → smallest $J_0(\beta)$ required for smallest power

→ $\beta = 2 = n \cdot 0.1$ → $n = 20$

2.5: $\beta_2 = 7.5$ → $\Delta f = 37.5$ kHz

2.6: $n_2 [f_2 \pm n_1 f_1] \pm f_m$

→ 150 MHz $\pm f_m$ or 120 MHz $\pm f_m$

→ {119.995M, 120.005M, ~~149.995M~~, ~~150.005M~~}

→ $-119.995 + 120.005 = 10$ kHz

mid 2020 :

1) $V_i = \text{sinc}(2000t)^{+\cdots} = \frac{\sin(2000\pi t)}{2000\pi t} + \cos(2\pi f_c t)$

$$V_o = \frac{\sin(2000\pi t)}{2000\pi t} + \cos(2\pi f_c t) + \frac{1}{2}\left[\frac{-\cos(4000\pi t)}{4000\pi t} + 1 + \cos(4\pi f_c t) + \boxed{\frac{\sin(2000\pi t)\cos(2\pi f_c t)}{1000\pi t}}\right]$$

taking the fourier transform of the boxed term

$\rightarrow \frac{1}{2000}\text{Rect}\left(\frac{f}{2000}\right) * \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$

$\rightarrow \frac{1}{4000}\text{Rect}\left(\frac{f-f_c}{2000}\right) + \frac{1}{4000}\text{Rect}\left(\frac{f+f_c}{2000}\right)$

$\rightarrow \frac{1}{4000}$ E

2) O $\because$ no carrier term $\because$ DSB-SC

4) bandwidth in $m^2(t) = 2\,BW_{m(t)} = 2k$

$\rightarrow BW_{(t)} = 4k$

5) $H(f) = j2\pi f \rightarrow h(t) = \frac{d}{dt}$

$\rightarrow \frac{dS(t)}{dt} = -\left[20\pi\times10^6 + 15000\pi\cos(30000\pi t)\right]\cdot\sin\cdots$

envelope detector removes sine and rectifies

$\rightarrow 20\pi\times10^6 + 15\times10^3\pi\cos(30000\pi t)$

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6) $\frac{\sin(1000\pi t)}{1000\pi t} + \cos(2\pi f_c t) + \left[\frac{-\cos(2000\pi t)}{1000\pi t} + 1 + \cos(4\pi f_c t)\right.$

$\left. + \frac{\sin(1000\pi t) \cdot \cos(2\pi f_c t)}{500\pi t}\right] \rightarrow 2\,\mathrm{sinc}(1000t)\cos$

$f_m = 1000 \rightarrow BW = 2f_m$ ask unde

7) $S(t) = A_c\left[1 + k_a m(t)\right] \cos 2\pi f_c t$

$S(t) = m(t) + \underbrace{\cos(2\pi f_c t)}_{HPF} + m^2(t) + \cos^2(2\pi f_c t) + \underbrace{2m(t)\cos(2\pi f_c t)}_{HPF}$

$\rightarrow \boxed{2m(t)\cos(2\pi f_c t)}$ $\boxed{k_a = 2}$

8) Carson's rule : $BW = 2\Delta f + 2f_m$

$= 2\Delta f\left(1 + \frac{1}{\beta}\right), \beta = 1$

$\rightarrow \Delta f = f_m = \cancel{15}k \rightarrow 60\,kHz$

1% rule : $2n_{max} f_m = 6f_m = 90\,kHz$

9) $\rightarrow 2 \cdot \frac{2.5\sin(2000\pi t)}{2000\pi t} = \frac{5\sin(2000\pi t)}{2000\pi t}$

or $5 \cdot \mathrm{sinc}(2000\pi t) \geq 1$

$\rightarrow \frac{\sin(2000\pi t)}{2000\pi t} \geq 0.2$

$\rightarrow \sin(2000\pi t) \geq 400\pi t$

$2000\pi\cos(2000\pi t) \geq 400\pi$

$\rightarrow \cos(2000\pi t) \geq 0.2$

$\rightarrow t =$

Phase reversal when $\sin = 0$

$\rightarrow \sin(2000\pi t) = 0 \rightarrow t =$

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$|k_a m(t)| > 1$ for phase reversal

$\rightarrow \left|5 \cdot \frac{\sin(2000\pi t)}{2000\pi t}\right| > 1$ suppress sin

$\rightarrow \left|\frac{5}{2000\pi t}\right| > 1 \rightarrow t > \frac{5}{2000\pi}$

$\rightarrow t > \frac{1.5}{2000}$

10) $\because x(t) = Re\{\tilde{x}(t)\, e^{j2\pi f_c t}\} = Rect\left(\frac{t - T/2}{T}\right)\cos(2\pi f_c t)$

$\rightarrow \tilde{x}(t) = Rect\left(\frac{t - T/2}{T}\right)$

11) passing $[m(t) \cdot \cos(3f_c \cdot 2\pi \cdot t)]$ only $\rightarrow$ DSB-SC

recover using coherent detector

12) $\cos((4000 + 10\times10^3)\pi t) \rightarrow w = 2\pi f = 14k\pi \rightarrow f = 7\,kHz$

13) $P_{total} = 0.5$, $\beta = 1$, $[J_0(\beta)]^2 = 0.58553$

$\rightarrow P_{carrier} = 0.29276552$

$\rightarrow P_{USB} = \frac{0.5 - 0.293}{2} = 0.1036 \approx 0.104$

14) $s(t) = \sum_{n=-\infty}^{\infty} J_n(0.5) \cos(2\pi(f_c + n f_m)t)$, $f_m = 15\,kHz$

$n_m = 4 \rightarrow \beta_{out} = 2$, $f_{out} = 80\pi \times 10^6$

$\rightarrow s(t) = \sum_{n=-\infty}^{\infty} J_n(2) \cos(2\pi(80\times10^6 + n15k)t)$

15) $x(t)[1 + e^{-j2\pi f T}] \rightarrow \frac{1}{j2\pi f}[1 + e^{-j2\pi f T}]$ $[1 + e^{-j2\pi f T}$

$\rightarrow \left[\left[\frac{1}{j2\pi f}\right]\left[1 + e^{-j2\pi f T}\right]\right]^2$

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Spring 2021 mid:

$$x(t) = Re\{\tilde{x}(t)\, e^{j2\pi f_c t}\} = \sin(2\pi f_c t)$$

$\rightarrow \tilde{x}(t) = j\,u(t)$

(or) $\tilde{x}(t) = e^{-j\frac{\pi}{2}} = -j\,u(t)$

4) $8 \times 2 \times 2 = 32\,kHz$

6) $A\cos(f_1 + f_2)$ $P_{total} = \frac{A^2}{2} = 8W$

$P_{USB} = \frac{P_{total}}{2} = 4W$

5) $107k + 15k = 122k$

7) $m(t) + \cos(2\pi f_c t) + A\,m^2(t) + A\cos^2(2\pi f_c t) + 2A\,m(t)\ldots$

$\rightarrow |2A\,m(t)| < 1$

$\rightarrow |m(t)| < 0.125$

8) $BW_T = 2n_{max} f_m$ & $f_m = 6\,kHz$

& $\beta = 0.5 \rightarrow n_{max} = 2$

$\rightarrow BW = 24\,kHz$

10) $G(f - f_c) \rightleftharpoons g(t)\, e^{j2\pi f_c t}$

$X(f) \rightleftharpoons g(t)\, e^{j4\pi t}$

$\rightarrow \left(\frac{1}{2}\delta(-t) + \frac{-1}{2j\pi t}\right) e^{j4\pi t}$

11) envelope detector or ~~coherent detector~~ Costas

12) $\frac{1}{2\pi \cdot 1000} \frac{d}{dt}\left[\cos(2\pi f_1 t + \sin(2\pi f_2 t))\right]$

$\rightarrow \frac{1}{2\pi 1000} \cdot \left[2\pi f_1 + 2\pi f_2 \cos(2\pi f_2 t)\right]$

$\rightarrow \frac{1}{1000}\left[f_1 + f_2 \cos(2\pi f_2 t)\right]$

$\Rightarrow 112 + 3\cos(2\pi f_2 t)$

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mid spring 2021:

Q1) $\beta = 0.5 \rightarrow P_{carrier} = \frac{A_c^2}{2}\left[J_0(0.5)\right]^2$

but $n = 4 \rightarrow \beta_{out} = 2 \rightarrow P_{carrier} = \frac{A_c^2}{2}\left[J_0(2)\right]^2$

$= \boxed{0.25667}$

Q2) $V_o(t) = \cancel{6\,m(t)} + \underbrace{48\cos(2\pi f_c t)}_{BPF} + 2\left[\cancel{m^2(t)} + \cancel{A_c^2\cos^2(2\pi f_c t)} + \underbrace{2\,m(t)A_c\cos(2\pi f_c t)}_{BPF}\right]$

modulated carrier: $48\cos(2\pi f_c t)$

$\rightarrow P_{carrier} = \frac{(48)^2}{2} = \boxed{1152\ W}$

Q3) $\because x(t) = Re\{\tilde{x}(t)\,e^{j2\pi f_c t}\} = \sin(2\pi f_c t)$

$\rightarrow \boxed{\tilde{x}(t) = -j}$

Q4) $BW_m = 8 \rightarrow BW_{m^2} = 16 \rightarrow BW_{s(t)} = \boxed{32\ kHz}$

Q5) $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\rightarrow s(t) = \cos(2\pi(f_2+f_1)t) \rightarrow \boxed{f = 122\ kHz}$

Q6) $A\cos(2\pi(f_1+f_2)t) \rightarrow P_{total} = A^2/2$

$P_{USB} = \frac{P_{total}}{2} = 4^2/4 = \boxed{4\ W}$

Q7) $V_o(t) = m(t) + \cos(2\pi f_c t) + A\left[m^2(t) + \cos^2(f_c 2\pi t) + 2\,m(t)\cos(f_c 2\pi t)\right]$

$\rightarrow [1 + 2A\,m(t)]\cos(2\pi f_c t)$

$\rightarrow |2A\,m(t)| < 1$ for no phase reversal

$\rightarrow |m(t)| < \frac{1}{2\cdot 4} \rightarrow \boxed{|m(t)| < 0.125}$

Q8) $\because \beta = 0.5 \rightarrow n_{max} = 2$ & $f_m = 6k$

$\rightarrow BW = 2n_{max}\cdot f_m = \boxed{24\ kHz}$

Q10) frequency shift : $G(f-f_0) \rightleftharpoons g(t)\, e^{j2\pi f_0 t}$, $f_0 = 2$

$\wedge$ $U(f) \rightleftharpoons \frac{1}{2}\delta(t) + \frac{-1}{j2\pi t}$

$\rightarrow$ $\boxed{x(t) = \left[\frac{-1}{j2\pi t} + 0.5\delta(t)\right] e^{j4\pi t}}$

Q12) $\because$ $\frac{d\,g(t)}{dt} \rightleftharpoons j2\pi f\, G(f)$ $\rightarrow$ $\frac{jf}{1000} G(f) \rightleftharpoons \frac{1}{2000\pi}\frac{d\,g(t)}{dt}$

$\rightarrow$ output $= \frac{1}{2000\pi}\cdot\left[2\pi f_1 + 2\pi f_2 \cos(2\pi f_2 t)\right]$

note sinusoidal component was removed by envelope detector

$\rightarrow$ output $= 112 + 3\cos(\text{---})$

Q13) $6m(t) + 6A_c\cos(2\pi f_c t) + 4\left[m^2(t) + A_c^2\cos^2(2\pi f_c t) + 2A_c m(t)\cos(2\pi f_c t)\right]$

$\rightarrow$ $V_o(t) = 6A_c\cos(2\pi f_c t) + 8A_c m(t)\cos(2\pi f_c t)$

$\rightarrow$ $V_o(t) = A_c\left[6 + 8m(t)\right]\cos(2\pi f_c t)$

$\therefore$ $V_o(t) = \frac{A_c}{6}\left[1 + \frac{8}{6}m(t)\right]\cos\text{---}$

$k_a = 1.333\ V^{-1}$

Q14) first time shift $\rightarrow$ $1 + e^{-j2\pi f} \rightarrow G(f)$

then integration $\rightarrow$ $\frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$

$\because$ $G(0) = 1 + e^0 = 2$ $\rightarrow$ $\boxed{H(f) = \frac{1}{j2\pi f}\left[1 + e^{-j2\pi f}\right] + \delta(f)}$

Q15) $\because$ $\beta = 0.5$ $\wedge$ $P_{carrier} = \frac{A_c^2}{2}\cdot\left[J_0(\beta)\right]^2$

$\rightarrow$ $\boxed{P_{carrier} = 7.4029\ W}$

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HW5:

Q13) $(SNR)_c = \frac{P_{modulated}}{P_{noise\ in\ message\ BW}} = \frac{4^2/2}{N_0 \cdot 4kHz} = \frac{2\times10^{-3}}{N_0}$

Q14) $2\cdot\frac{N_0}{2}\cdot 200\times10^3 = \cancel{50\times10^6 N_0} \to 2\times10^5 N_0$

Q15) $N_0 \cdot 2B$ & $B = 100k \Rightarrow 2\times10^5 N_0$

Q16) $2B$ & $B = 100k \Rightarrow$ 200k samples/second

Q17) $S_{N_0}(f) = \frac{N_0 f^2}{A_c^2}$ for $|f| \le W$

or 0 e.w

$\to P_{N_0} = \int_{-W}^{W} S_{N_0}(f)\,df = \frac{N_0}{A_c^2}\int_{-W}^{W} f^2\,df$

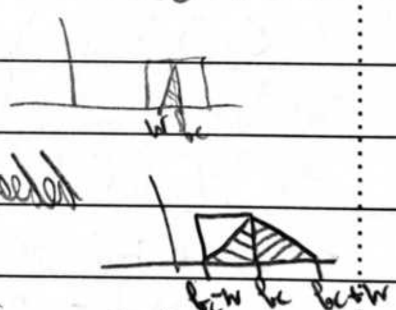
$\to \frac{N_0}{A_c^2}\cdot\frac{1}{3}\left[f^3\right]_{-W}^{W} \Rightarrow \frac{2N_0 W^3}{3A_c^2}$

$\therefore A_c = 4$, $W = 4kHz \to P_{N_0} = 2.6667\, N_0 \times 10^9$ W

Q18) $\therefore$ lower sideband $\to f_c - \frac{W}{2} = f_c - 2k$

Q19) $N_0 \cdot W = 4k\, N_0$

noise bandwidth = W



Q20) $\frac{T}{T_s} < 0.1 \to$ equalizer not needed

$T_s = \frac{1}{8k} = 0.125$ ms $\to T_{max} = 12.5\ \mu s$

Q23) $|q_{max}| = \frac{\Delta}{2}$ & $\Delta = \frac{2m_{max}}{L}$ & $L=4$ & $m_{max}=2$

$\to |q_{max}| = 0.5$

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old final : full AM

Q2-1 : $\because$ $A_c[1 + k_a m(t)]\cos(2\pi f_c t)$

$\rightarrow 2[1 + 0.5\cos(2\pi f_m t)]\cos(2\pi f_c t)$

min: $2[1 + 0.5(-1)]\cos(2\pi f_c t)$

$\rightarrow$ min $= 1$

Q2-2: $\frac{(A_c k_a)^2 1^2}{2} / 2 = 0.25 = \frac{1}{4} = P_{USB} = P_{LSB}$

$P_{SB\,total} = \frac{1}{2}$

Q2-3 : $|k_a m(t)| < 1 \rightarrow |m(t)| < \frac{1}{0.5} (=2)$

Q2-4 : practically VSB are demodulated using envelope detectors

distortion can be reduced by reducing k_a

1- reduce percentage modulation to reduce k_a

2- by increasing the width of the vestigal sideband to reduce $m(t)$

Q3-1

$\because \beta = 0.1 << 1$ rad $\rightarrow J_0(\beta) \simeq 1$, $J_1(\beta) \simeq \frac{0.1}{2}$

& $\beta \leq 0.3$ rad, the effect of residual AM & harmonic distortion is negligible

$\rightarrow$ 1% Rule : $BW = 2 n_{max} \cdot f_m$, $\boxed{J_2(\beta) \simeq 0}$

$\rightarrow n_{max} = 1 \rightarrow BW = 2 f_m$, $f_m = 1k$

$\rightarrow BW = 2kHz$

Q3-2 : $\because \beta_{out} = 1$, & $\Delta f = f_m \cdot \beta$

$\rightarrow \Delta f = 1kHz$

Q3-3 : ~~19k + 1k = 20~~ FM stereo multiplexing

$m(t) = m_l(t) + \cancel{m_r(t)} + [m_l(t) - \cancel{m_r(t)}]\cos(4\pi f_c t)$

$+ \underbrace{k\cos(2\pi f_c t)}_{\text{pilot signal}}$ $\rightarrow f_c = 19k$

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$\rightarrow m(t) = m_1(t) + [m_2(t)]\cos(4\pi f_c t)$

$\rightarrow \max f = f_m + 2f_c = 398 Hz$

Q3-4:

- in FM stereo multiplexing, the low pass filter within the de-multiplexer (in the receiver) is used to pass the sum of the signals picked up by both microphones

Q3-5: $m(t) = \cos(20\pi t)$, $c(t) = A_c\cos(2\pi f_c t)$ & $k_f = 10$

$$s(t) = A_c\cos\left(2\pi f_c t + 2\pi k_f\int_0^t m(t)\,dt\right)$$

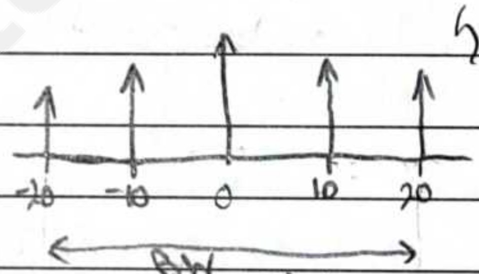
$$\Rightarrow A_c\cos\left(2\pi f_c t + \frac{2\pi k_f}{20\pi}\sin(20\pi t)\right)$$

$\rightarrow \beta_f = 1$, Carson's rule: $BW = 2\Delta f(1+\frac{1}{\beta})$

& $\Delta f = f_m\beta \rightarrow BW = 40 Hz$

$\therefore \Delta f = 10 Hz$

$\hookrightarrow$ deltas

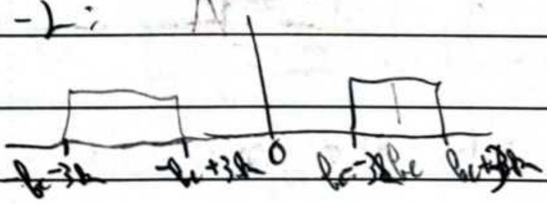


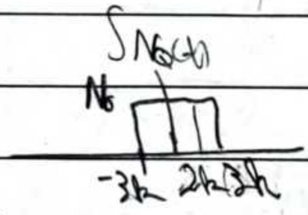
Q4-1: $(SNR)_c = \frac{P_{\text{~~transmitted~~}}}{P_{\text{noise in mBW}}} \rightarrow \frac{2^2/2}{2BW\cdot N_0/2}$

$\therefore BW = f_m$ & $f_m = 1k \rightarrow 2BW = 2k$

$\rightarrow \frac{2}{1000\,N_0}$

Q4-2:





$= N_0$

Q4-3: $2B$ & $B = 3k \rightarrow 6k$ samples/second

Q4-4: $(SNR)_o = \frac{P_{message,out}}{P_{noise}}$, $P_{message,out}$: $(1000)^2 P$

$\rightarrow (SNR)_o = \frac{1\times10^6 P}{P_{noise}}$

$P_{noise} = \frac{N_0}{A_c^2}\int_{-W}^{W} f^2 \, df = \frac{N_0}{3A_c^2}\left[f^3\right]_{-W}^{W}$

$\rightarrow P_{noise} = \frac{2N_0 W^3}{3A_c^2}$, $W = 1k$, $A_c = 2$

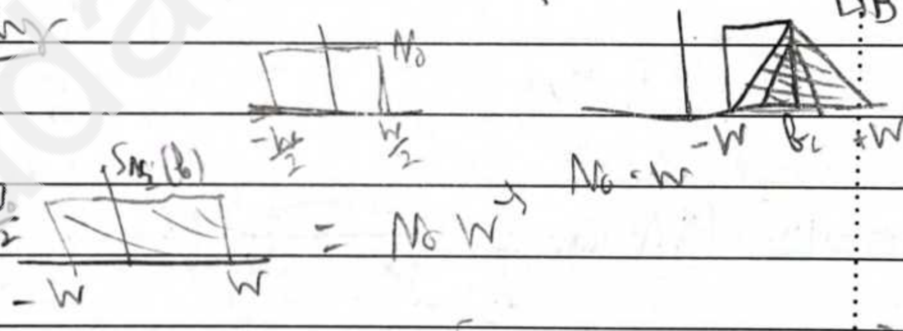
$\rightarrow \frac{N_0 \, 10^9}{6}$ $\rightarrow (SNR)_o = \frac{6P}{N_0 \times 10^3}$

Q4-5:

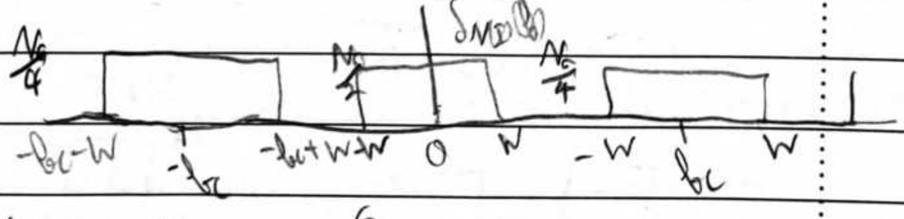
- The pre-emphasis & de-emphasis in FM are used to protect the high frequency components of the message signal.

Q4-6: the power in the noise at the output of the FM receiver (using a frequency discriminator followed by LPF) is due to the quadrature component of filtered noise only

Q4-7



Q4-8: after product modulator: $\frac{1}{2}\left[S_{N_I}(f-f_c) + S_{N_I}(f+f_c)\right]$



at $2f_c + W/2$, $S_{N_I} = 0$ $S_{N_I}(f)$

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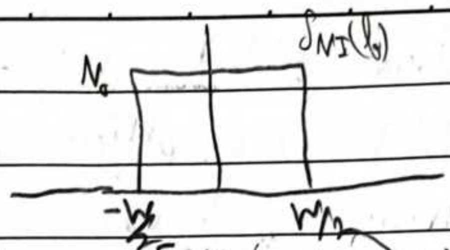
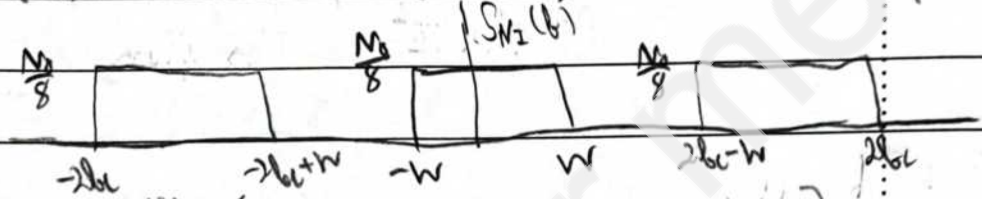
Q4-8: LSB



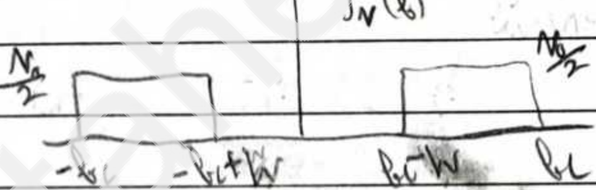
$\rightarrow S_{N_I}(f) \cdot \cos(\pi(2f_c + \frac{W}{2})t) = \frac{1}{8}\left[S_{N_I}(f-(2f_c+\frac{W}{2})) + S_{N_I}(f+2f_c+\frac{W}{2})\right]$

$\frac{1}{2}$ from cosine, $\frac{1}{2}$ from $\frac{(A_c)^2}{2}$ & $\frac{1}{2}$ from in-phase

$\rightarrow$



$\therefore S_{N_I}(f) = \frac{1}{4}\left[S_N(f-f_c) + S_N(f+f_c)\right]$



centered at $f_c - \frac{W}{2}$ to pass lower sideband

- would be centered at $f_c + \frac{W}{2}$ to pass USB

Q4-9: $\int_{-W}^{W} \frac{N_0}{8}\,df = \frac{N_0}{8} \cdot 2W = \frac{N_0 W}{4}$

$\therefore$ only in-phase will be output

quadrature component is suppressed in SSB

Q4-10: figure of merit for SSB & DSB-SC is the same $\rightarrow$ both have the same noise performance

Q5-1: $\frac{T}{T_s} \leq 0.1 \rightarrow T \leq \frac{0.1}{20k} = 5\,\mu s$

~~$\frac{2\mu s}{50k}$~~ $\leq 2\mu s \cdot$ ~~$20k$~~ $= 0.04$

Q5-2: amplitude distortion: $|T \operatorname{sinc}(fT)\, e^{-j\pi fT}|$

$\rightarrow$ amplitude distortion: $2\mu \operatorname{sinc}(0)$ $\therefore$ DC

$f=0$ for DC, $\operatorname{sinc} 0 = 1 \rightarrow$ amplitude distortion $= 2\times10^{-6}$

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Q5-3 : phase distortion : $-\pi f T$ & $f=0 \rightarrow$ phase distortion $=0$

Q5-4 : PPM is more efficient than PWM or PDM in terms of power

Q5-5 : $f_{sampling} = 2W = 8000$ samples / second

$$\therefore M_\delta(f) = \underbrace{\boxed{f_{sampling}}}_{}^{\text{scalar}} \cdot \sum_{n=-\infty}^{\infty} M(f - f_{sampling} \cdot n)$$

$\rightarrow$ scaled by $f_{sampling} = 8000$

Q5-6 :

in PPM, so $|m(t)|_{max} < \frac{T_s}{2}$ is a sufficient condition to avoid overlap between pulses

$\frac{T_s}{2} = \frac{1}{2 \cdot 10k} = 50$ usec

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Q1) $\hat{m}(t) = \sin(2\pi f_m t)$

$\rightarrow m(t)\cos(2\pi f_c t) - \sin(2\pi f_m t)\sin(2\pi f_c t) =$

$\cancel{\frac{1}{2}\cos(2\pi(f_c - f_m)t)} + \frac{1}{2}\cos(2\pi(f_c + f_m)t) = \cos(2\pi(f_c + f_m)t)$

$\cancel{-\frac{1}{2}\cos(2\pi(f_c - f_m)t)} + \frac{1}{2}\cos(2\pi(f_c + f_m)t)$

output : $S(t)\cdot\cos(2\pi(f_c+3)t) = \cos(2\pi(f_c+f_m)t)\cdot\cos(2\pi(f_c+3)t)$

$\rightarrow$ output : $\frac{1}{2}\cos(2\pi(f_m - 3)t) + \cancel{\frac{1}{2}\cos(2\pi(2f_c+f_m+3)t)}$

$\rightarrow f_{output} = f_m - 3 = \boxed{1212\ Hz}$

Q2) FM stereo multiplexing : $m(t) = m_l(t) + m_r(t) + \overbrace{k\cos(2\pi f_c t)}^{pilot}$
$+ [m_l(t) - m_r(t)]\cos(4\pi f_c t)$

$\therefore$ pilot frequency $= 19\ kHz = f_c$

spectrum : $2f_c + f_l$ or $2f_c + f_r = \boxed{49\ kHz}$

Q3) $P_{m,in} = 1.5$, $P_{m,out} = k_f^2 \cdot P_{m,in}$

$\& k_f = 0.5 \rightarrow \boxed{P_{m,out} = 0.375}$

Q4) max envelope $= 7V$, min $= -5\ V$

Q5) $P = \frac{1}{4}\cdot\frac{N_0 W}{2} = N_0 \cdot \frac{12k}{8} = \boxed{1500\ N_0}$

Q6) $\frac{N_0}{2}\cdot 2W_{s(t)} = N_0 \cdot 189k = 1.89\times10^5\ N_0$

Q7) $f_{max} = \frac{\Delta}{2}$ $\& \Delta = \frac{2m_{max}}{L}$, $m_{max} = 4 \& L = 2^4$

$\therefore \Delta = \frac{1}{2}$ $\&$ $\boxed{f_{max} = 0.25}$

Q10) USB → center frequency: $f_c + \frac{W}{2} = 103 + 2.5$

∴ center frequency = $\boxed{105.5\ kHz}$

Q11) power = $\frac{(m_{max})^2}{3\cdot 2^{2R}} = \frac{3^2}{3\cdot 2^{12}} \approx \boxed{7.3\times 10^{-4}}$

Q12) ∵ $S_{N_o}(f) = \frac{N_0 f^2}{A_c^2}$ for $|f|\le W$, 0 o.w.

→ $P_{n_o(t)} = \frac{N_0}{A_c^2}\int_{-W}^{W} f^2\, df = \frac{2N_0 W^3}{3A_c^2}$

$W = 6\,kHz \wedge A_c = 4 \rightarrow \boxed{P_{n_o(t)} = 9\times 10^9 N_0}$

Q13) $12\cos\left(2\pi f_c t + \frac{2\pi k_f\cdot 5}{2\pi f_m}\cdot m(t)\right)$

→ $\beta_f = \frac{95}{6}$

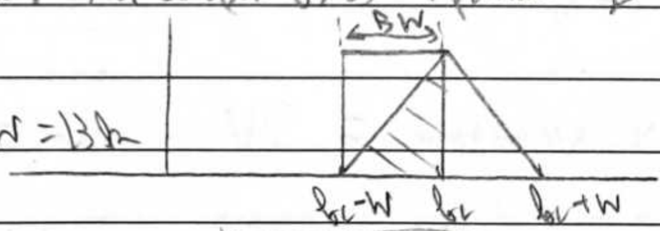
∵ Carson's rule: $2\Delta f\left(1+\frac{1}{\beta_f}\right) \wedge \Delta f = f_m\cdot\beta_f$

→ $BW = 2f_m\left(1+\frac{95}{6}\right) = \boxed{202\ kHz}$

Q14) $\frac{T}{T_s}\le 0.1 \wedge T_s = \frac{1}{18k} \rightarrow T \le 5.555\ \mu sec$

Q16) modulated carrier: $A_1\cos(2\pi f_c t) \rightarrow$ power $= \frac{A_1^2}{2} = \boxed{2W}$

Q17) LSB → BW = W = 13k



Q18) $f_{c,out} = 4 f_{c,in} = \boxed{3392\ kHz}$

Q20) $\frac{N_0}{2}\cdot 2W \wedge W = BW$ of BPF $\rightarrow P = 197k\cdot N_0$

Q21) ∵ $|V(t)| = \frac{k_f}{f_v}|m(t)| \wedge |m(t)|=1,\ f_v = 38k$

∵ $\beta_f = 1 \rightarrow \frac{2\pi k_f}{2\pi f_m} = 1 \rightarrow k_f = f_m$ Hz/V

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$\therefore \quad |V(t)| = \frac{10k}{38k} \cdot 1 = 0.263157...\ V$

Q22) $P_{noise,BPF} = \frac{N_0}{2} \cdot 2W$, $W = 15k\ Hz \rightarrow \boxed{P_{n,BPF} = 15k N_0}$

Q23) $\because \quad m(t) = m_L(t) + m_R(t) + [m_L(t) - m_R(t)] \cos(4\pi f_c t)$
+ pilot

coherent detection frequency $= 2f_c \xrightarrow{\text{detects}} \boxed{[m_L(t) - m_R(t)]}$

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Q1) $\frac{N_0}{2} \cdot 2W = N_0 \cdot 192k$

Q2) $\Delta f = f_m \cdot \beta \wedge \beta = 0.5 \wedge f_m = 4k$
$\Rightarrow \Delta f = 2kHz$

Q3) $P_{s(t)} = \frac{A_c^2}{2} \cdot [J_0(\beta)]^2 \wedge \beta = 0.5$
$\Rightarrow P_{s(t)} = 4.232$

Q4) $\sigma_Q^2 = \frac{m^2_{max}}{3} \cdot 2^{-2R} = \frac{4^2}{3 \cdot 2^{6}} = 0.0833$

Q5) $A\cos(2\pi(f_1+f_2)t) \Rightarrow P_{USB} = \frac{1}{2} \cdot \frac{A^2}{2} = \frac{2.6^2}{4}$

Q6) $\frac{A_t^2}{2} = \frac{2^2}{2}$

Q7) $USB \Rightarrow f_c + \frac{W}{2} = 111 kHz$

Q8) $2N_0W = 3.2e^4 N_0$

Q10) $6\,m(t) + 6A_c\cos(2\pi f_c t)$
$+ 3[m^2(t) + A_c^2\cos^2(2\pi f_c t) + 2A_c m(t)\cos(2\pi f_c t)]$

$\Rightarrow 6A_c\cos(2\pi f_c t) + 6A_c m(t)\cos(2\pi f_c t)$
$\Rightarrow 6A_c[1 + m(t)]\cos(2\pi f_c t)$

Q11) $\frac{T}{T_s} \leq 0.1 \Rightarrow T \leq 3.8462\ ms$

Q12) $T = \frac{T_s}{2} = \frac{1}{2f_s} = 8.0645\ ms \quad delay = \frac{T}{2} = 4\ ms$

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Q13) $N_0 W = 20k\, \cancel{N_0}$

Q14) $5\, m(t) + 5 A_c \cos(2\pi f_c t) + 2\left[m^2(t) + A_c^2 \cos^2(2\pi f_c t) + 2 m(t) A_c \cos(2\pi f_c t)\right]$

$\rightarrow 5 A_c \cos(2\pi f_c t) + 4\, m(t) A_c \cos(2\pi f_c t)$

$\rightarrow 5 A_c \left[1 + \frac{4}{5} m(t)\right] \cos(2\pi f_c t)$

$\rightarrow P_{carrier} = \frac{(5 A_c)^2}{2} = 1012.5$

Q16) power in output noise: $\frac{W N_0}{2} = 7.5k\, N_0$

Q17) $\frac{f_k}{1000} \rightleftharpoons \frac{1}{1000 \cdot 2\pi} \frac{d}{dt} y(t)$

$\rightarrow \frac{1}{2000\pi}\left[2\pi f_{c1} + 2\pi f_\Delta \cos(2\pi f_2 t)\right]$ ~~Discrim~~ envelope detector

$\rightarrow \frac{1}{\cancel{2000\pi}}\left[\cancel{2\pi} \cdot 113k + \cancel{2\pi} \cdot 10k \cos(2\pi f_2 t)\right]$

$\rightarrow \boxed{113 + 10 \cos(2\pi f_2 t)}$

Q18) $|q|_{max} = \frac{\Delta}{2}$, $\Delta = \frac{2\, m_{max}}{2^n}$

$\rightarrow |q|_{max} = 0.0046875$

Q19) $S_{N_0}(f) = \frac{N_0 f^2}{A_c^2}$ for $|f| \leq W$, 0 o.w

$\rightarrow$ power $= \frac{N_0}{A_c^2} \int_{-W}^{W} f^2\, df = \frac{2 N_0 W^3}{3 A_c^2}$

$\rightarrow \frac{2 N_0 \cdot (7000)^3}{3 \cdot 4^2} = 1.43 e^{10}\, N_0$

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Q2φ) input of demodulator = output of BPF

$P' = \frac{A_c^2 \cdot P_m}{2} = 85.05 \ W$

Q24) $m(t) = m_L(t) + m_R(t) + [m_L(t) - m_R(t)] \cos(4\pi f_c t)$

+ pilot & $f_c = 19k$

extends to $2 \cdot 19k + 20k = 58 \ kHz$