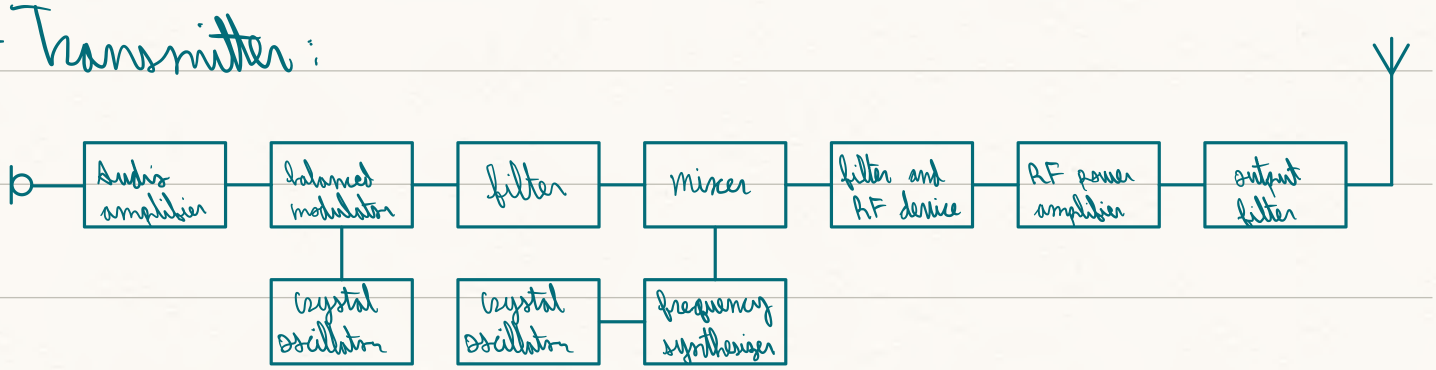
Chapter 1

* RF (communication) electronics: branch of electronics that deals with the study and design of devices, circuits, and systems operating in the radio frequency band

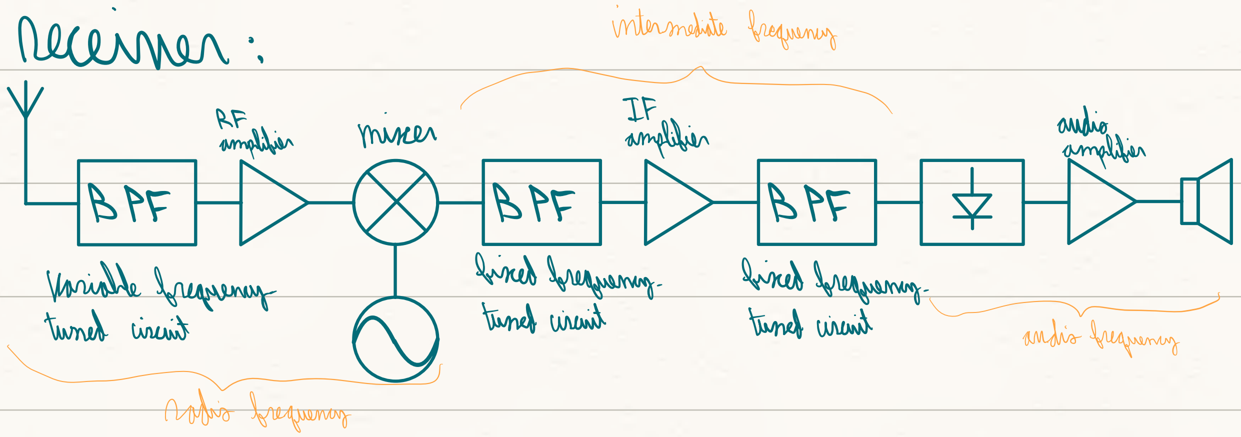
*RF bands: electromagnetic signals that cover a specific wide range of frequenci
these bands are allocated by the ITU

* Communication system components:

\+ Transmitter:



\+ Receiver:



- at high frequencies, stray capacitances and inductances begin having noticable effects and altering the impedances, thereby changing the matching conditions.
- wires and board tracks begin having large resistances at high frequencies due to the skin effect. additionally, they begin radiating electromagnetic waves as the wires' electrical length becom short relative to the wavelength.
- finite wavelength effects produce propagation delays and phase shifts, which may result in destructive interference between signals from different paths.
- since low-power signal are received, the noise generated at the receiver must be taken into account, as it determines the minimum power the transmitter should transmit.

Chapter 2

- noise cannot be removed from the received signal, but it can be minimized by using low-noise devices and proper filters
- another problem at the receiver is intermodulation distortion, which is caused by nonlinear devices.

* noise: random unwanted electrical signal that is added (AWGN) to the desired signal.

\+ noise is generated by:

- internal sources: components within the system (resistors, diodes, transistors, etc.)
- external sources: sun, stars, motors, etc.

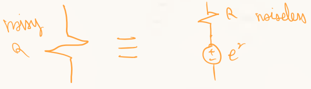
- noise is defined by its statistical properties such as mean, pdf, etc.
- total average noise delivered to a load: $P = \int_{f_k}^{f_n} \underbrace{P(f)}_{\text{spectral density function (W/Hz)}} \, df$

\+ types of internal noise:

- thermal noise
- shot noise
- flicker noise

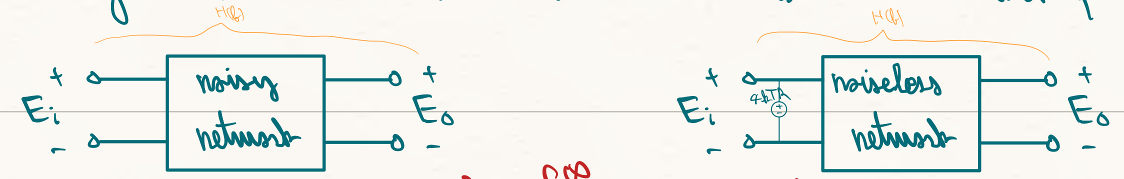
* Thermal noise:

- generated in resistors, ideal capacitors and conductors do not generate
- mean-square spectral density: $e^2 = 4\underbrace{k}_{\text{boltzmann constant}}\overbrace{T}^{\text{temperature (K)}}\overbrace{R}^{\text{resistance}}$ V^2/Hz or W/Hz
- noisy resistor is equivalent to noiseless resistance plus noisy voltage source



- total mean-square power : $E^2 = \int_0^\infty 4kT\,R(f)\,df$ V² or W

- noisy network can be connected to a noiseless network plus noise source



$$E_o^2 = \int_0^\infty 4kTR\,|H(f)|^2\,df$$

- all the noise power is given by R :

$$E_o^2 = 4kTR\cdot\overbrace{B_n}^{\text{noise bandwidth}} \quad \because \int_0^\infty |H(f)|^2\,df = B_n$$

* shot noise : due to random fluctuations in current passing pn junctions

- power spectral density : $i_n^2 = 2\overbrace{q}^{\text{electron charge}}I_o$ A²/Hz

- total mean-squared shot noise : $I_n^2 = 2qI_o\overbrace{B}^{\text{bandwidth}}$ A²

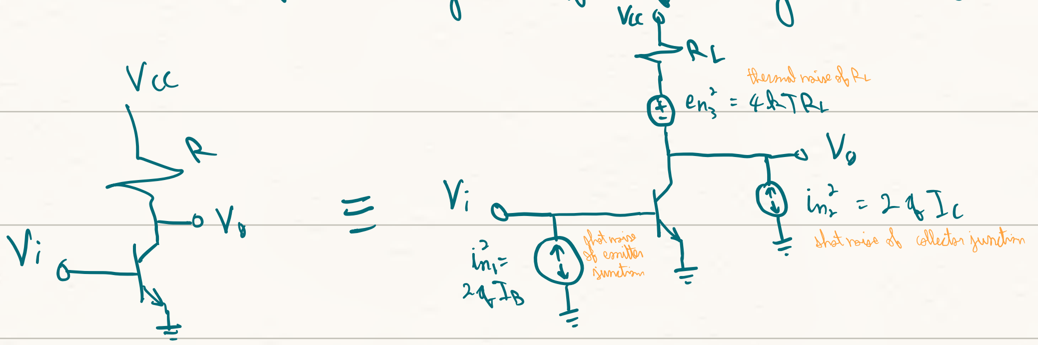
- thermal and shot noise are called "white" since their power spectral densities are independent of frequency

- thermal noise is additive since it is independent of voltage whereas shot noise is multiplicative.

* flicker noise : low frequency phenomenon where the power spectral density is inversely proportional to frequency

- in a transistor amplifier, noise is generated by the resistor (thermal) and pn junction (shot)

- total noise power can be found by connecting all noises to voltages and referring them to the amplifier's input.



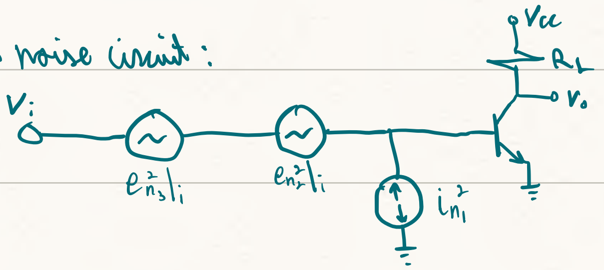
- equivalent thermal noise at input: $e_{n_3}^2\big|_i = \frac{4kTR_L}{A_V^2}$ $\xrightarrow{}$ voltage Amplification

- equivalent collector shot noise at input: $\underbrace{e_{n_2}^2}_{\text{voltage}}\big|_i = \frac{i_{n_2}^2 \cdot R_L}{A_V^2}$ → converted to voltage

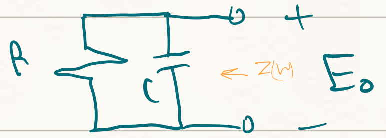
$\because A_V = g_m R_L$ (for common emitter)

$\therefore e_{n_3}^2\big|_i = \frac{4kT}{g_m^2 R_L}$ ∧ $e_{n_2}^2\big|_i = \frac{2qI_c}{g_m^2 \cdot R_L}$

equivalent noise circuit:



- Example 3.1:



noise is generated by real part of impedance

$$Z(\omega) = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{j\omega RC + 1} \times \frac{1 - j\omega RC}{1 - j\omega RC}$$

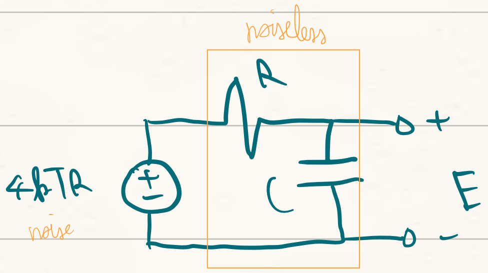
$$\to Z(\omega) = \boxed{\frac{R}{1 + \omega^2 R^2 C^2}} - \frac{j\omega RC}{1 + \omega^2 R^2 C^2}$$

∴ $\overline{E_o^2} = \int_0^\infty 4kT\,R(f)\,df = \int_0^\infty 4kT \cdot \frac{R}{1+4\pi^2 R^2 C^2 f^2}\,df$

→ $\overline{E_o^2} = \frac{kT}{C}$

Or (second approach)

∴ $E_o = V_C$



→ Voltage division: $E_o = 4kTR \cdot \frac{1/j\omega C}{R + 1/j\omega C}$

∴ $H(f) = \frac{1}{1+j\omega RC}$

& $G(f) = H(f) \cdot H^*(f) = \frac{1}{1+j\omega RC} \cdot \frac{1}{1-j\omega RC}$

→ $\frac{1}{1+\omega^2 C^2 R^2}$

→ $\overline{E_o^2} = 4kTR \int_0^\infty \frac{1}{1+4\pi^2 f^2 C^2 R^2}\,df = \frac{kT}{C}$

- the noise power was found independent of R since it is dependent on the noise bandwidth, which is inversely proportional to R, as follows:

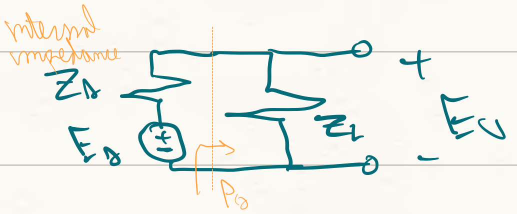
$$B_n = \int_0^\infty G(f)\,df = \frac{kT}{C} \cdot \frac{1}{4kTR} = \frac{1}{4RC}$$

Therefore, increasing R will increase the noise's power spectral density but decrease its bandwidth.

- The 3-dB bandwidth can be found from the frequency at which the transfer function squared (i.e., $G(f)$) is equal to $\frac{1}{2}$

→ $f_{3\text{-dB}}: \frac{1}{1+\omega^2 R^2 C^2} = \frac{1}{2}$ → $4\pi^2 f^2 R^2 C^2 = 1$

* Available power: maximum power that can be delivered to a load



- for max. power transfer: $R_s = R_L$

$\rightarrow Z_L = Z_s^*$ (matching)

if matched → voltage is evenly divided (i.e., $E_o = \frac{E_s}{2}$)

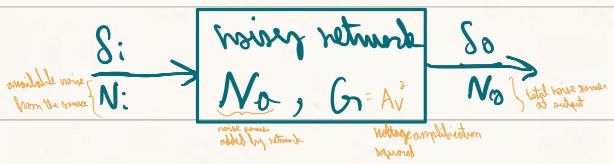
$\therefore P_s = \frac{(E_s/2)^2}{R_L} = \frac{E_s}{4R_L} = \frac{E_s}{4R_s}$ only valid for matched

- noise factor is: $F = \frac{\text{available output noise power}}{\text{available output noise power due to source}}$

$1 \le F < \infty \rightarrow 0 \le NF < \infty$

- noise figure: noise factor in dB → $NF = 10 \log_{10}(F)$

$$F = \frac{N_o}{\underbrace{(N_i)_o}_{\text{referred to output}}} = \frac{\overbrace{(N_o)_i}^{\text{referred to input}}}{N_i} = \frac{N_i + \overbrace{N_a}^{\text{noise added by network is normally referred to the input}}}{N_i}$$



- in terms of input and output SNR:

$$\therefore F = \frac{(N_o)_i}{N_i} \cdot \frac{S_o}{S_o} \quad \& \quad S_o = G S_i \rightarrow F = \frac{\overset{N_o/G}{(N_o)_i}\, G S_i}{N_i S_o} \quad \therefore F = \frac{S_i/N_i}{S_o/N_o}$$

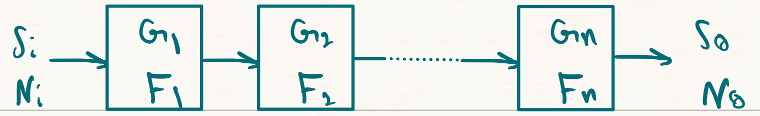
- noise factor is always equal to (noiseless network) or greater than one. $\frac{(S/N)_i}{(S/N)_o} \ge 1 \quad (S/N)_i \ge (S/N)_o$

- the maximum noise power (per Hz) available from a source:

$$N_i = \frac{\cancel{4} k T \cancel{R_s}}{\boxed{\cancel{4R_s}}_{\text{matched}}} = kT \qquad \therefore F = 1 + \frac{N_a}{N_i} = 1 + \frac{N_a}{kT}$$

- Therefore, the noise added by the network is: $N_a = (F-1)kT$

+ for cascaded networks :



- total power gain is the product of all gains : $G = G_1 \cdot G_2 \cdots G_n \rightarrow S_o = GS_i$

- total noise factor is : $F = \dfrac{N_i + \left. N_a \right\} \text{noise added by all stages}}{N_i}$

↳ N_a : noise added by each stage referred to the input of the network.

$$\rightarrow F = \frac{N_i + (F_1-1)N_i + (F_2-1)\frac{N_i}{G_1} + (F_3-1)\frac{N_i}{G_1G_2} + \cdots + (F_n-1)\frac{N_i}{G_1 \cdot G_2 \cdots G_{n-1}}}{N_i}$$

$$\therefore F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1 \cdot G_2} + \cdots + \frac{F_n-1}{G_1 \cdot G_2 \cdots G_{n-1}}$$

- since the denominator keeps getting larger, then the contribution of each stage keeps getting smaller. Therefore, the first stage gives the largest contribution, implying that F_1 is the most important noise factor to minimize. Thus, low-noise amplifiers (LNA) are usually used at the input.

+ noise Temperature : often used to provide higher accuracy wider range and less rounding than noise power per Hz

$$\therefore F = 1 + \frac{N_a}{N_i} = 1 + \frac{kT_n}{kT} \rightarrow F = 1 + \frac{T_n}{\underbrace{T}_{\text{room temperature}}}$$

$$\therefore T_n = (F-1) \cdot T$$

✱ receiver sensitivity (noise floor N_f) : the required available input signal power to achieve a certain output SNR

$$\therefore \left(\frac{S}{N}\right)_i = F \cdot \left(\frac{S}{N}\right)_o \rightarrow S_i = FN_i\,(S/N)_o$$

$$\therefore \underset{= \text{ noise floor } N_f}{\overset{\text{sensitivity}}{(S_i)_{min}}} = FkTB \underbrace{\left(\frac{S}{N}\right)_o}_{\text{specified}}$$

- If the input S_i is less than the sensitivity, the output will not satisfy the performance measure (noisy, distorted, etc.)
- the minimum detectable signal is the sensitivity in V

for matched: $S_i = \frac{E_i^2}{4R_s} \rightarrow E_i = \sqrt{4R_s S_i}$

- example 3.3:

$G = 12 + 10 = 22$ dB, $F_1 = 10^{0.2}$, $F_2 = 10^{0.6}$

$\rightarrow F = 10^{0.2} + \frac{10^{0.6}-1}{10^{1.2}} = 1.973$

- example 3.4:

∵ $(N_o)_i = N_i + N_a$ & $N_a = (F-1)\cdot N_i$

$\rightarrow (N_o)_i = FkTB \rightarrow N_o = FkTB \cdot G = FkTB \cdot G_1 \cdot G_2$

∴ $N_o = 3.39 \times 10^{-15}$ W

- example 3.5:

$F = [1, 1.6] \rightarrow T_e = [0, 174]$ K

∵ $T_e = (F-1)T$

- example 3.6: ∵ NF = 8 dB, $R_s = 50\ \Omega$, $(SNR)_o = 0$ dB, B = 2.1 kHz

∵ $S_i = FkTB \cdot \left(\frac{S}{N}\right)_o = NF + (kTB)_{dB} + (SNR)_{o,dB} = -161.7$ dBW

∴ $S_i = 6.3 \times 10^{-17}$ W $\rightarrow E_i = 0.1\ \mu V$

- example 3.7:

If $(SNR)_o = 10$ dB $\rightarrow S_i = 152.7$ dBW $= 5.37 \times 10^{-16}$ W

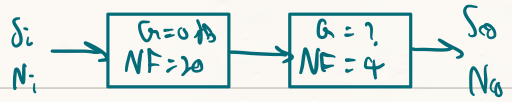
$\rightarrow E_i = 0.327\ \mu V$

- example 3.8: $\therefore$ $NF = 4\,dB$, $R_s = 50\Omega$, $B = 3\,kHz$, $(SNR)_o = 10\,dB$

a) $S_i = 4 + (kTB)_{dB} + 10 = -155.2\ dBW$

$$E_i = \sqrt{R_s \cdot 4 \cdot 10^{\frac{-155.2}{10}}} = 2.45 \times 10^{-8}\ V$$

b) antenna noise figure = 20 dB :



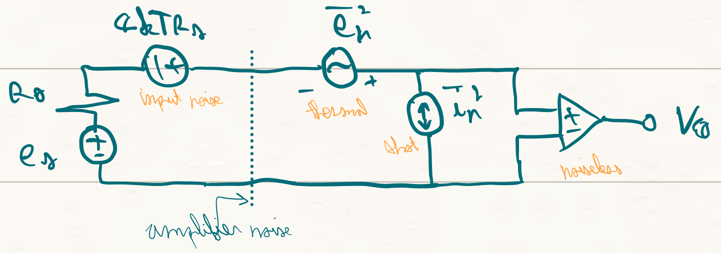
$\therefore$ $G = 0 + ?$ $\wedge$ $NF_{total} = \left(F_1 + \frac{F_2 - 1}{G_1}\right)_{dB} = 20.06$

$\therefore$ $S_i = NF + (kTB)_{dB} + (SNR)_o = -139\ dBW$

$\rightarrow$ $E_i = 1.56\ \mu V$

* design of low noise networks :

- any linear noisy amplifier can be represented as a noiseless amplifier in addition to two noise sources at its input (thermal and shot)



$\rightarrow$ $N_i = 4kTR_s$, $N_a = \overline{e}_n^2 + \overline{i}_n^2 R_s^{\boxed{2}}$ power

$$\wedge \quad F = \frac{N_i + N_a}{N_i} = \frac{4kTR_s + \overline{e}_n^2 + \overline{i}_n R_s^2}{4kTR_s}$$

- since R_s is the only variable, it must be reduced to reduce the noise.

- to find the minima, differentiate with respect to the variable

$$\rightarrow \frac{dF}{dR_s} = \frac{1}{4kT}\left[\overline{i_n^2} - \frac{\overline{e_n^2}}{R_s^2}\right] = 0$$

$$\rightarrow R_s^2 = \frac{\overline{e_n^2}}{\overline{i_n^2}} \qquad \therefore \overset{\text{optimum}}{R_s} = \frac{\overline{e_n}}{\overline{i_n}}$$

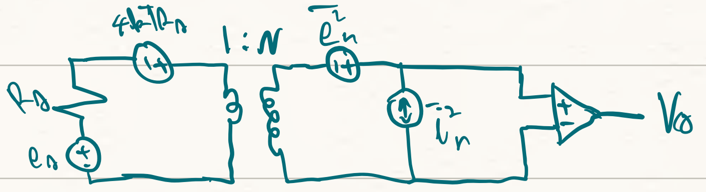
- minimizing the noise factor does not necessarily maximize the output SNR because of mismatching.

- since all noises are referred to the input, the output SNR can be found as follows: $\left(\frac{S}{N}\right)_o = \frac{e_s^2}{4kTR_s + \overline{e_n^2} + \overline{i_n^2}\cdot R_s^2}$

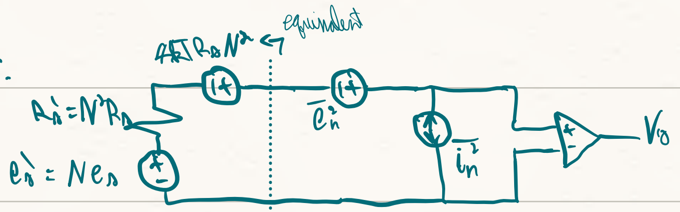
- hence the maximum SNR is when R_s is equal to zero

- Therefore, there is a conflict between minimizing SNR and maximizing it.

- This conflict is resolved by adding a transformer between the source and noisy network to isolate R_s



- taking the equivalent circuit of the above transformer referred to the N turns side:



$$\rightarrow (SNR)_o = \frac{N^2 e_s^2}{4kTR_sN^2 + \overline{e_n^2} + i_n^2\cdot \underbrace{N^4\cdot R_s^2}_{(R_s')^2}}$$

- the only variable in the above equation is N, assuming R_s is fixed for matching. Thus, the maximum SNR is found from:

$$\frac{d(SNR)_o}{dN} = 0 \rightarrow R_s' = N^2 R_s = \frac{e_n}{i_n} \quad \therefore \overset{\text{optimum}}{N} = \sqrt{\frac{e_n}{i_n} \cdot \frac{1}{R_s}}$$

- adding this transformer therefore maximizes the $(SNR)_o$ and minimizes the noise factor.

* intermodulation distortion:

- since some sections of a receiver may operate in the saturation region then the receiver is not entirely linear.

- the non-linearity can be described by the following taylor series expansion:

$$y(x) = \underbrace{\overset{\text{voltage gain}}{k_1} f(x)}_{\text{linear}} + \underbrace{k_2 f^2(x) + k_3 f^3(x)}_{\text{non-linear}}$$

- if two adjacent signals enter the receiver, then:

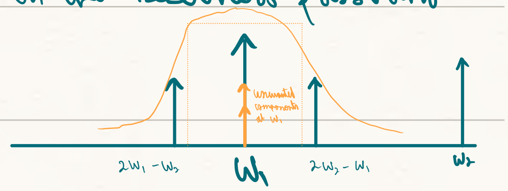
$$f(x) = A_1 \cos(w_1 t) + A_2 \cos(w_2 t)$$

$$\rightarrow y(t) = k_1 [A_1 \cos(w_1 t) + A_2 \cos(w_2 t)] + k_2 [A_1 \cos(w_1 t) + A_2 \cos(w_2 t)]^2 + k_3 [A_1 \cos(w_1 t) + A_2 \cos(w_2 t)]^3$$

$$\rightarrow y(t) = k_1 A_1 \cos(w_1 t) + k_1 A_2 \cos(w_2 t) + k_2 [A_1^2 \overbrace{\cos^2(w_1 t)}^{\frac{1+\cos(2\theta)}{2} = \cos^2(\theta)} + 2A_1 A_2 \cos(w_1 t) \cdot \cos(w_2 t) + A_2^2 \cos^2(w_2 t)] + k_3 [A_1 \cos(w_1 t) + A_2 \cos(w_2 t)] \cdot [A^2 \cos^2(w_1 t) + 2A_1 A_2 \cos(w_1 t) \cos(w_2 t) + A_2^2 \cos^2(w_2 t)]$$

$$\rightarrow \quad y(t) = k_1 A_1 \cos(\omega_1 t) + k_1 A_2 \cos(\omega_2 t) + k_2\Big[\frac{A_1^2}{2} + \frac{A_1^2}{2}\cos(2\omega_1 t) + A_1A_2\big[\cos((\omega_1-\omega_2)t) + \cos((\omega_1+\omega_2)t)\big] + \frac{A_2^2}{2} + \frac{A_2^2}{2}\cos(2\omega_2 t)\Big] + k_3\Big[\frac{A_1^2}{2} + \frac{A_1^2}{2}\cos(2\omega_1 t) + A_1A_2\big[\cos((\omega_1-\omega_2)t) + \cos((\omega_1+\omega_2)t)\big] + \frac{A_2^2}{2} + \frac{A_2^2}{2}\cos(2\omega_2 t)\Big]\cdot\big[A_1\cos(\omega_1 t) + A_2\cos(\omega_2 t)\big]$$

- this gives many frequency components with various amplitudes, but the most dangerous are the ones closest in frequency to our desired signal, as they may be included in the receivers passband.



+ gain compression:

- by expanding the k_3 term, some components at ω_1 can be found:

$$k_3\Big[\frac{A_1^2}{2} + \frac{A_1^2}{2}\cos(2\omega_1 t) + A_1A_2\big[\cos((\omega_1-\omega_2)t) + \cos((\omega_1+\omega_2)t)\big] + \frac{A_2^2}{2} + \frac{A_2^2}{2}\cos(2\omega_2 t)\Big]$$

$$\cdot\big[A_1\cos(\omega_1 t) + A_2\cos(\omega_2 t)\big]$$

$$\rightarrow k_3\Big[\underline{\frac{A_1^3}{2}\cos(\omega_1 t)} + \overset{\frac{A_1^3}{4}\cos(\omega_1 t)}{\underline{\frac{A_1^3}{2}\cos(2\omega_1 t)\cos(\omega_1 t)}} + A_1^2A_2\big[\cos((\omega_1-\omega_2)t)^{\cdot\cos(\omega_1 t)} + \cos((\omega_1+\omega_2)t)\big]^{\cdot\cos(\omega_1 t)}$$

$$+ \underline{\frac{A_1A_2^2}{2}\cos(\omega_1 t)} + \frac{A_1A_2^2}{2}\cos(2\omega_2 t)\cos(\omega_1 t)\Big] + k_3\Big[\frac{A_1^2A_2}{2}\cos(\omega_2 t) +$$

$$\frac{A_1^2A_2}{2}\cos(2\omega_1 t)\cos(\omega_2 t) + A_1A_2^2\Big[\overset{\frac{1}{2}\cos(\omega_1 t)}{\underline{\cos((\omega_1-\omega_2)t)\cos(\omega_2 t)}} + \overset{\frac{1}{2}\cos(\omega_1 t)}{\underline{\cos((\omega_1+\omega_2)t)\cdot}}$$

$$\underline{\cos(\omega_2 t)}\Big] + \frac{A_2^3}{2}\cos(\omega_2 t) + \frac{A_2^3}{2}\cos(2\omega_2 t)\cos(\omega_2 t)\Big]$$

$$\rightarrow k_3\Big[\frac{A_1^3}{2}\cos(\omega_1 t) + \frac{A_1^3}{4}\cos(\omega_1 t) + \frac{A_1A_2^2}{2}\cos(\omega_1 t) + A_1A_2^2\cos(\omega_1 t)\Big]$$

$$\therefore \omega_1 \text{ components}: \quad k_3\Big[\frac{3}{4}A^3\cos(\omega_1 t) + \frac{3}{2}A_1A_2^2\cos(\omega_1 t)\Big]$$

- these components will be added to the signal at ω_1 and distort it:

$$A_1' \cos(\omega_1 t) = \overbrace{k_1 A_1 \cos(\omega_1 t)}^{\text{desired}} + \overbrace{k_3\left(\frac{3}{4}A_1^3 + \frac{3}{2}A_1A_2^2\right)\cos(\omega_1 t)}^{\text{undesired}}$$

- k_3 is usually negative, thus it will weaken the received signal
- if k_3 is positive it will distort the signal.
- if the adjacent signals amplitude is strong, the desired signal may be lost, especially since the undesired signal level is proportional with the square of the adjacent signal's amplitude.

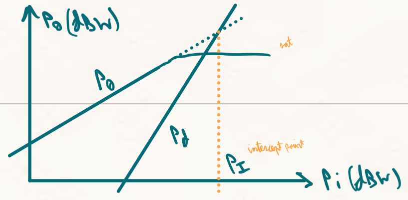
+ single tone compression:
  - occurs when the adjacent signal is negligible and the desired signal compresses itself.

    ratio of received signal to desired signal

    gain compression factor: $\frac{A_1'}{k_1 A_1} = \frac{k_1 \cancel{A_1} + \frac{3}{4} k_3 A_1^{\cancel{3}\,2}}{k_1 \cancel{A_1}}$
  - the signal can therefore cancel itself completely if $k_1 = -\frac{3}{4} k_3 A_1^2$

+ intermodulation distortion (IMD)
  - if ω_1 and ω_2 are adjacent, then either one (only one) of the intermodulation components $(2\omega_1 - \omega_2)$ or $(2\omega_2 - \omega_1)$ may be within the receivers passband of ω_1.

    $$y(t) = \overbrace{k_1 A_1 \cos(\omega_1 t)}^{\text{desired signal}} + \overbrace{\frac{3}{4} k_3 A_1^2 A_2 \cos((2\omega_1 - \omega_2)t)}^{\text{undesired intermodulation distortion component}}$$
  - intermodulation distortion ratio: $\overset{\text{undesired / desired}}{IMR} = \frac{\frac{3}{4} k_3 A_1^2 A_2}{k_1 A_1} = \frac{3 k_3 A_1 A_2}{4 k_1}$



+ intercept point:

power

- the value of input signal power that gives an output power equal to the IMD

$$\therefore \overset{\text{IMD power}}{P_d} = \frac{(3/4\, k_3 A_1^2 A_2)^2}{2}, \quad \text{if } A_1 \approx A_2 \rightarrow P_d = \frac{(3/4\, k_3 A_1^3)^2}{2}$$

$$\& \ \overset{\text{input power}}{P_i} = \frac{A_1^2}{2} \longrightarrow P_d = (k_d P_i)^3 \rightarrow P_d \propto P_i^3$$

$$\rightarrow P_{IMR} = \frac{P_d}{P_o} \quad \& \quad P_o = \frac{(k_1 A_1)^2}{2} \rightarrow P_o = k_1^2 P_i$$

$$\therefore P_{IMR} = (\overset{\text{IMR constant}}{k_i} P_i)^2 \quad \text{s.t.,} \ k_i = \frac{k_d^3}{k_1^2}$$

- to find the intercept point: $P_d = P_o \rightarrow k_i P_i^2 = 1$ & $P_i = P_I$

$$\therefore P_I = \frac{1}{k_i} \longrightarrow P_{IMR} = \left(\frac{P_i}{P_I}\right)^2$$

- if k_3 is 0, then the intercept point P_I is ∞.

+ dynamic range:

in dB — input — noise floor

- the range from the minimum signal level (sensitivity) to the maximum input signal level.

- the maximum input signal level depends on distortion, rather than noise.

- the maximum input signal is the input signal at which the intermodulation distortion, referred to the input, is equal to the sensitivity.

$$(S_i)_{max} \ @ \ (P_d)_i = (S_i)_{min}$$

$$\therefore P_{IMR} = \frac{P_d}{P_o} = \frac{P_d}{P_i \cdot k_1^2} = \frac{P_d/k_1^2}{P_i} = \frac{(P_d)_i}{P_i} = \left(\frac{P_i}{P_I}\right)^2$$

$$\rightarrow \frac{\overset{\text{noise floor}}{N_f}}{(S_i)_{max}} = \left(\frac{(S_i)_{max}}{P_I}\right)^2 \longrightarrow (S_i)_{max} = \sqrt[3]{P_I^2 N_f}$$

∴ dynamic range (DR): $\frac{(S_i)_{max}^{\text{W or mW}}}{(S_i)_{min} = N_f} = \left(\frac{P_I}{N_f}\right)^{2/3}$ W or mW

- in normal power units, the dynamic range is a ratio, whereas in dB it is $(S_i)_{max}\,\text{dBW} - (S_i)_{min}\,\text{dBW} = \frac{2}{3}[P_I - N_f]$ dB

- example 3.10: ∴ $R_s = 10k$, $\overline{e_n^2} = 8\times10^{-16}$ V^2/Hz, $\overline{i_n^2} = 4\times10^{-25}$ A^2/Hz

$$\rightarrow F = \frac{4kTR_s + \overline{e_n^2} + \overline{i_n^2}R_s^2}{4kTR_s} = 6.56$$

- example 3.11: ∴ $(R_s)_{opt} = \sqrt{\frac{\overline{e_n^2}}{\overline{i_n^2}}} = 29.8\ k\Omega$

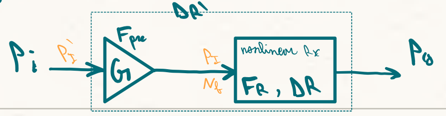
$$\rightarrow F = 4.35$$

- example 3.14: ∴ $P_I = 20$ dBm & $N_f = -123$ dBm

$$\rightarrow DR = \frac{2}{3}[143] = 95.3\ dB$$

+ if a linear amplifier is connected before the non-linear receiver:

∴ $DR = \left(\frac{P_I}{N_f}\right)^{2/3}$



for overall system, $DR' = \left(\frac{P_I'}{N_f'}\right)^{2/3}$ & $P_I' = \frac{P_I}{G}$

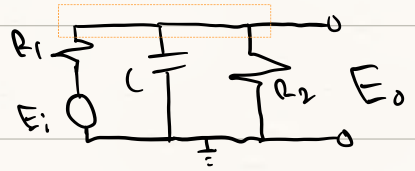
∴ $F_{total} = F_{pre} + \frac{F_R - 1}{G}$ $\rightarrow$ $N_f' = F_{total}\,\overbrace{kTB\left(\frac{S}{N}\right)_o}^{\text{do not change}}$

$$\rightarrow DR' = \left(\frac{P_I}{G\,N_f'}\right)^{2/3}$$

MOHAMMAD SANAD ALTAHER 130806

EE 524: homework #1

3.2:



applying KCL: $\frac{E_o}{R_2} + E_o \cdot j\omega C = \frac{E_i - E_o}{R_1}$

$$\rightarrow E_o\left[\frac{1}{R_2} + j\omega C + \frac{1}{R_1}\right] = E_i \cdot \frac{1}{R_1}$$

$$\therefore H(\omega) = \frac{E_o}{E_i} = \frac{1}{\frac{R_1}{R_2} + j\omega C R_1 + 1} = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C}$$

$$\rightarrow |H(\omega)|^2 = H(\omega) \cdot H^*(\omega) = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C} \cdot \frac{R_2}{R_1 + R_2 - j\omega R_1 R_2 C}$$

$$\rightarrow |H(\omega)|^2 = \frac{R_2^2}{(R_1 + R_2)^2 + \omega^2 R_1^2 R_2^2 C^2}$$

$$B_n = \int_0^\infty |H(\omega)|^2 \, df = \boxed{\frac{1}{4 R_1 R_2 (R_1 + R_2) C}} \quad X \quad \boxed{\frac{R_1 + R_2}{4 R_1 R_2 C}}$$

& $f_{3\text{-}dB}$ at $|H(\omega_{3\text{-}dB})|^2 = \frac{1}{2} \max\left[|H(\omega)|^2\right]$

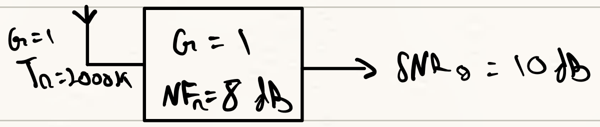
$\because |H(\omega)|^2$ max at $\omega = 0$ $= \left(\frac{R_2}{R_1 + R_2}\right)^2$

$$\rightarrow |H(\omega_{3\text{-}dB})| = \frac{\cancel{R_2^2}}{(R_1 + R_2)^2 + \omega_{3dB}^2 R_1^2 R_2^2 C^2} = \frac{1}{2}\left(\frac{R_2}{R_1 + R_2}\right)^2$$

$$\rightarrow 4\pi^2 f_{3\text{-}dB}^2 \cdot R_1^2 \cdot R_2^2 \cdot C^2 = (R_1 + R_2)^2$$

$$\therefore \boxed{f_{3\text{-}dB} = \frac{R_1 + R_2}{2\pi R_1 R_2 C}}$$

3.7: a)

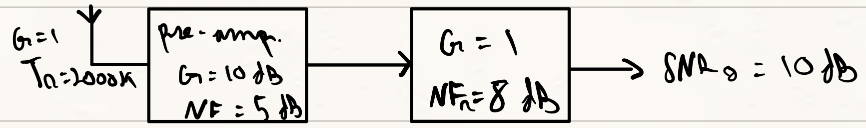


$\because T_n = 2000$ K $\rightarrow F = 7.896$ & $F_n = 10^{0.8}$

$$\rightarrow F_{total} = 7.896 + \frac{10^{0.8}-1}{1} \doteq 13.2$$

$$\therefore (S_i)_{min} \doteq F_{total}\, kTB \left(\frac{S}{N}\right)_o = \boxed{1.585 \times 10^{-15} \text{ W}}$$

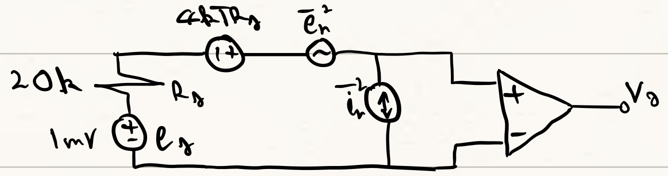
h)



$$F_{total} = 7.896 + \frac{10^{0.5}-1}{1} + \frac{10^{0.8}-1}{10} = 10.59$$

$$\rightarrow (S_i)_{min} = F_{total}\, kTB_{receiver}\left(\frac{S}{N}\right)_o \doteq \boxed{1.271 \times 10^{-15} \text{ W}}$$

3.11: a)



$$\left(\frac{S}{N}\right)_o = \frac{e_s^2}{4kTR_s + \overline{e_n^2} + \overline{i_n^2} \cdot R_s^2}$$

$\because f_c = 1kHz \rightarrow \overline{e_n^2} = 8 \times 10^{-16}\ V^2/Hz$ & $\overline{i_n^2} = 9 \times 10^{-26}\ A^2/Hz$

$$\rightarrow \boxed{\left(\frac{S}{N}\right)_o = 676 \times 10^6}$$ per Hz

b) $R_s = \frac{e_n}{i_n} = 29.814k\Omega \rightarrow \boxed{\left(\frac{S}{N}\right)_o = 481 \times 10^6}$

3.13: a) $\because (S_i)_{min} = F kTB \left(\frac{S}{N}\right)_o = \boxed{7.57 \times 10^{-16} \text{ W}}$

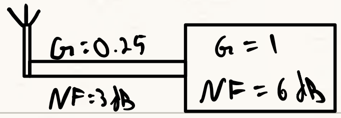
b) $\because P_I = 20$ dBm $= 100$ mW $\rightarrow DR = \left(\frac{P_I}{N_f}\right)^{2/3}$

$$\therefore DR = 2.59 \times 10^9 = \boxed{94.14 \text{ dB}}$$

c) $F_{total} = \cancel{0}$ 1 $+ \frac{10^{0.8}-1}{1000}$ voltage gain $= \cancel{0}.531$ 1.053 $\rightarrow N_f = \cancel{6.395 \times 10^{-17}}$ W 1.25 × 10^{-16} W

$P' = 1mW \rightarrow \boxed{\cancel{DR = 101.3} \text{ dB}}$ 86 dB

3.16:

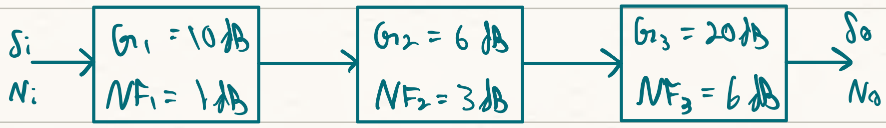


$F_{total} = 10^{0.3} + \frac{10^{0.6} - 1}{0.25} = 13.92$ $\therefore (S_i)_{min} = 1.67 \times 10^{-15}$ W

if antenna Ta = 3000K $\rightarrow F = 11.35 \rightarrow F_{total} = 11.35 + 10^{0.3} - 1 + \frac{10^{0.6} - 1}{0.25}$

$\therefore F_{total} = 24.27 \rightarrow \boxed{(S_i)_{min} = 2.914 \times 10^{-15} \text{ W}}$

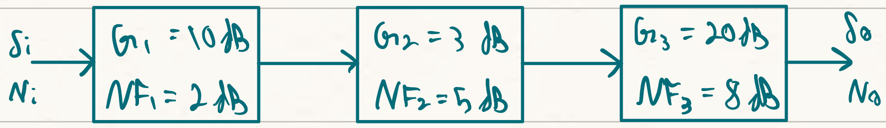
+ quiz #1 practice:



$G_{total} = 36$ dB , $F_{total} = 10^{0.1} + \frac{10^{0.3} - 1}{10} + \frac{10^{0.6} - 1}{10 \cdot 10^{0.6}} = 1.433$

$(S_i)_{min} = F_{total}\, kTB \left(\frac{S}{N}\right)_o = 1.147 \times 10^{-13}$ W $= -99.4$ dBm

$\rightarrow (E_i)_{min} = \sqrt{4 R_s (S_i)_{min}} = 4.79\ \mu V$



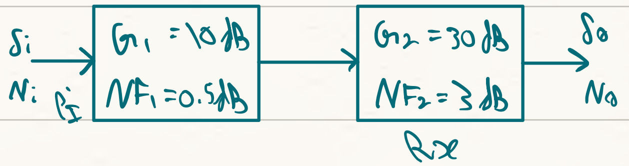
$G_{total} = 33$ dB $= 1995.3$

$F_{total} = 10^{0.2} + \frac{10^{0.5} - 1}{10} + \frac{10^{0.8} - 1}{10 \cdot 10^{0.3}} = 2.07$

$\rightarrow NF_{total} = 3.16$ dB

& $(S_i)_{min} = F\, kTB \cdot 10^{3} = 8.28 \times 10^{-14}$ W $= -100.8$ dBm

$\rightarrow (E_i)_{min} = \sqrt{4 R_s \cdot (S_i)_{min}} = 4.07\ \mu V$



a) $(S_i)_{min, Rx} = 10^{0.3} \cdot kT \cdot 10k \cdot 10^{2} = 7.985 \times 10^{-15}$ W

$\because P_{I, Rx} = 20$ dBm & $DR = \left(\frac{P_I}{N_f}\right)^{2/3}$ $\because (S_i)_{max} = \sqrt[3]{P_I^2 N_f}$

$\rightarrow DR = 5.39 \times 10^{8} = 87.3$ dB

b) $F_{total} = 10^{0.05} + \frac{10^{0.3} - 1}{10} = 1.22 \rightarrow (S_i)_{min} = 4.88 \times 10^{-15}$ W

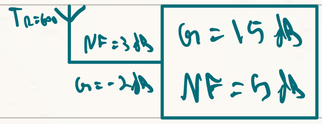
$$\therefore P_I' = \frac{10^2 \text{ mW}}{10} = 10 \text{ mW} \rightarrow DR' = \left(\frac{10 \text{ mW}}{4.88 \times 10^{-15} \text{ W}}\right)^{2/3}$$

$$\rightarrow DR' = 82.1 \text{ dB}$$

+ first exam practice:

11/2012

Q1)



$$F_{antenna} = 1 + \frac{600}{290} = 3.07$$

$$\rightarrow F_{total} = 3.07 + \frac{10^{0.3} - 1}{1} + \frac{10^{0.5} - 1}{10^{-0.2}} = 7.49$$

a) $(S_i)_{min} = F_{total}\, kTB \left(\frac{S}{N}\right)_o = 2.94 \times 10^{-14}$ W

$\rightarrow (S_i)_{min} = -105.2$ dBm

$\wedge (E_i)_{min} = 2.445\ \mu V$

b) $\because (SNR)_o = 20$ dB $\wedge (S_i)_{min}$

$\rightarrow S_o = (S_i)_{min} \cdot 10^{1.3} \rightarrow N_o = 5.47 \times 10^{-15}$ W

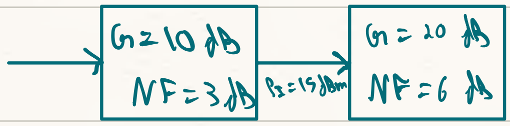
or $N_o = (F\, kTB) \cdot G_1 G_2$

Q2) a) $\because DR = \left(\frac{P_I}{N_o}\right)^{2/3} \rightarrow DR = \frac{2}{3}[15 + 100] = 76.7$ dB

$\because FkTB\left(\frac{S}{N}\right)_o = -100$ dBm

$\rightarrow B \cdot \left(\frac{S}{N}\right)_o = 6.28 \times 10^6$

b)

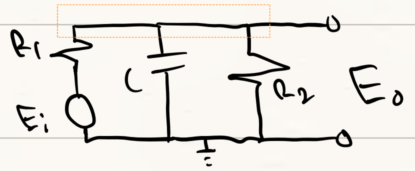


$P_I' = 5$ dBm, $F_{total} = 10^{0.3} + \frac{10^{0.6} - 1}{10} = 2.29 \rightarrow 3.59$ dB

$\rightarrow (S_i)_{min} = F_{total}\, kT\boxed{B \cdot \left(\frac{S}{N}\right)_o} = -102.4$ dBm $\rightarrow DR' = 71.6$ dB

+ homework #1 revisited:

3.2: initial solution:



Applying KCL: $\frac{E_0}{R_2} + E_0 \cdot j\omega C = \frac{E_i - E_0}{R_1}$

$$\rightarrow E_0\left[\frac{1}{R_2} + j\omega C + \frac{1}{R_1}\right] = E_i \cdot \frac{1}{R_1}$$

$$\therefore H(\omega) = \frac{E_0}{E_i} = \frac{1}{\frac{R_1}{R_2} + j\omega C R_1 + 1} = \frac{R_2}{R_1+R_2+j\omega R_1 R_2 C}$$

$$\rightarrow |H(\omega)|^2 = H(\omega)\cdot H^*(\omega) = \frac{R_2}{R_1+R_2+j\omega R_1R_2C}\cdot\frac{R_2}{R_1+R_2-j\omega R_1R_2C}$$

$$\rightarrow |H(\omega)|^2 = \frac{R_2^2}{(R_1+R_2)^2+\omega^2R_1^2R_2^2C^2}$$

prof. mansouri solution ↓

$$B_n = \int_0^\infty |H(\omega)|^2 \, df = \boxed{\frac{1 \; R_2^2}{4R_1R_2(R_1+R_2)C}} \qquad \boxed{\frac{R_1+R_2}{4R_1R_2C}}$$

∧ $f_{3\text{-}dB}$ at $|H(\omega_{3\text{-}dB})|^2 = \frac{1}{2}\max\left[|H(\omega)|^2\right]$

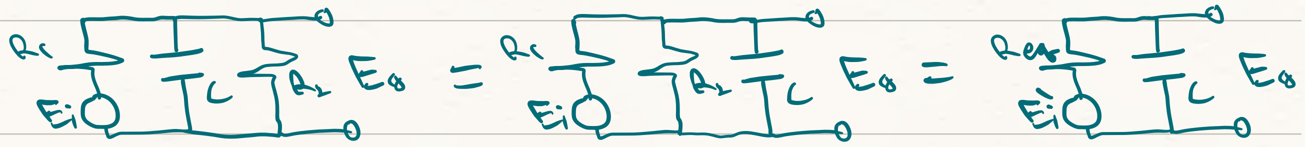
$$\because |H(\omega)|^2 \text{ max at } \omega = 0 = \left(\frac{R_2}{R_1+R_2}\right)^2$$

$$\rightarrow |H(\omega_{3\text{-}dB})| = \frac{\cancel{R_2^2}}{(R_1+R_2)^2 + \omega_{3dB}^2 R_1^2 R_2^2 C^2} = \frac{1}{2}\left(\frac{R_2}{R_1+R_2}\right)^2$$

$$\rightarrow 4\pi^2 f_{3\text{-}dB}^2 \cdot R_1^2 \cdot R_2^2 \cdot C^2 = (R_1+R_2)^2$$

$$\therefore \boxed{f_{3\text{-}dB} = \frac{R_1+R_2}{2\pi R_1R_2C}}$$

+ by following prof. mansour's method of finding the Thevenin equivalent :



$$\therefore R_{eq} = R_1 // R_2 = \frac{R_1R_2}{R_1+R_2} \wedge E_i' = E_i \cdot \frac{R_2}{R_1+R_2}$$

- from the solved example: $H(\omega) = \frac{1}{j\omega RC+1}$

$$\rightarrow \frac{E_o}{E_i'} = \frac{1}{j\omega R_{eq}C+1} = H'(\omega)$$

$$\therefore \frac{E_o}{E_i'} = \frac{E_o}{E_i} \cdot \frac{R_1+R_2}{R_2} \rightarrow H'(\omega) = H(\omega) \cdot \frac{R_1+R_2}{R_2}$$

$$\therefore H(\omega) = \frac{R_2}{R_1+R_2} \cdot \frac{1}{j\omega \frac{R_1R_2}{R_1+R_2} C + 1} = \boxed{\frac{R_2}{j\omega R_1R_2C + R_1+R_2}}$$

$$\rightarrow H(\omega) \cdot H^*(\omega) = \frac{R_2^2}{(R_1+R_2)^2 + (\omega R_1R_2C)^2}$$

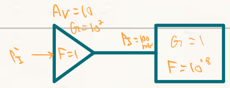
$$\therefore \int_0^\infty G(f)\,df = \boxed{\frac{R_2}{4R_1(R_1+R_2)C}}\ \text{Hz} = B_n$$

3.13: a) $\because (S_i)_{min} = FkTB \cdot \left(\frac{S}{N}\right)_o = 10^{0.8} \cdot k \cdot T \cdot 3k \cdot 10 = 7.57 \times 10^{-16}\ W$

$\rightarrow (E_i)_{min} = \sqrt{4R_s (S_i)_{min}} = 3.84 \times 10^{-7}\ V$

b) $\because N_f = 7.57 \times 10^{-16}\ W$, $P_I = 100\ mW$

$\rightarrow DR = 94.14\ dB$



c) noiseless $\rightarrow NF = 0\ dB \rightarrow F = 1$

$F' = 1 + \frac{10^{0.8} - 1}{100} = 1.0531$, $P_I' = \frac{P_I}{100} = 1mW$

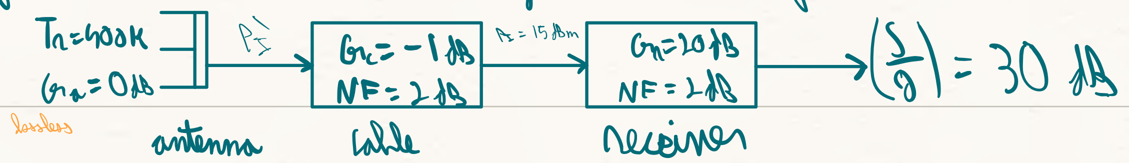
$\rightarrow N_f' = F' kTB \left(\frac{S}{N}\right)_o = 1.264 \times 10^{-16}\ W$

$\therefore DR' = 89.9\ dB$

+ first exam practice continued:

12/2020:

Q1: first draw the block diagram of the system:



a) $G_{total}(dB) = \overset{0}{G_a(dB)} + \overset{-1}{G_c(dB)} + \overset{20}{G_r(dB)} = \boxed{19 \text{ dB}}$

$$F_{total} = F_a + \frac{F_c - 1}{G_a} + \frac{F_r - 1}{G_a G_c}$$

$\because T_a = 500 K \rightarrow F_a = 1 + \frac{T_a}{T} = 1 + \frac{500}{290} = 2.72$

$\rightarrow F_{total} = 2.72 + \frac{10^{0.2} - 1}{1} + \frac{10^{0.2} - 1}{10^{-0.1}} = 4.04$

$\therefore \boxed{NF_{total} = 6.06 \text{ dB}}$

$\because T_n = (F - 1)T \rightarrow \boxed{T_{n,total} = 882 \text{ K}}$

b) $\because (S_i)_{min} = F_{total} \cdot k \cdot T \cdot B \cdot \left(\frac{S}{N}\right)_o = \boxed{8.08 \times 10^{-14} \text{ W}}$

$\rightarrow (S_i)_{min} = -100.9 \text{ dBm}$

& $(E_i)_{min} = \sqrt{4 \cdot R_o \cdot (S_i)_{min}} = \boxed{4.02 \ \mu V}$

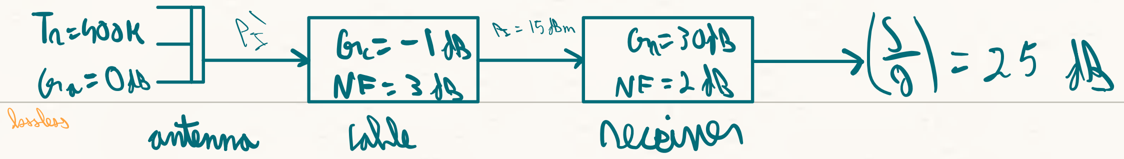
c) $\because \left(\frac{S}{N}\right)_i = F \left(\frac{S}{N}\right)_o \rightarrow \left(\frac{S}{N}\right)_i = 36.1 \text{ dB}$

& $N_o = F_{total} kTB \cdot G_a \cdot G_c \cdot G_r = \boxed{6.42 \times 10^{-15} \text{ W}}$

d) $\because DR = \left(\frac{P_I'}{N_f}\right)^{2/3} = \left(\frac{P_I}{G_c \cdot N_f}\right)^{2/3} = \boxed{77.95}$

& $(S_i)_{max} = DR \cdot N_f = \boxed{5.04 \times 10^{-6} \text{ W}}$

12/2021:



Q1:

a) $G_{total} = 29$ dB, $F_{total} = 4.46 \rightarrow NF_{total} = 6.49$ dB

& $T_{n,total} = 3.46 \cdot 290 = 1003.4$ K

b) $(S_i)_{min} = 5.64 \times 10^{-14}$ W $= -102.5$ dBm

& $(E_i)_{min} = 3.36$ uV, $\left(\frac{S}{N}\right)_i = NF_{total} + \left(\frac{S}{N}\right)_o = 31.49$ dB

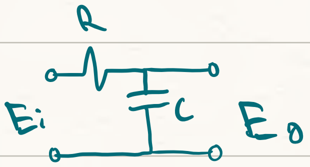
c) $N_o = 1.42 \times 10^{-13}$ W $= -98.5$ dBm

d) $DR = \frac{2}{3}\left[P_I - (S_i)_{min}\right] = 78.3$ dB

& $(S_i)_{max} = -24.2$ dBm $= 3.83$ uW

11/2022:

Q1: a) RC LPF:



$\because E_o = E_i \cdot \frac{1}{j\omega RC + 1} \rightarrow H(\omega) = \frac{1}{j\omega RC + 1}$

$\therefore |H(\omega)|^2 = \frac{1}{1 + (\omega RC)^2}$

& $B_n = \int_0^\infty |H(f)|^2 \, df = \frac{1}{4RC} = 250$ kHz

usually true, but not sure.

assuming signal bandwidth = 3-dB bandwidth

$\rightarrow f_{3\text{-}dB}$ at $\frac{1}{1 + 4\pi^2 f^2 R^2 C^2} = \frac{1}{2} \rightarrow f = \frac{1}{2\pi RC} = 159$ kHz

b) $\cos(2\pi f_{LO} t) \cdot \cos(2\pi f_{RF} t) = k \cos(2\pi f_{IF} t) \rightarrow \overset{IF}{BPF}$

$\rightarrow \cos[2\pi (f_{LO} - f_{RF}) t] = \cos(2\pi f_{IF} t) \rightarrow f_{LO} = 110.7 \text{ MHz}$

- the local oscillator frequency could be chosen as $f_{RF} - f_{IF}$ however it is usually taken as $f_{RF} + f_{IF}$ to further separate the local oscillator frequency and intermediate frequency.

c) 1- amplifiers: amplify received signals and reduces noise contributions of later stages.

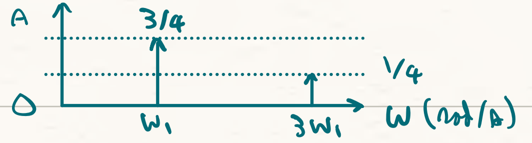
2- oscillators: enable modulation, multiplexing, mixing, etc

3- filters: isolate the desired signals

d) $\cos^3(w_1 t) = \cos(w_1 t) \cdot \cos^2(w_1 t) = \cos(w_1 t) \cdot [\frac{1}{2} + \frac{1}{2}\cos(2w_1 t)]$

$\rightarrow \frac{1}{2}\cos(w_1 t) + \frac{1}{2}\cos(w_1 t) \cdot \cos(2w_1 t)$

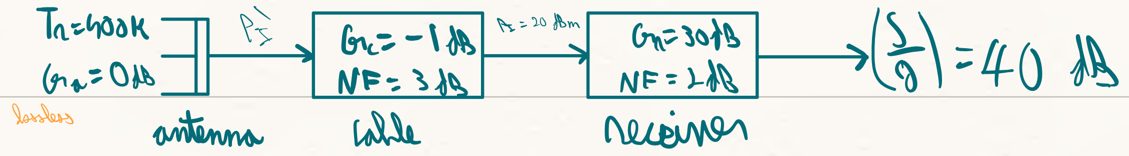
$\rightarrow \frac{1}{2}\cos(w_1 t) + \frac{1}{4}[\cos(3w_1 t) + \cos(w_1 t)] = \frac{3}{4}\cos(w_1 t) + \frac{1}{4}\cos(3w_1 t)$



e) 1- mean or mean-square, 2- variance or standard deviation

3- probability density function, 4- power spectral density

Q2:



a) (check previous solutions for details) $G_{total} = 29$ dB

$F_{total} = 4.46 \rightarrow NF_{total} = 6.49$ dB

$\rightarrow T_{n,total} = 1003.4$ K

b) $(S_i)_{min} = 3.57 \times 10^{-12}$ W $= -84.5$ dBm

$\wedge$ $(E_i)_{min} = 26.7$ uV

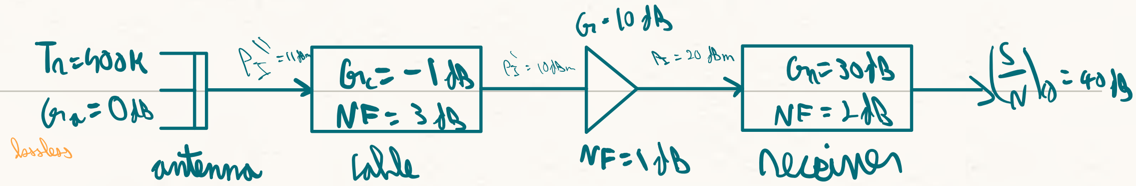
c) $N_i = kTB = 8 \times 10^{-17}$ W, $N_a = (F-1)N_i = 2.768 \times 10^{-16}$ W

$\wedge$ $N_o = (N_i + N_a) \cdot G_{total} = FkTB\,G_{total} = 2.84 \times 10^{-13}$ W

d) $\frac{S_i}{N_i} = 62500 = 47.96$ dB $\wedge$ $\frac{S_o}{N_o} = \frac{S_i \cdot G_{total}}{N_o} = 41.46$ dB

e) $P_I' = 21$ dBm $\rightarrow DR' = 70.3$ dB $\wedge (S_i)_{max} = -14.17$ dBm

f)



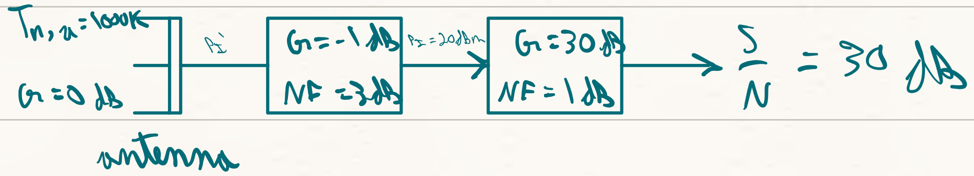
$$F_{total} = 1 + \frac{500}{290} + 10^{0.3} - 1 + \frac{10^{0.1} - 1}{10^{-0.1}} + \frac{10^{0.2} - 1}{10^{-0.1} \cdot 10^{1}} = 4.12$$

$\rightarrow N_f = 3.29 \times 10^{-12}$ W $= -84.8$ dBm

$\rightarrow DR'' = \frac{2}{3}(11 + 84.8) = 63.9$ dB

10/2014:

Q1:



a) $F_{total} = \frac{1000}{290} + 10^{0.3} + \frac{10^{0.1} - 1}{10^{-0.1}} = 5.77$

$G_{total} = 29$ dB

$T_{n,total} = 4.77 \times 290 = 1383.3\ K$

b) $(S_i)_{min} = F_{total}\, kTB \cdot 10^3 = 2.31 \times 10^{-13}\ W = -126.4\ dBW$

$\rightarrow (E_i)_{min} = 6.79\ \mu V$

$\wedge\ \frac{(S_i)_{min}}{N_i} = F_{total} \left(\frac{S}{N}\right)_o = 37.6\ dB = 5770:1$

c) $\because F = \frac{N_i + N_a}{N_i} \rightarrow N_a = N_i(F-1) = 1.91 \times 10^{-16}\ W$

$N_o = F N_i \cdot G_{total} = 1.83 \times 10^{-13}\ W$

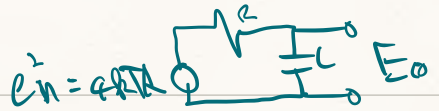
d) $DR = \frac{2}{3}\left[P_I^{\prime} - -126.4 + 30\right] = 78.3\ dB$

$\rightarrow (S_i)_{max} = DR + N_o = -18.1\ dBm$

11/2013 :

Q1 : a) resistors : thermal noise , diode and transistor : shot noise

b)



$\rightarrow H(\omega) = \frac{1}{j\omega RC + 1} \rightarrow |H(\omega)|^2 = \frac{1}{1 + (\omega RC)^2}$

$\rightarrow E_o^2 = e_n^2 \cdot \int_0^\infty \frac{1}{1 + 4\pi^2 f^2 R^2 C^2}\, df = e_n^2 \cdot \frac{1}{4RC}$

$\therefore E_o^2 = \frac{kT}{C}\ W\ (V^2)$

Chapter 7:

- amplifiers are the most important devices in communication systems.

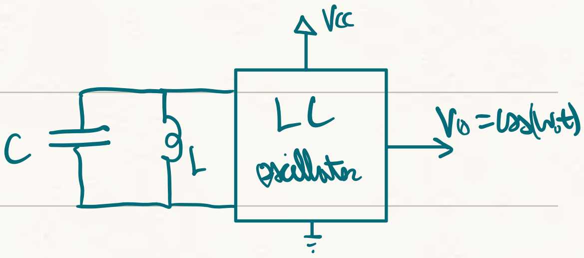
no input, only output harmonic oscillators produce sinusoidal AC signals

- electronic oscillators convert DC power to periodic AC signals at specific freq.
- harmonic oscillators are used in communication systems to produce carriers, which

which allows bandpass channels like free-space to be used frequency multiplexing (FDM)

can be used in modulators, demodulators, multiplexers, demultiplexers, upconverters and downconverters.

+ major types of oscillators :

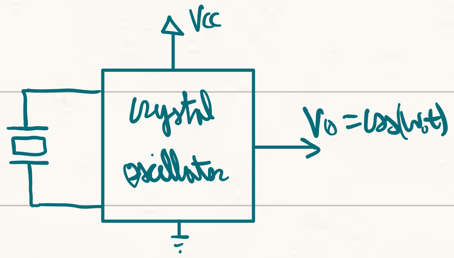


1- LC oscillators (conventional) :

- frequency isn't precise since L and C aren't precise and may vary

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}} \rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

2- Crystal oscillator :

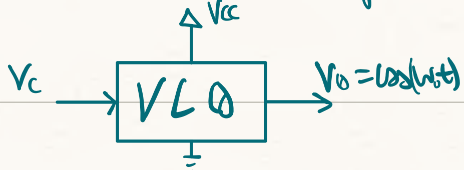


- very precise.

3- Voltage controlled oscillators (VCO):

- output voltage is controlled by a DC input control voltage (Vc)

$$\rightarrow f_0 = f(V_c)$$

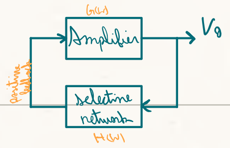


- used in FM modulators and phase-locked loops (PLL)

- oscillators are represented as unstable networks made of amplifiers with positive feedback from frequency selective networks.

- oscillators have AC outputs without an input, instead noise triggers the oscillation.

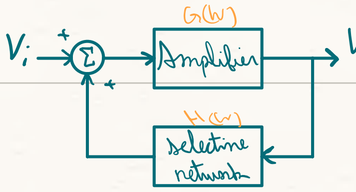
only DC power



- an input is assumed to derive the oscillation conditions

$$\rightarrow \quad V_o = [V_i + H(\omega) V_o]\, G(\omega)$$

$$\therefore \quad V_o = \underbrace{\frac{G(\omega)}{1 - G(\omega) H(\omega)}}_{\text{closed-loop gain}} \cdot V_i$$



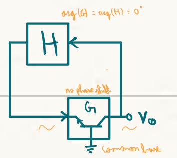
- If $G(\omega) H(\omega) = 1$, the closed loop gain becomes $\overset{\text{unstable}}{\text{infinite}}$, implying that there will be an output even for $V_i = 0$.

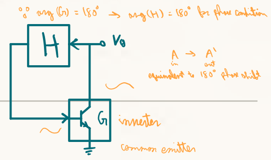
- the frequency selective network $H(\omega)$ will have one frequency (ω_o) at which the condition $G(\omega) H(\omega) = 1$ is satisfied.

+ Therefore, there are two conditions for $G(\omega_o) H(\omega_o) = 1$:

  - magnitude condition: $|G(\omega_o) H(\omega_o)| = 1$

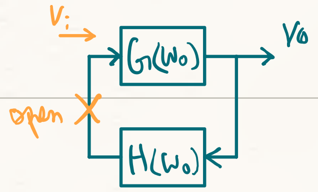
  - phase condition: $\arg(G(\omega_o) H(\omega_o)) = 0$ or $2n\pi \;\forall n = \text{integer}$



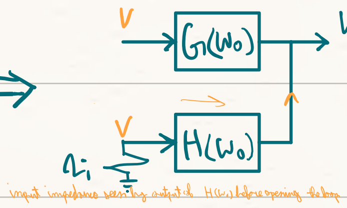


- three different approaches to analyse oscillators will be explored.

1- open feedback loop: first approach



$\Rightarrow$

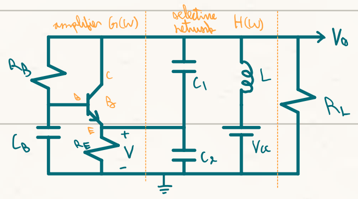


find $G = \frac{V_o}{V}$, $H = \frac{V}{V_o}$

for $|GH| \overset{\text{to give some noise margin}}{\geq} 1$, $\arg(GH) = 0$ $\underline{2n\pi}$

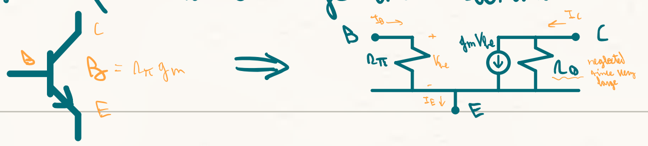
+ common base oscillator analysis:

  - capacitor C_B is added to short the base and



make the amplifier common base in AC analysis, provided $\frac{1}{\omega_0 C_B} \simeq 0$

- the first step is to change the transistor to its hybrid T model.



$\rightarrow I_E = I_C + I_B$

$\rightarrow r_\pi = \frac{kT}{q} \cdot \frac{\beta}{I_C} \approx \frac{0.026\beta}{I_C}\Big|_{300K}$

Transconductance

$\rightarrow g_m = \frac{I_C}{kT/q} \approx \frac{I_C}{0.026}\Big|_{300K}$

$\therefore \beta = r_\pi g_m$

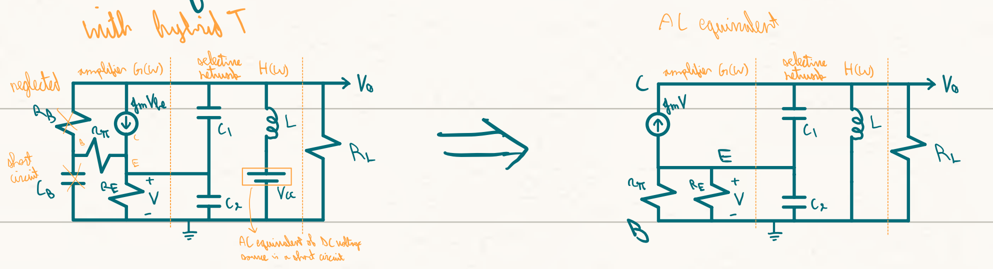
$\because g_m V_{be} = g_m r_\pi I_b = \beta I_b$

$\rightarrow I_C \approx \beta I_B$ if r_o is very large

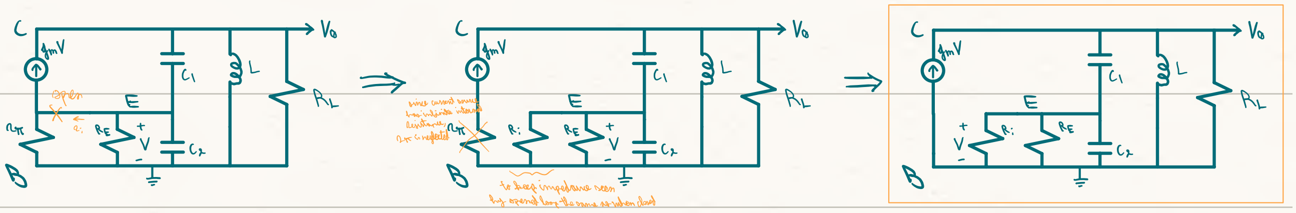
- important equations to recall:

$$g_m \approx \frac{I_C}{0.026}, \quad I_C \approx \beta I_B, \quad \beta = r_\pi g_m$$

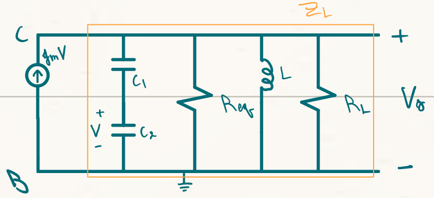
- the AC equivalent circuit of the common-base oscillator becomes:



- next, the feedback loop is opened: where $r_i = \frac{1}{g_m}$



- The analysis of the AC equivalent circuit can be simplified using capacitive transformation:



$$\rightarrow R_{eq} = \frac{r_i R_E}{r_i + R_E} \cdot \left(\frac{C_1 + C_2}{C_1}\right)^2$$

only if $\frac{1}{[\omega(C_1+C_2)]^2} \ll \left(\frac{r_i R_E}{r_i + R_E}\right)^2$

- finally, $H(\omega)$ and $G(\omega)$ are found as follows:

$\rightarrow H(\omega) = \frac{V}{V_o} \rightarrow V = V_o \cdot \frac{1/j\omega C_2}{1/j\omega C_1 + 1/j\omega C_2}$ voltage division across capacitors $\rightarrow \frac{V}{V_o} = \frac{C_1}{C_2 + C_1}$

& $G(\omega) = \frac{V_o}{V} \rightarrow V_o = \overset{\text{current}}{g_m V} \cdot Z_L \rightarrow G(\omega) = g_m Z_L$

$\therefore \frac{1}{Z_L} = \frac{1}{R_L} + \frac{1}{j\omega L} + \frac{1}{R_{eq}} + j\omega C_{eq}$ s.t. $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

- now we must satisfy the oscillation conditions:

since H is real

+ $\arg(GH) = 0°$ $\wedge$ $\arg(H) = 0°$ $\rightarrow$ $\arg(G) = 0°$

  - for $\arg(G) = 0°$, G must be real $\rightarrow$ $\frac{1}{Z_L}$ is real

    $\therefore \cancel{j}\omega_o C_{eq} = \frac{\cancel{j}}{\omega_o L} \rightarrow \omega_o = \sqrt{\frac{1}{LC_{eq}}} \rightarrow \omega_0 = \sqrt{\frac{1}{L \cdot \frac{C_1 C_2}{C_1+C_2}}}$ phase condition

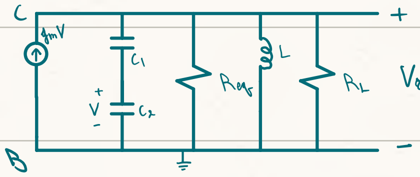
    at resonance

    $\rightarrow$ at ω_o, $Z_L = \frac{R_L R_{eq}}{R_L + R_{eq}}$

+ $|GH| \geq 1$, at resonance: $G(\omega_o) = g_m \frac{R_L R_{eq}}{R_L + R_{eq}}$

  $\therefore |GH| = g_m \frac{R_L R_{eq}}{R_L + R_{eq}} \cdot \frac{C_1}{C_1 + C_2} \geq 1$ magnitude condition

+ example 7.1: 20 MHz common-base oscillator with $\beta = 100$ and $I_C = 1mA$



$R_{eq} = \frac{r_i R_E}{r_i + R_E} \cdot \left(\frac{C_1 + C_2}{C_1}\right)^2$, $r_i = \frac{1}{g_m}$

only if $\frac{1}{[\omega(C_1+C_2)]^2} << \left(\frac{r_i R_E}{r_i + R_E}\right)^2$

we need $20 MHz = \sqrt{\frac{1}{L \cdot \frac{C_1 C_2}{C_1 + C_2}}}$ and $g_m \frac{R_L R_{eq}}{R_L + R_{eq}} \cdot \frac{C_1}{C_1 + C_2} \geq 1$

$\therefore g_m = \frac{I_C}{0.026} \simeq 40 mA$ $\wedge$ $r_i = \frac{1}{g_m} = 25 \Omega$

$\wedge$ $R_E >> r_i \rightarrow R_{eq} \simeq r_i \left(\frac{C_1 + C_2}{C_1}\right)^2$

$\rightarrow \frac{1}{r_i^2} << [\omega_o(C_1 + C_2)]^2$ ①

$\rightarrow$ choose $R_L >> R_{eq} \rightarrow |GH| \simeq g_m \cdot R_{eq} \cdot \frac{C_1}{C_1 + C_2} = g_m \cdot r_i \cdot \frac{C_1 + C_2}{C_1}$

$\therefore |GH| \simeq \frac{C_1 + C_2}{C_1} \geq 1$, if $\frac{C_1 + C_2}{C_1} = 3 \rightarrow C_2 = 2C_1$

from ①

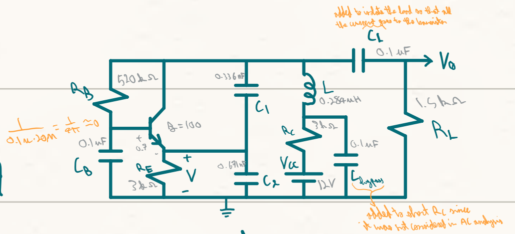
$\wedge$ $\frac{10}{r_i^2} < [\omega_o(C_1 + C_2)]^2 \rightarrow 3C_1 \geq 1.01 nF$

$$\therefore \quad C_1 \geq 0.336\,nF \;\; \& \;\; C_2 \geq 0.671\,nF$$

$$\because \quad \omega_0 = \sqrt{\frac{1}{L C_{eq}}} \rightarrow L = \frac{1}{\omega^2 \cdot \frac{C_1 C_2}{C_1+C_2}} = 0.283\,mH$$

found through trial and error

- Resistors R_B, R_C, and R_E are chosen to make $I_C = 1mA$ and to satisfy the other assumptions.


added to isolate the load so that all the current goes to the transistor

$\frac{1}{0.1u \cdot 20M} = \frac{1}{4\pi} \simeq 0$

added to short R_C since it was not considered in AC analysis

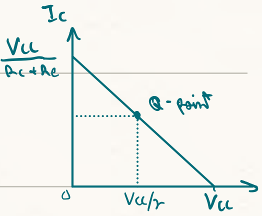
since V_{CE} not known

$$\rightarrow -V_{CC} + I_C R_C + I_B R_B + 0.7 + I_E \cdot R_E = 0 \;\; \& \;\; I_E \simeq I_C, \; I_B \simeq \frac{I_C}{\beta}$$

$$\rightarrow V_{CC} = I_C \left[R_C + \frac{R_B}{\beta} + R_E\right] + 0.7$$

$$\rightarrow \overset{3k}{R_C} + \frac{\overset{520k}{R_B}}{\underset{100}{\beta}} + \overset{3k}{R_E} = \frac{V_{CC} - 0.7}{I_C} = 11.3\,k\Omega$$

- I_C must be chosen to keep the transistor at the center of the active region to have a symmetric output and avoid trimming

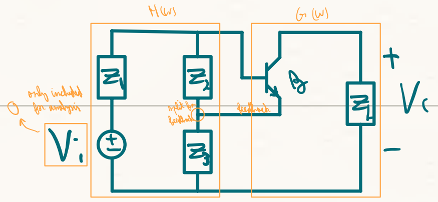


- large I_C is desirable as it increases the transconductance (g_m) but a large I_C also increases power consumption, and hence there must be a tradeoff.

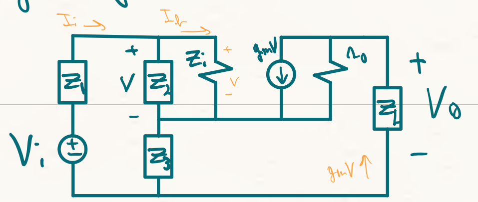
2- Circuit analysis : second approach

- a resonant circuit must be used as the frequency selective network to determine the operating frequency of the oscillator.

- This second approach requires three impedances as the selective network and a transistor with feedback to provide gain for the oscillator.



⇒



equivalent
AC
circuit

- assuming r_o is very large (o.c): KVL

1) $\underset{\to 0}{\cancel{-V_i}} + I_i Z_1 + V + Z_3(I_i + g_m V) = 0$

2) $V = I_i \cdot (Z_2 // Z_i) = I_i \frac{Z_i Z_2}{Z_i + Z_2} \rightarrow I_i Z_2 - V \frac{Z_i + Z_2}{Z_i} = 0$

- for oscillation, I_i and I_{fb} must be non-zero even when V_i is zero, which is only possible if the determinant is zero

from (1): $I_i (Z_1 + Z_3) + V(1 + g_m Z_3) = 0$

from (2): $I_i \; Z_2 \quad - V\left(\frac{Z_i + Z_2}{Z_i}\right) = 0$

$$\rightarrow \begin{vmatrix} Z_1 + Z_3 & 1 + g_m Z_3 \\ Z_2 & -\frac{Z_i + Z_2}{Z_i} \end{vmatrix} = 0$$

or ∴ $I_i = V \cdot \frac{Z_i + Z_2}{Z_i Z_2}$

$$\rightarrow Z_2 \times \underbrace{\left[(Z_1 + Z_3)\frac{Z_i + Z_2}{Z_i Z_2} + 1 + g_m Z_3\right]}_{\text{must equal zero}} \cdot \overset{\text{not zero}}{V} = 0 \times Z_2$$

$$\rightarrow (Z_1 + Z_3) \cdot \left(1 + \frac{Z_2}{Z_i}\right) + Z_2 + g_m Z_2 Z_3 = 0$$

$$\rightarrow Z_1 + Z_2 + Z_3 + \frac{Z_2}{Z_i}(Z_1 + Z_3) + g_m Z_2 Z_3 = 0$$

∴ $r_\pi = Z_i$ for BJT : multiply by Z_i

$$\rightarrow \underbrace{(Z_1 + Z_2 + Z_3) r_\pi}_{\text{imaginary}} + \underbrace{Z_2(Z_1 + Z_3)}_{\text{real}} + \underbrace{\overbrace{\beta}^{g_m r_\pi} Z_2 Z_3}_{\text{real}} = 0$$

∵ imaginary × imaginary = real

- to reduce power consumption, above impedances should be purely reactive (i.e., no real part)
- for an equation involving complex numbers to equal zero, both real and imaginary parts must equal zero.

Z_3 must have an opposite sign to Z_1
→ if Z_1 inductive, Z_3 capacitive

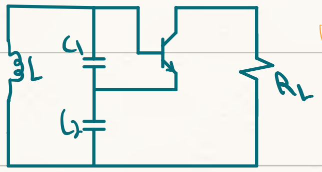
$$\therefore \quad \cancel{Z_2}(Z_1 + Z_3) + \beta \cancel{Z_2} Z_3 = 0 \rightarrow Z_1 = -Z_3(\beta + 1)$$

$-Z_3(\beta+1) \rightarrow Z_3(-\beta-1+1) + Z_2 = 0$

Z_2 and Z_3 both must have the same sign
→ both inductive or capacitive

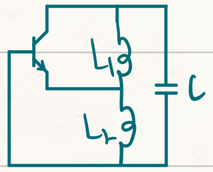
$$\wedge \quad r_\pi(\overset{\sim}{Z_1} + Z_2 + Z_3) = 0 \rightarrow Z_2 = \beta Z_3$$

- Therefore, the two possible configurations following this approach are: (a) two capacitors + one inductor (colpitts), (b) two inductors + one cap. (hartley)



$\omega_0 = \frac{1}{\sqrt{L \frac{C_1 C_2}{C_1 + C_2}}}$

(a)
colpitts



$\omega_0 = \frac{1}{\sqrt{(L_1 + L_2) \cdot C}}$

(b)
hartley

- practically, the impedances have resistive components. if we reanalyze the colpitts oscillator with $Z_1 = r + j\omega L$:

inductor usually has much more significant resistance than capacitor

$Z_2 = \frac{1}{j\omega C_1}$
$Z_3 = \frac{1}{j\omega C_2}$

$$\left(r + j\omega L + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right) r_\pi + \frac{1}{j\omega C_1}\left(r + j\omega L + \frac{1}{j\omega C_2}\right) - \beta \frac{1}{\omega^2 C_1 C_2} = 0$$

$$\text{real} \rightarrow \quad r r_\pi + \frac{L}{C_1} - \frac{1}{\omega_0^2 C_1 C_2} - \frac{\beta}{\omega_0^2 C_1 C_2} = 0$$

$$\text{imaginary} \rightarrow \quad r_\pi\left(\omega_0 L - \frac{1}{\omega_0 C_1} - \frac{1}{\omega_0 C_2}\right) - \frac{r}{\omega_0 C_1} = 0$$

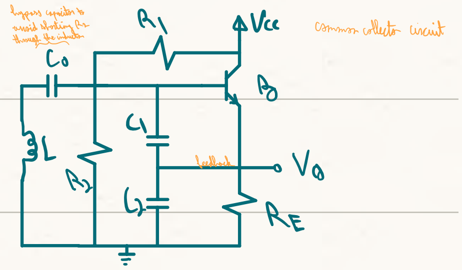
∴ magnitude condition: $R r_\pi \le \frac{\beta+1}{\omega_0^2 C_1 C_2} - \frac{L}{C_1}$

& phase condition: $\omega_0 = \frac{1}{\sqrt{L \cdot \frac{C_1' C_2}{C_1' + C_2}}}$ s.t., $C_1' = \frac{C_1}{1 + \frac{R}{r_\pi}}$

- hence, the internal resistance changes the resonant frequency, as observed from the phase condition. however $C_1' \approx C_1$ if $r_\pi >> R$
- the magnitude condition indicates that the gain of the transistor has to be large enough to overcome the resistive losses.
- there is another constraint on the capacitors, as they need to be much larger than the stray/parasitic capacitance of the transistor.

unloaded quality factor for inductor: $Q = \frac{\omega L}{R}$

example 7.2: 5MHz colpitts, 10 μH inductor with $Q_u = 100$, $\beta = 100$



∴ $\omega_0 = 10\pi$ MHz, $Q_u = \frac{\omega_0 L}{R}$

→ $R = \overset{3.14}{\pi}$ Ω

& phase condition: $\omega_0 = \frac{1}{\sqrt{L C_{eq}}} \to C_{eq} = 1.013 \times 10^{-10} = 101.3$ pF

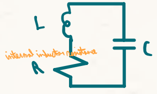
if $C_1' = C_1 = C_2 \to C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \to C_1 = C_2 = 202.6$ pF

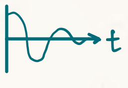
→ magnitude condition: $R r_\pi \le \frac{\beta+1}{\omega_0^2 C_1 C_2} - \frac{L}{C_1} \to r_\pi \le 0.778$ MΩ

∴ $r_\pi = \frac{0.026\beta}{I_C} \to I_C = 3.34$ μA

3 - negative Resistance: (third approach)

- ideally, an excited LC circuit will oscillate forever. in reality, the inductor has a large resistance (found from its Q-factor) that causes the oscillations to attenuate.

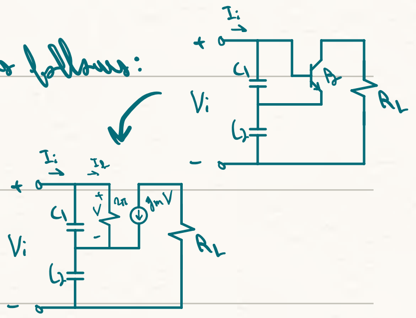




$\omega_0 = \frac{1}{\sqrt{LC'}} \approx \frac{1}{\sqrt{LC}}$

- to counter the attenuation, a "$\overset{\text{equivalent input resistance of an active circuit}}{\text{negative resistance}}$" is inserted such that $|r_i| > |R|$

- the input impedance of the AC equivalent active circuit is found as follows:



$$\rightarrow -V_i + \overbrace{V}^{(I_i - I_b)X_{C_1}} + (I_i + \overbrace{g_m V}^{\beta I_b}) X_{C_2} = 0$$

$$\rightarrow V_i = I_i(X_{C_1} + X_{C_2}) - I_b(X_{C_1} - \beta X_{C_2})$$

$$\& \ (I_b - I_i) X_{C_1} + I_b r_\pi = 0 \rightarrow -I_i X_{C_1} + I_b(r_\pi + X_{C_1}) = 0$$

$$\therefore I_b = I_i \cdot \frac{X_{C_1}}{r_\pi + X_{C_1}} \rightarrow V_i = I_i\left[(X_{C_1} + X_{C_2}) - \frac{X_{C_1}}{r_\pi + X_{C_1}} \cdot (X_{C_1} - \beta X_{C_2})\right]$$

$$\rightarrow Z_i = \frac{V_i}{I_i} = \frac{r_\pi(X_{C_1} + X_{C_2}) + (\beta + 1) X_{C_1} X_{C_2}}{r_\pi + X_{C_1}}$$

if $X_{C_1} << r_\pi$ and $\beta >> 1$

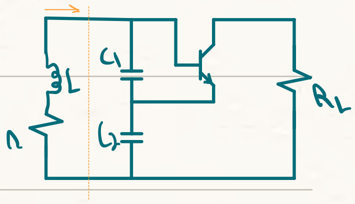
$$\rightarrow Z_i = \overset{\frac{1}{j\omega C_1}}{X_{C_1}} + \overset{\frac{1}{j\omega C_2}}{X_{C_2}} + \overbrace{g_m}^{\beta / r_\pi} \overset{\frac{1}{j\omega C_1}}{X_{C_1}} \overset{\frac{1}{j\omega C_2}}{X_{C_2}}$$

$$\therefore Z_i = \left(\frac{C_2 \cdot 1}{C_2 \cdot j\omega C_1} + \frac{C_1 \cdot 1}{C_1 \cdot j\omega C_2}\right) \boxed{- \frac{g_m}{\omega^2 C_1 C_2}}$$

real part is negative implying active circuit
$-r_i$

$$\rightarrow Z_i = -r_i + \frac{1}{j\omega C_i} \rightarrow \frac{C_1 C_2}{C_1 + C_2}$$

- Therefore, if a $\overbrace{\text{coil}}^{\text{inductor}}$ with resistance r is connected across the above equivalent circuit, it becomes a Colpitts oscillator with the following oscillation conditions:



- magnitude: $r_i > r$

$$\rightarrow \frac{g_m}{\omega_0^2 C_1 C_2} > r$$

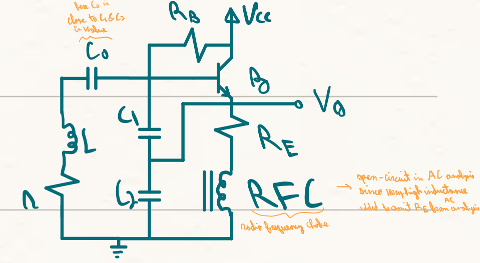
equivalent to second approach:

$$\frac{1}{r_\pi} \cdot r \cdot r_\pi \leq \frac{1}{r_\pi} \cdot \frac{\overbrace{\beta + 1}^{\approx \beta}}{\omega_0^2 C_1 C_2} - \overbrace{\frac{L}{C} \frac{1}{r_\pi}}^{\text{neglected}} \rightarrow r \leq \frac{g_m}{\omega_0^2 C_1 C_2}$$

- phase condition: $\omega_0 = \dfrac{1}{\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}}$

+ Clapp - gouriet oscillator :

- modified version of the colpitts oscillator, where the bypass capacitor (C_0) is made small enough to be included in the resonant circuit to allow for more freedom in designing the oscillator.



C_0 was much larger than C_1 & C_2 in colpitts here C_0 is close to C_1 & C_2 in value

→ open-circuit in AC analysis since very high inductance added to omit R_E from $\overset{AC}{\text{analysis}}$

radio frequency choke

$$\rightarrow \quad W_0 = \frac{1}{\sqrt{L \cdot \frac{C_0 C_i}{C_0 + C_i}}} \quad \& \quad C_i = \frac{C_1 C_2}{C_1 + C_2}$$

example 7.4: $f_0 = 1 MHz$, $X_L = 800\,\Omega$, $Q_u = 200$, $g_m = 6mS$

$\therefore X_L = W_0 L \rightarrow L = 127.3\ uH$, $\quad r = \frac{W_0 L}{Q_u} = 4\ \Omega$

$\therefore$ phase condition: $\frac{1}{\sqrt{L \cdot \frac{C_0 C_i}{C_0 + C_i}}} = W_0$, $\quad C_i = \frac{C_m}{2}$, $C_m = C_1, C_2$

→ need two equations to find C_m and C_0, $\therefore g_m = \frac{I_C}{0.026} \rightarrow$ fixed

assuming equal

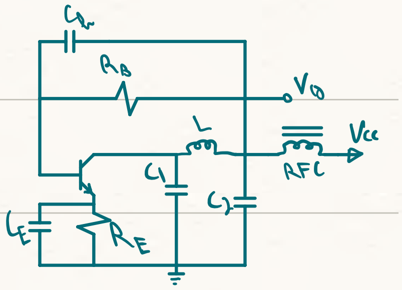
from magnitude condition: $\frac{g_m}{(W_0 C_m)^2} \geq r \rightarrow C_m = 6.16\ nF$

from phase condition

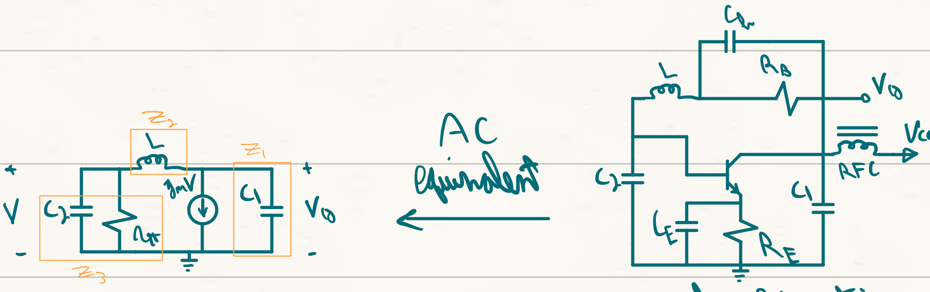
$\rightarrow \quad C_0 = 212.7\ pF$

+ pierce oscillator : common emitter

- bias resistors are shorted and do not shunt the tuned circuit, which gives a more stable frequency.



first configuration



second configuration

- find $G(W)$ and $H(W)$ to find oscillation conditions.

$$H(\omega) = \frac{V}{V_0} : \text{Voltage division, } V = V_0 \cdot \boxed{\frac{Z_3}{Z_3 + Z_2}}^{\,H(\omega)}$$

$$G(\omega) = \frac{V_0}{V} : \quad V_0 = I_{Z_1} \cdot Z_1 \,, \; -I_{Z_1} = g_m V \cdot \frac{Z_2 + Z_3}{Z_1 + Z_2 + Z_3}$$

$$\therefore \; G(\omega) = -g_m Z_1 \cdot \frac{Z_2 + Z_3}{Z_1 + Z_2 + Z_3}$$

$$\text{for } GH \geq 1 \rightarrow \frac{-g_m Z_1 Z_3}{Z_1 + Z_2 + Z_3} \geq 1$$

$$\because \; Z_1 = \frac{1}{j\omega C_1} \,, \; Z_2 = j\omega L \,, \; Z_3 = \frac{r_\pi}{r_\pi j\omega C_2 + 1}$$

$$\rightarrow GH = \frac{-\underbrace{\boxed{g_m r_\pi}}_{\beta}}{(j\omega r_\pi C_2 + 1)(j\omega C_1)\left[\frac{1}{j\omega C_1} + j\omega L + \frac{r_\pi}{j\omega r_\pi C_2 + 1}\right]}$$

$$\rightarrow GH = \frac{-\beta}{1 - \omega^2 L C_1 + \underbrace{j\omega r_\pi [C_1 + C_2 - \omega^2 L C_1 C_2]}_{\text{imaginary part}}}$$

$$\because \; \mathrm{Im}\{GH\} = 0 \rightarrow j\omega r_\pi [C_1 + C_2 - \omega^2 L C_1 C_2] = 0$$

$$\rightarrow \omega_0 = \frac{1}{\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}} \quad \text{(phase condition)}$$

$$\wedge \quad \frac{\beta}{\underbrace{\omega_0^2 L C_1 - 1}_{\omega_0^2 L C_1 > 1}} > 1 \quad \text{(magnitude condition)}$$

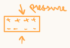
+ Crystal oscillators:

higher Q-factor → more stable and precise

- frequency stability and preciseness depend on the Q-factor.
- because of the inductors' resistances, the Q-factor of LC resonant circuits is less than 1k.

Q-factor up to 10^6

- crystals, such as quartz or ceramics, are electromechanical devices that exploit piezoelectric effect:



reverse piezoelectric effect:

expands and contracts at a specific frequency

the reverse piezoelectric effect to generate voltage at a rate depending on the crystal's resonant frequency.

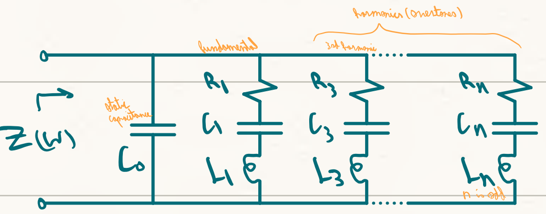
much more precise and stable than LC resonators

- a crystal is essentially a thin wafer between two conductive plates. since the crystal material is a dielectric, the crystal behaves as a static capacitor with capacitance :

$$\frac{\overbrace{\epsilon_0 \epsilon_r}^{\text{permittivity of material}} \overbrace{A}^{\text{area of plates}}}{t \text{ thickness}}$$

- the fundamental resonant frequency of a crystal is inversely proportional to its thickness.
- the equivalent electric circuit of a crystal is made up of many resonant circuits in parallel with each other, with each being an odd harmonic of the fundamental frequency.

and the static capacitance



- the crystals cannot oscillate at multiple modes simultaneously. They are often made to oscillate at one of the harmonics to achieve higher frequencies; however, the generated signal becomes weaker at higher harmonics.
- the static capacitance is typically in the pF range, whereas the nth capacitance is in the fF range.

pico 10^{-12}

femto 10^{-15}

- The resonant frequencies at the fundamental mode are found from:

$$Z(\omega) = X_{C_0} // [R_1 + X_{C_1} + X_{L_1}] = \frac{\frac{1}{j\omega C_0}\left[R_1 + \frac{1}{j\omega C_1} + j\omega L_1\right]}{R_1 + \frac{1}{j\omega C_0} + \frac{1}{j\omega C_1} + j\omega L}$$

if R_1 is assumed to be very small $\rightarrow$ $Z(\omega) = \frac{\frac{1}{j\omega C_0}\left[\frac{1}{j\omega C_1} + j\omega L_1\right]}{\frac{1}{j\omega C_0} + \frac{1}{j\omega C_1} + j\omega L}$

- hence resonance occurs when $Z(\omega) = 0$ or $Z(\omega) = \infty$

series resonance

1- for $Z(\omega) = 0 \rightarrow \cancel{\frac{1}{j\omega C_0}} \cdot \left[\frac{\overset{-j}{1}}{j\omega C_1} + j\omega L_1\right] = 0 \rightarrow \frac{1}{\omega_s C_1} = \omega_s L_1$

$$\therefore \quad \omega_s = \frac{1}{\sqrt{L_1 C_1}}$$

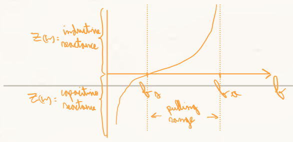
if $C_1 << C_0 \rightarrow \omega_a \approx \omega_s$

parallel resonance (anti-resonance)

2- for $Z(\omega) = \infty \rightarrow \frac{-j}{\omega_a C_0} + \frac{-j}{\omega_a C_1} + j\omega_a L_1 = 0 \rightarrow \omega_a = \frac{1}{\sqrt{L_1 \frac{C_0 C_1}{C_0 + C_1}}}$

f_a is generally used in defining crystals rather than f_s

- a relation between f_a and f_s can be derived to give:



$\because \frac{C_1}{C_0} << 1$ given $C_1 << C_0$

$$f_a = \left(1 + \frac{C_1}{C_0}\right)^{1/2} \cdot f_s \xrightarrow{\text{binomial approximation}} f_a \approx f_s \cdot \left(1 + \frac{C_1}{2C_0}\right)$$

between f_s and f_a

* pulling range: the region in which the crystal oscillator operates

example 9.6: $C_0 = 5.1$ pF, $C_1 = 21$ fF, $R_1 = 25\ \Omega$ (from table)

$\therefore Q_u = \frac{\omega_a L_1}{R_1}$, $j\omega_a L_1 = \frac{j}{\omega_a C_0} + \frac{j}{\omega_a C_1}$

very large

must take five or more significant figures

$\rightarrow L_1 = 0.037278$ H $\rightarrow Q_u = 53403.2$

- crystals are used in oscillators as either parallel or series resonators

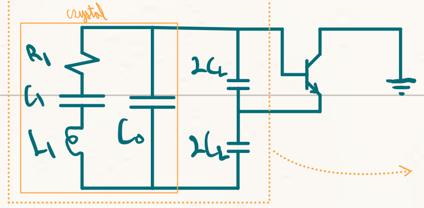
- if the parallel mode is chosen, the inductor in single inductor oscillators (e.g., colpitts) is removed and replaced by the crystal.

- the magnitude oscillation conditions are the same for crystal oscillat except R_n for the crystal replaces R of the previous inductor

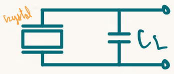
as LC oscillators.

- for a colpitts crystal oscillator, the magnitude condition is : $\dfrac{g_m \overbrace{X_{C_1} X_{C_2}}^{\text{colpitts capacitances}}}{\underset{\text{crystal (inductor) resistance}}{R_1}} \geqslant 1$

- The resonance frequency is found by replacing the crystal with its equivalent circuit:



C_1 and C_2 of the oscillator were assumed equal → $C_L = \frac{C_1 C_2}{C_1+C_2} = \frac{C_1}{2} = \frac{C_2}{2}$
to avoid confusion with C_1 of the crystal, C_1 and C_2 were replaced by $2C_L$ each



loaded anti-resonant frequency

$$\rightarrow \omega_0 L_1 = \frac{1}{\omega_0 C_1} + \underset{\text{added since parallel}}{\frac{1}{\omega_0 (C_0 + C_L)}} \rightarrow f_0 = \frac{1}{2\pi \cdot \sqrt{L_1 \cdot \frac{C_1 \cdot (C_0 + C_L)}{C_1 + C_0 + C_L}}} = f_a'$$

$$\therefore \text{ if } C_L = \infty \rightarrow f_0 = \frac{1}{2\pi\sqrt{L_1 C_1}} \rightarrow f_a' = f_s$$

$$\wedge \text{ if } C_L = 0 \rightarrow f_0 = \frac{1}{2\pi \cdot \sqrt{L_1 \cdot \frac{C_1 C_0}{C_1 + C_0}}} \rightarrow f_a' = f_a$$

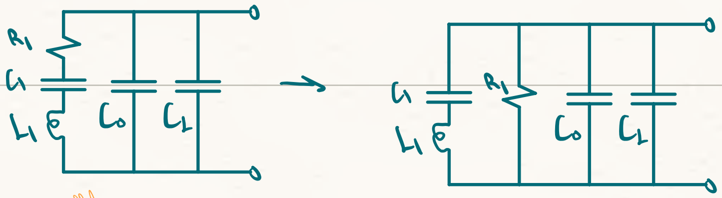
$f_s \leq f_a' = f_0 \leq f_a$

- Therefore, the operating frequency is always within the pulling range.

- since C_L is in parallel, the $\overset{\text{loaded}}{\text{series}}$ resonance frequency is unaffected by it. $f_s' = f_s$

- The capacitor in parallel with the crystal also affects the loaded Q-factor of the oscillator.

- To find the loaded quality factor, R_1 is made parallel with the resonant circuit : $R_p = R_1 \left(1 + \frac{X_p^2}{R_1^2}\right)$, $X_p = \overset{\text{loading}}{\frac{1}{\omega (C_0 + C_L)}}$



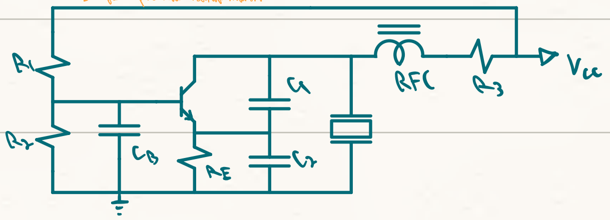
$$\rightarrow Q_L = \frac{\overset{\text{parallel}}{R_p}}{\omega L_1} \approx \frac{\omega L_1}{R_1}$$

increasing C_L decreases frequency stability

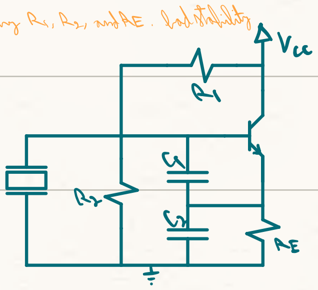
- hence, increasing C_L decreases R_p and, in-turn, the Q-factor.

- the loaded Q factor of a crystal reduces when it is shunted by a resistor. $Q_u = \frac{R_p}{\omega L_1}$, $Q_L = \frac{R_p // R_L}{\omega L_1} < Q_u$

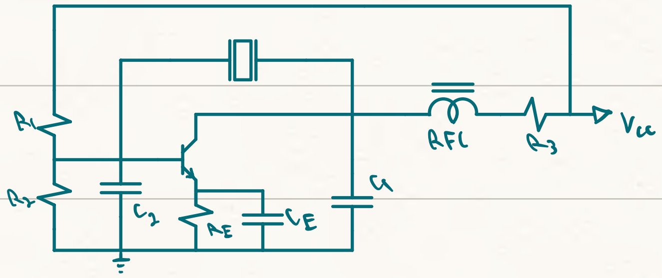
common-base with medium stability, crystal is loaded by R_E after capacitive transformation



common collector where the crystal is loaded by R_1, R_2, and R_E. bad stability



common emitter (Pierce) oscillator. crystal is not loaded by any resistor and hence stability is very high

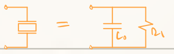


+ series mode crystal oscillators:

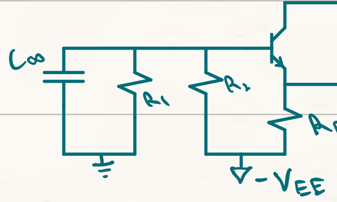
- in parallel mode, the inductor is simply replaced by the crystal whereas in series mode, all the components are left unchanged and the crystal is added to the feedback path.
- the crystal in the feedback path will act as a short circuit if the operating frequency matches the crystals series resonance frequency.

$$\rightarrow f_s = f_0 \quad \therefore \quad \overset{f_s}{\frac{1}{2\pi\sqrt{L_1 \underbrace{C_1}_{\text{C1 of crystal, not to be confused with C1 of the LC circuit}}}}} = \overset{f_o}{\frac{1}{2\pi\sqrt{L C_{eq}}}}$$

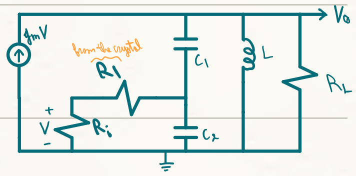
at resonance



common-base series-mode oscillator



$\Rightarrow$



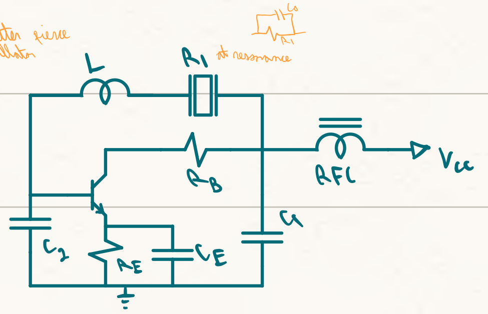
$$\rightarrow V = V_o \cdot \frac{C_1}{C_1 + C_2} \cdot \frac{R_i}{R_i + R_1}$$

same analysis as conventional oscillators

the inductor is in the feedback path

- for a pierce oscillator, the crystal is inserted in series with the inductor

common emitter pierce series-mode oscillator

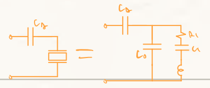


$$\rightarrow f_o = f_s = \frac{1}{2\pi\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}}$$

- the Q factor of the above circuit is affected by the crystal's resistance, such that: $Q = \frac{W_s \cdot L}{\underset{\text{inductor resistance}}{R} + \underset{\text{crystal resistance}}{R_1}}$

$f_a = f_a'$

- if a capacitor is placed in series with the crystal the parallel resonance does not change, but the series resonance changes when the capacitor is taken into account.



$$f_s' = \frac{1}{2\pi\sqrt{L_1 \cdot \frac{C_1(C_0 + C_s)}{C_1 + C_0 + C_s}}}, \quad \begin{array}{l} \text{if } C_s = 0 \rightarrow f_s' = f_a \\ \text{if } C_s = \infty \rightarrow f_s' = f_s \end{array}$$

+ voltage controlled oscillators:

- a local oscillator is required at the receiver to demodulate signals.

- one way to vary an oscillator frequency is to change the capacitors' values.

- a capacitor whose value depends on the voltage across it can be created by using a reverse biased pn junction.

Varactor or Varicap diode

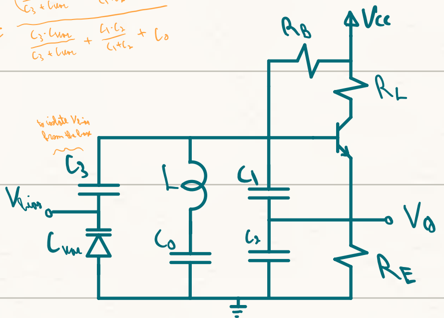
- increasing the voltage across a reverse biased pn junction increases the depletion regions thickness, thereby decreasing the capacitance

$C = \frac{\varepsilon A}{t}$ $C \propto \frac{1}{V_{bias}}$

- a conventional LC oscillator can be converted to a VCO by inserting a varactor diode.

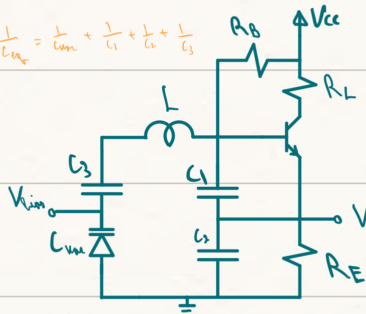
- the tuning range of a VCO depends on the capacitance range of the varactor diode and its location in the LC circuit.

$$C_{eq} = \frac{\left(\frac{C_3 \cdot C_{var}}{C_3 + C_{var}} + \frac{C_1 \cdot C_2}{C_1 + C_2}\right) \cdot C_0}{\frac{C_3 \cdot C_{var}}{C_3 + C_{var}} + \frac{C_1 \cdot C_2}{C_1 + C_2} + C_0}$$



Clapp oscillator with varactor diode in parallel

$$\frac{1}{C_{eq}} = \frac{1}{C_{var}} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Clapp oscillator with varactor diode in series (all capacitors in series)

$$\rightarrow f_{0,min} = \frac{1}{2\pi\sqrt{L \cdot C_{eq,max}}} \quad , \quad f_{0,max} = \frac{1}{2\pi\sqrt{L \cdot C_{eq,min}}}$$

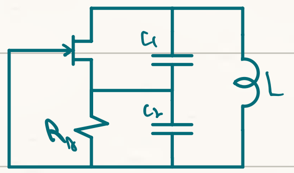
at C_{var} max at C_{var} min

- tuning range: $f_{0,min} \rightarrow f_{0,max}$ (large range is better)

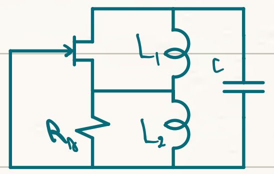
+ field effect transistor oscillator:

- all BJTs in previous oscillators can be converted to FETs.

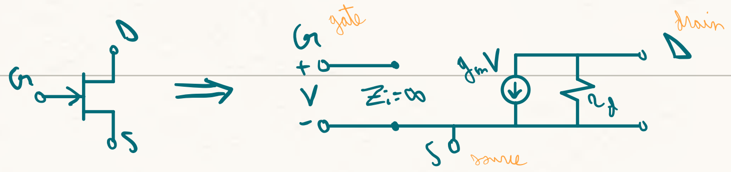
colpitts



hartley

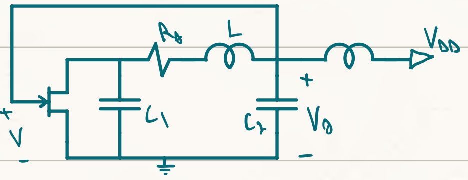


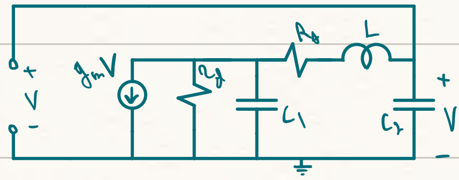
- equivalent model of FET is the same as BJT with $Z_i = \infty$:



$$g_m = g_{m0} \cdot \left(\frac{I_D}{I_{DSS}}\right)^{1/2}$$

- the FET pierce oscillator shown below is analyzed as follows:





$\because$ $V = V_0 \rightarrow H = 1$, if $r_d >> \frac{1}{\omega C_1}$ assume open circuit

$$\rightarrow V_0 = -g_m \cdot \overset{V_0}{V} \cdot \overset{\text{current division}}{\boxed{\frac{X_{C_1}}{X_{C_1} + R_s + X_L + X_{C_2}}}} \cdot X_{C_2}$$

$$\rightarrow 1 = \frac{-g_m \cdot X_{C_1} \cdot X_{C_2}}{X_{C_1} + X_{C_2} + X_L + R_s} = \frac{g_m \cdot \frac{1}{\omega^2 C_1 C_2}}{R_s + j\left(\omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2}\right)}$$

for $GH = 1$: $\omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2} = 0$

$\therefore$ phase condition : $\omega_0 = \frac{1}{\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}}$

& magnitude condition : $\frac{g_m}{R_s \cdot \omega_0^2 \cdot C_1 C_2} \geq 1$

MOHAMMAD SANAD ALTAHER 130806

EE 524: homework #2

7.1 : $\because$ $H = \frac{V_1}{V_o}$ $\wedge$ $G = \frac{V_o}{V_1}$, $V_1 = -g_m V_o \cdot 1k$

$\rightarrow H = -g_m \cdot 1k$ $\wedge$ $V_o = -g_m \cdot V_1 \cdot Z_L \rightarrow G = -g_m \cdot Z_L$

s.t. $Z_L = X_L // X_C // R_L \rightarrow \frac{1}{Z_L} = \frac{1}{j\omega L} + j\omega C + \frac{1}{R_L}$

$\therefore G = -g_m \cdot \frac{1}{\frac{1}{j\omega L} + j\omega C + 1/R_L}$

$$\rightarrow GH = \frac{g_m^2 \cdot 1k}{1/j\omega L + j\omega C + 1/5k} \geq 1$$

* phase condition: $\arg\{GH\} = 0 \rightarrow \text{Im}\{GH\} = 0$

$$\rightarrow \frac{1}{j\omega L} + j\omega C = 0 \rightarrow \boxed{f_o = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}}$$

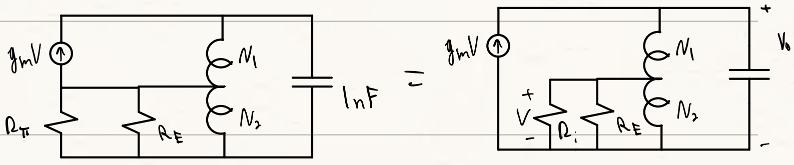
* magnitude condition: $\frac{g_m^2 \cdot 1k}{1/5k} \geq 1 \rightarrow \boxed{g_m^2 \cdot 5\times10^6 \geq 1}$

✓

from phase condition: $\boxed{L = 2.533\ \mu H}$ for 10 MHz

from magnitude condition: $\boxed{g_m \geq 0.45\ mS}$ ✓

7.3 : the AC equivalent circuit is :



$\rightarrow V_o = g_m V \cdot Z_L \rightarrow G = g_m \cdot Z_L$

$$\wedge\ V = V_o \cdot \frac{X_{L_2}//R_E//R_i}{X_{L_1} + X_{L_2}//R_E//R_i} \rightarrow H = \frac{X_{L_2}//R_E//R_i}{X_{L_1} + X_{L_2}//R_E//R_i}$$

$$\rightarrow \frac{GH}{g_m} = (X_{L_2}//R_E//R_i + X_{L_1}) // X_C \geq 3$$

2mA $g_m = 76.9\ mS$

After calculating I_C, r_i is found as $12.9\ \Omega \rightarrow R_E // r_i \simeq r_i$

$$\rightarrow GH = \left(\frac{j\omega L_2 \cdot r_i}{j\omega L_2 + r_i} + j\omega L_1\right) // \frac{1}{j\omega C} \cdot g_m$$

- assuming that $r_i >> \omega L_2$

$$\rightarrow GH = (j\omega L_1 + j\omega L_2) // \frac{1}{j\omega C} \cdot g_m$$

$$H = \frac{L_2}{L_1 + L_2}$$

$$\rightarrow \boxed{GH = \frac{L_T/L \cdot g_m}{j\omega(L_1+L_2) + \frac{1}{j\omega C}}}$$

$$G = \frac{r_i R_E}{r_i + R_E} \cdot \left(\frac{L_1+L_2}{L_2}\right)^2$$

inductive transformation

for $\text{Im}\{GH\} = 0 \rightarrow \omega = \frac{1}{\sqrt{(L_1+L_2) \cdot C}}$

$\therefore L_{total} = L_1 + L_2 = 1.013\ \mu H$ for $5\ MHz$

magnitude condition: $\frac{L_T \cdot g_m}{C} \geq 3$

however, $\omega L_2 << r_i \rightarrow L_2 = 41\ nH$

$\therefore \frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 \rightarrow \boxed{\frac{N_1}{N_2} = 4.87}$

7.4: phase condition for a colpitts FET oscillator: $\omega = \frac{1}{\sqrt{L \frac{C_1 C_2}{C_1 + C_2}}}$

magnitude condition: $\frac{g_m}{R_s \omega_o^2 C_1 C_2} \geq 2.5$

from the phase condition: $C_{eq} = 2.533 \times 10^{-10}\ F$

$\therefore R_s = \frac{\omega_o \cdot L}{Q_L} = \frac{\pi}{5}\ \Omega$ X R_s is source

from mag. condition: $C_1 C_2 \leq 8.06 \times 10^{-19}$

$\therefore \boxed{C_1 = C_2 = 0.51\ nF}$ satisfies both conditions

$C_1 = 379.5\ pF$, $C_2 = 759\ pF$

7.6: phase condition of a colpitts: $\omega_0 = \frac{1}{\sqrt{LC_{eq}}}$

→ for 3.5 MHz: $C_{eq} = 1.38$ nF ✓

magnitude condition: $r r_\pi \le \frac{1+\beta}{\omega_0^2 C_1 C_2} - \frac{L}{C_1}$

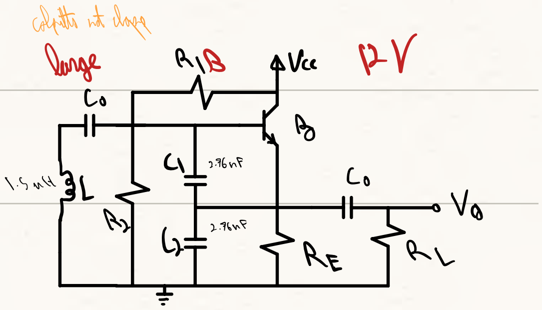
assuming $C_1 = C_2 = 2C_{eq} = 2.76$ nF & $r = \frac{\omega_0 L}{Q_u} = 0.22\Omega$

→ $r_\pi \le 122.15$ kΩ

∵ $r_\pi = \frac{0.026\beta}{I_C}$ → $I_C \ge \frac{0.026\beta}{122.15k}$

∴ $I_C \ge 2.13 \times 10^{-8}$ A

2×10^{-5} A



homework #2 redone:

7.1: ∵ L∥C → $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$ → L = 2.53 uH

∵ $H = \frac{V_1}{V_o} = -g_m \cdot 1k$

& $G = \frac{V_o}{V_1} = -g_m \cdot 5k$

→ $G \cdot H \ge 1$ → $g_m^2 \cdot 5\times10^6 \ge 1$

∴ $g_m \ge 0.447$ mS

7.3: equivalent:

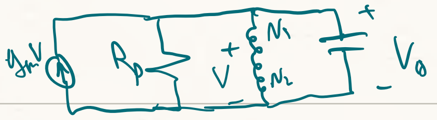
$r_i = \frac{1}{g_m}$, $g_m = \frac{I_C}{0.026}$

to find Ic: $-12 + \frac{I_C}{\beta} \cdot 360k + 0.7 + I_C \cdot 2k = 0$

→ $I_C = 2.02$ mA → $r_i = 12.9\ \Omega$

$\rightarrow R_i // R_E \approx R_i$

inductive Transformation :



$$R // L_2 + L_1 = R_p // (L_1 + L_2)$$

$$\frac{1}{L_1 + \frac{R\cdot L_2}{R+L_2}} = \frac{1}{R_p} + \frac{1}{L_1+L_2} \rightarrow \frac{1}{R_p} = \frac{R+L_2}{L_1(R+L_2)+R\cdot L_2} - \frac{1}{L_1+L_2}$$

$$\rightarrow \frac{1}{R_p} = \frac{(R+L_2)\cdot(L_1+L_2) - L_1(R+L_2)-RL_2}{(L_1(R+L_2)+RL_2)\cdot(L_1+L_2)}$$

$$\rightarrow R_p = \frac{(L_1(R+L_2)+RL_2)\cdot(L_1+L_2)}{(R+L_2)\cdot(L_1+L_2) - L_1(R+L_2) - RL_2}$$

$$\rightarrow R_p = \frac{(L_1R + L_1L_2 + RL_2)(L_1+L_2)}{\cancel{RL_1} + \cancel{RL_2} + \cancel{L_1L_2} + L_2^2 - \cancel{RL_1} - \cancel{L_1L_2} - \cancel{RL_2}}$$

$$\rightarrow R_p = \frac{L_1^2R + L_1^2L_2 + 2RL_1L_2 + L_1L_2^2 + L_2^2R}{L_2^2}$$

if $L_1^2\cdot L_2$ and $L_1L_2^2$ are neglected

$$\rightarrow \boxed{R_p \approx R\cdot\left(\frac{L_1+L_2}{L_2}\right)^2}$$ inductive transform

$\therefore L_1 + L_2 = L_T$ and $\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 \rightarrow L_2 = L_1\cdot\left(\frac{N_2}{N_1}\right)^2$

$$\rightarrow R_p \approx R_i \cdot \left(\frac{\cancel{L_1}\left(1+\left(\frac{N_2}{N_1}\right)^2\right)}{\cancel{L_1}\left(\frac{N_2}{N_1}\right)^2}\right)^2 = R_i\cdot\left(\frac{N_1^2+N_2^2}{N_1^2}\cdot\frac{N_1^2}{N_2^2}\right)^2$$

$$\rightarrow R_p \approx R_i \cdot \left(1+\left(\frac{N_1}{N_2}\right)^2\right)^2$$

$\therefore f_0 = \frac{1}{2\pi}\cdot\frac{1}{\sqrt{L_T\cdot C}} \rightarrow L_T = 1.01\,\mu H$ for 5 MHz

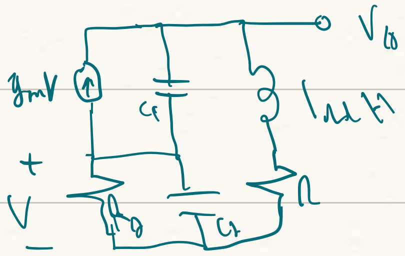
∴ $H = \frac{V}{V_o} = \frac{L_2}{L_1+L_2}$ & $G = \frac{V_o}{V} = g_m \cdot R_p$

→ $G \cdot H = g_m \cdot \Omega_i \cdot \frac{L_1+L_2}{L_2} = \frac{L_1+L_2}{L_2}$

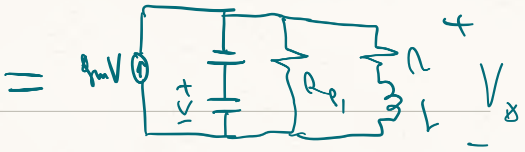
∴ $G \cdot H \geq 3 \rightarrow L_2 = \frac{1}{3} L_T \approx 0.33$ mH

→ $L_1 \approx 0.67$ mH → $\frac{N_1}{N_2} = \sqrt{2}$

7.4: equivalent circuit :



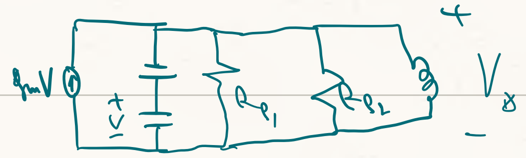
transform R_E to R_p



$R_{p_1} = R_E \cdot \left(\frac{C_1+C_2}{C_1}\right)^2$

∴ $f_o = 10$ MHz $= \frac{1}{2\pi}\sqrt{\frac{1}{L \cdot \frac{C_1C_2}{C_1+C_2}}}$ → $C_{eq} = 0.2533$ nF

→



$R_{p_2} = Q \cdot \omega L$

→ $R_{p_2} = 6.283k$

→ $G = g_m \cdot (R_{p1} // R_{p2}) \geq 2.5$

If $R_{p2} >> R_{p1}$ → $G = g_m \cdot R_{p1} \geq 2.5$

assuming $R_{p_1} = R_E // \Omega_7 \cdot \left(\frac{C_1+C_2}{C_1}\right)^2$

→ $G \cdot H = g_m \cdot \Omega_i \cdot \frac{C_1+C_2}{C_1} \geq 2.5$

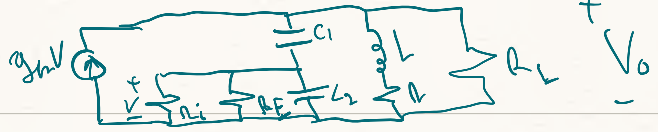
→ $C_2 \geq 1.5 C_1$

→ $\frac{1.5 C_1^2}{2.5 C_1} = 0.2533$ nF

$C_2 = 0.633$ nF → $C_1 = 0.422$ nF

If $G \cdot H \geq 3$ → $C_1 = 0.38$ nF & $C_2 = 0.76$ nF

7.6:



→

$$R_{p_1} = r_i \cdot \left(\frac{C_1+C_2}{C_1}\right)^2 , \quad R_{p_2} = Q \cdot WL$$

→ $G = g_m \cdot \left(R_{p_1} // R_L // R_{p_2}\right)$ if $R_{p_1} << R_{p_2}$ and R_L

∧ $H = \frac{C_1}{C_1+C_2}$ → $GH \approx \frac{C_1+C_2}{C_1} \geq 1$

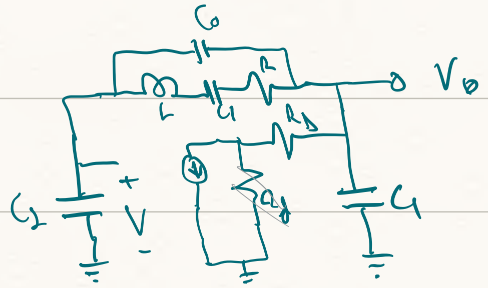
∧ $7\pi \text{ MHz} = \frac{1}{\sqrt{LC_{eq}}}$ → $C_{eq} = 1.38 \text{ nF}$

→ $I_C \geq 2.3 \times 10^{-5}$ A if $C_1 = C_2 = 2.76 \text{nF}$

→ $-12 + R_B \cdot \frac{I_C}{100} + 0.7 + R_E \cdot I_C = 0$

→ $\frac{R_B}{100} + R_E = \frac{12-0.7}{I_C}$

7.22: equivalent circuit:



Second practice:

12/2020

Q2: $R_1 = 40\ \Omega$, $C_1 = 3\ fF$, $C_0 = 6.2\ pF$

a) $W_s = \frac{1}{\sqrt{L_1 C_1}}$

$\therefore f_a = 26\ MHz = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L \frac{C_1 C_0}{C_1 + C_0}}} \rightarrow L = 12.44 mH$

$\rightarrow f_s = \frac{f_a}{\left(1 + \frac{C_1}{C_0}\right)^{1/2}} \rightarrow f_s = 25.9939\ Hz$

$\rightarrow$ pulling range: $f_a - f_s = 6288\ Hz$

$\rightarrow Q_u = \frac{W_s \cdot L}{R} = 51010$

b) $C_p = C_0 + 32 pF$

$f_a' = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{\underset{\text{must be very accurate}}{L} \cdot \frac{C_1 C_p}{C_1 + C_p}}} \approx 25.9947\ MHz$

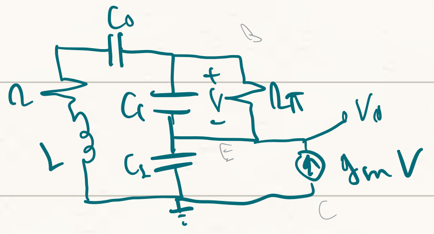
or f_s = no change since series

$\rightarrow f_a' = f_s \cdot \left(1 + \frac{C_1}{C_p}\right)^{1/2} = 25.9957\ MHz$

$f_s \leq f_a' \leq f_a$

Q3: a) $-10 + I_c \cdot \frac{R_B}{100} + 0.7 + I_c \cdot 10k = 0$

$\rightarrow I_c = 0.5\ mA$



b) $Z_1 = X_{C_1} \parallel R_\pi$, $Z_2 = X_{C_2}$, $Z_3 = R + X_L$

$$(Z_1 + Z_2 + Z_3) R_\pi + Z_1 (Z_2 + Z_3) + \beta Z_1 Z_2 = 0$$

$Z_1 \approx X_{C_1}$

$$\rightarrow (X_{C_1} + X_{C_2} + X_L) R_\pi + R R_\pi + X_{C_1} (X_{C_2} + X_L)$$
$$+ X_{C_1} \cdot R + \beta X_{C_1} X_{C_2}$$

$$\rightarrow (X_{C_1} + X_{C_2} + X_L) R_\pi + X_{C_1} R = 0 \quad \text{Phase}$$

$$\wedge \quad R R_\pi + X_{C_1} (X_{C_2} + X_L) + \beta X_{C_1} X_{C_2} = 0$$

$$\therefore \frac{R + R_\pi + R_\pi}{j w C_1} + j w L R_\pi = 0$$

$w^2 L = \frac{2}{C}$

$\rightarrow w = \frac{1}{\sqrt{L \frac{C}{2}}}$

$$\rightarrow (R + 2R_\pi) \cdot \frac{1}{w C_1} = w L \cdot R_\pi$$

$$\therefore \boxed{\frac{R + 2R_\pi}{R_\pi} = w^2 C_1 L}$$

$\approx R_\pi = \frac{0.026\beta}{I_C}$

$\rightarrow C_1 = 1.2665\ nF$
$= C_2$

$$\wedge \quad R R_\pi - \frac{1}{w^2 C_1 C_2} + \frac{L}{C_1} - \frac{\beta}{w^2 C_1 C_2} = 0$$

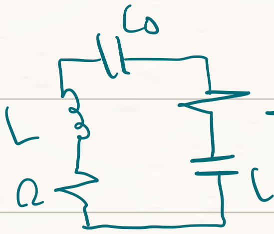
$$\therefore \boxed{R R_\pi = \frac{\beta + 1}{w^2 C_1 C_2} - \frac{L}{C_1}}$$

$$R = \frac{w_0 L}{Q_u} = \frac{\pi}{25}$$

$$\rightarrow R_\pi \leq \frac{25}{\pi} \cdot \left[\frac{\beta + 1}{w^2 C^2} - \frac{L}{C}\right] = 31102.5\ \Omega$$

$$\therefore I_C \geq \frac{0.026\beta}{31.1k\Omega} = 83.6\ mA$$

c)



$C_{eq} = \frac{C_1}{2}$

$\because R = \frac{\pi}{25}$

$$\wedge \quad W_0 = \frac{1}{\sqrt{L \cdot \frac{C_0 C_{eq}}{C_0 + C_{eq}}}}$$

$\rightarrow C'_{eq} = 6.3326 \times 10^{-6}$ F

$\therefore -R_i = \frac{-g_m}{\omega^2 C_1^2}$, $R_i \geq R$ mag

$\therefore g_m = \frac{I_C}{0.026} = 19.23$ mA

$\rightarrow C_1 \leq 3.113$ nF

assume $C_1 = C_2 = 3.113$ nF $\rightarrow C_{eq} = 1.5565$ nF

$\rightarrow C_0 = 1.07$ nF

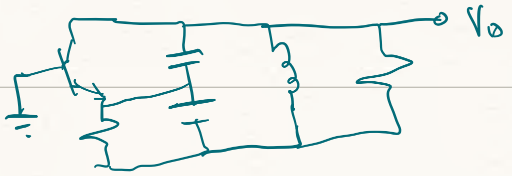
$$C_0 \cdot C'_{eq} + C_{eq} C'_{eq} = C_0 C_{eq}$$

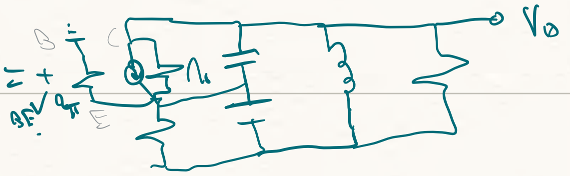
$$\rightarrow C_0 = \frac{C_{eq} C'_{eq}}{C_{eq} - C'_{eq}}$$

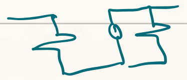
12/2021

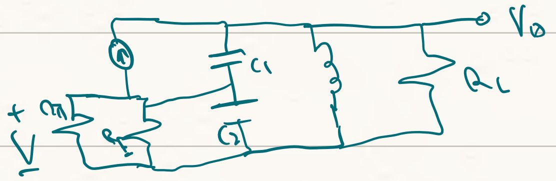
Q2: a) C_B is a bypass transistor that grounds the base

C_o is used to isolate the load from the DC bias of the transistor, only AC passes to output.

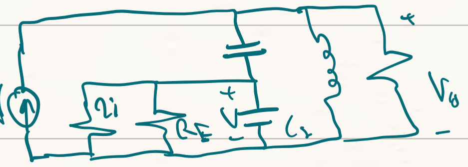






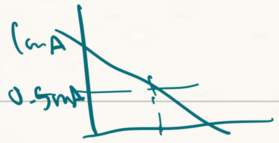


$\therefore R_{eq} =$



b) $\therefore -10 + I_C \cdot \frac{R_B}{100} + 0.7 + I_C R_E = 0$

$$\rightarrow \frac{R_B}{100} + R_E = \frac{9.3}{I_C}$$



$$\rightarrow \frac{10}{R_E} = 1\ mA \rightarrow R_E = 10k\Omega$$

$$\therefore R_B = 860\ k\Omega$$

c) $\because H = \frac{C_1}{C_1+C_2}$ and $G = g_m \frac{R_L R_{eq}}{R_L + R_{eq}} \simeq g_m R_{eq}$

$R_E >> r_i$

assuming $R_L >> R_{eq}$ & $R_{eq} \simeq r_i \cdot \left(\frac{C_1+C_2}{C_1}\right)^2$

$$\rightarrow GH = \frac{C_1+C_2}{C_1} = 2 \rightarrow C_1 = C_2$$

$$\frac{1}{(\omega(C_1+C_2))^2} << r_i^2 \rightarrow C_1 + C_2 = 2C_1 = 0.968\ nF$$

$$\rightarrow C_1 = C_2 = 0.484\ nF$$

$$\because \omega_0 = \frac{1}{\sqrt{L \frac{C_1C_2}{C_1+C_2}}} \rightarrow L = 1.047\ mH$$

Q3: a) $Z_3 = r + j\omega L$, $Z_2 = \frac{1}{j\omega C_2}$, $Z_1 = \frac{1}{j\omega C_1}$

$$\rightarrow (Z_1 + Z_2 + Z_3) r_\pi + Z_1(Z_2 + Z_3) + \beta Z_1 Z_2 = 0$$

$$\rightarrow \frac{r_\pi}{j\omega C_2} + \frac{r_\pi}{j\omega C_1} + j\omega L r_\pi + \frac{r}{j\omega C_1} = 0 \quad \& \ C_1 = C_2$$

$$\therefore \frac{2r_\pi + r}{\omega C_1} = \omega L r_\pi \rightarrow \boxed{\frac{2r_\pi + r}{r_\pi} = \omega^2 LC}$$

$$\& \quad r r_\pi + \frac{L}{C_1} - \frac{1}{\omega^2 C_1^2} - \frac{\beta}{\omega^2 C_1^2} = 0$$

$$\therefore r r_\pi \leq \frac{\beta+1}{\omega^2 C_1^2} - \frac{L}{C_1}$$

$$\because r_\pi >> r \rightarrow 2 = \omega^2 L C_1$$

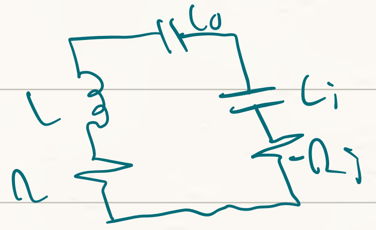
$$\therefore C_1 = C_2 = 4.05\ nF$$

$$\rightarrow r_\pi \leq \frac{Q_n}{\omega \cdot L} \cdot \left[\frac{\beta+1}{\omega^2 C_1^2} - \frac{L}{C_1}\right] = 38.93\ k\Omega$$

$\because \quad r_\pi = \frac{0.026\beta}{I_C} \rightarrow I_C \geq 66.78\, \mu A$

$\therefore \quad g_m \geq 2.57\ mS$

h)



$R_i \geq R = \frac{\pi}{20}\ \Omega$

$\sim \quad R_i = \frac{g_m}{(\omega_0 C_1)^2} \quad \because C_1 = C_2, \ g_m = \frac{I_C}{0.026}$

$\therefore \quad C_1 = C_2 = 90.9\ nF \rightarrow C_i = 45.45\ nF$

$\rightarrow \omega_0 = \frac{1}{\sqrt{L \frac{C_o C_i}{C_o + C_i}}} \rightarrow C_o = 1.939\ nF$

12/2022:

Q1: a) $Z_3 = r + j\omega L$, $Z_2 = Z_1 = \frac{1}{j\omega C_2} = \frac{1}{j\omega C_1}$

from $(Z_1 + Z_2 + Z_3)\cdot r_\pi + Z_1(Z_2 + Z_3) + \beta Z_1 Z_2 = 0$

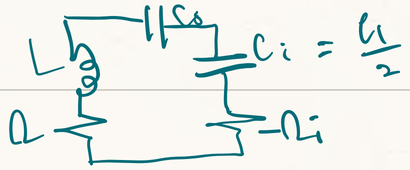
$\rightarrow$ phase condition: $\frac{r + 2r_\pi}{r_\pi} = \omega_0^2 \cdot LC_1 \simeq 2$

$\therefore \; C_1 = C_2 = 5.07 \times 10^{-9}$ F

from mag. condition: $r r_\pi \leq \frac{\beta+1}{(\omega_0 \cdot C_1)^2} - \frac{L}{C_1}$

& $r = \frac{\omega_0 L}{Q_u} = \frac{\pi}{25}$ Ω

$\rightarrow r_\pi \leq 7.963$ kΩ $\therefore I_C \geq 0.335$ mA

b) 

$\rightarrow r = \frac{\pi}{25}$ Ω

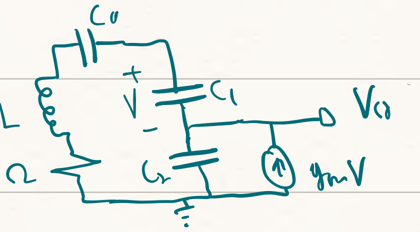
& $r_i \geq r$ (mag. condition)

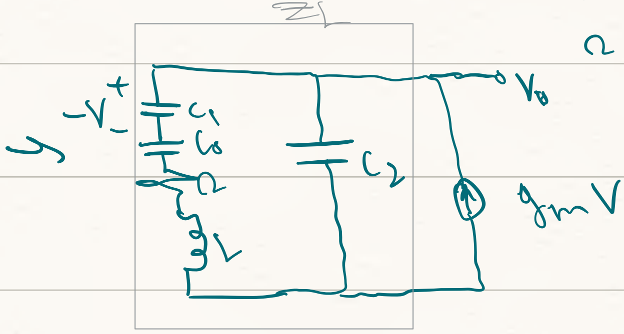
$\rightarrow \frac{g_m}{(\omega_0 C_1)^2} \geq \frac{\pi}{25} \rightarrow C_1 = C_2 \leq 6.23$ nF

& $f_0 = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L \cdot \frac{\frac{C_i}{2} \cdot C_0}{C_{1/2} + C_0}}} \rightarrow C_0 = 13.56$ nF

$\therefore -10 + I_C \cdot \frac{R_B}{100} + 0.9 + I_C \cdot R_E = 0 \rightarrow R_B = 860$ kΩ

C) common collector:





$C_{eq} \cdot C_0 = x C_{eq} + C_0 x$

$\rightarrow C_0 = \frac{x C_{eq}}{C_{eq} - x}$

$\rightarrow G = g_m \cdot Z_L$, $H = \frac{-X_{C_1}}{X_{C_1} + X_{C_0} + r + X_L}$

$$\wedge\ Z_L = \frac{X_{C_2}\cdot(X_{C_1}+X_{C_0}+r+X_L)}{X_{C_2}+X_{C_1}+X_{C_0}+r+X_L}$$

$$\therefore\ GH = g_m \cdot \frac{-X_{C_1}\cdot X_{C_2}}{X_{C_2}+X_{C_1}+X_{C_0}+r+X_L}$$

$$\rightarrow\ GH = \frac{g_m \cdot \frac{1}{w^2 C_1 C_2}}{\frac{1}{jwC_2}+\frac{1}{jwC_1}+\frac{1}{jwC_0}+jwL+r} \geq 1$$

real $\rightarrow$ $\operatorname{Im}\{GH\}=0 \rightarrow w = \dfrac{1}{\sqrt{L\cdot\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_0}}}}$

mag: $GH = \dfrac{g_m}{w^2 C_1 C_2 r} \geq 1$

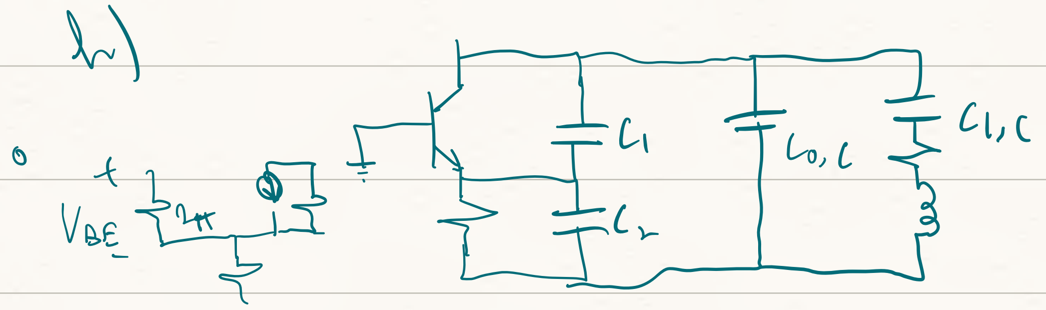
Q2: a) $\because$ 26 MHz $= f_a = \dfrac{1}{2\pi}\cdot\dfrac{1}{\sqrt{L\frac{C_1C_0}{C_1+C_0}}}$

$\rightarrow L = 11.51$ mH

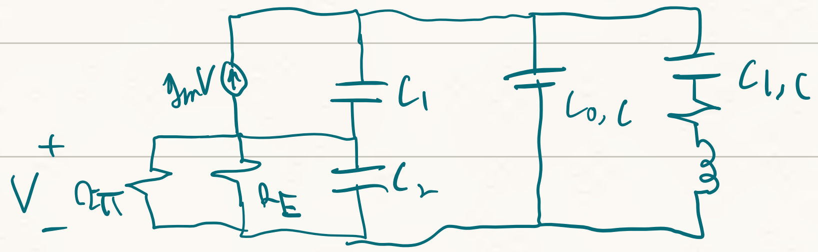
$\rightarrow f_s = 25.9937$ MHz

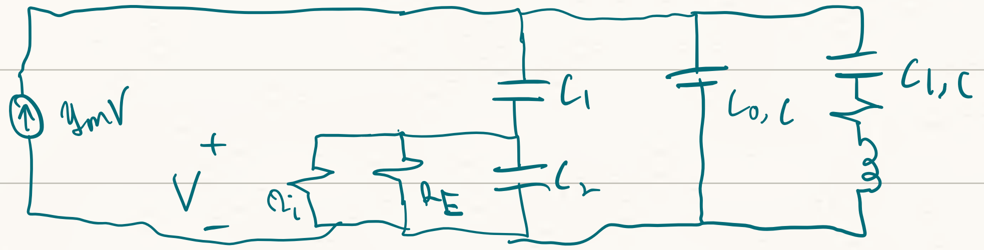
$\rightarrow$ pulling range = 6.3 kHz

b)

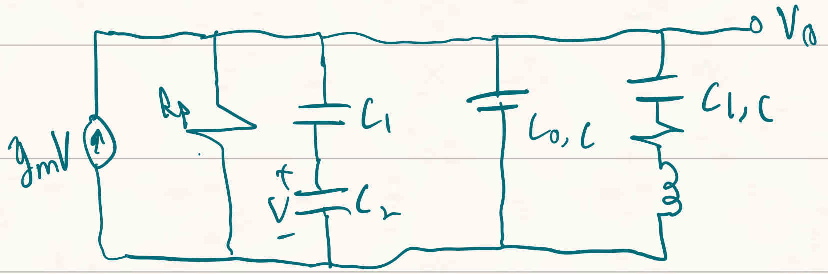


$\rightarrow$





$$\rightarrow \quad R_p \approx r_i \cdot \left(\frac{C_1+C_2}{C_1}\right)^2$$



$$\rightarrow \quad \omega = \frac{1}{\sqrt{L \cdot \frac{(C_L+C_0)\cdot C_1}{C_L+C_0+C_1}}}$$

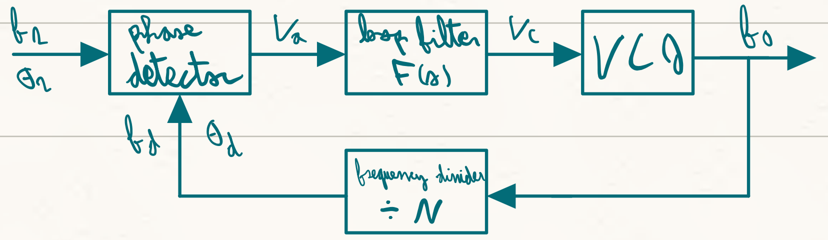
$$\therefore \quad f_o = 26.4439 \text{ MHz}$$

$$\rightarrow \quad \grave{f}_n = 26.4439 \cdot \left(1 + \frac{C_1}{C_0+C_L}\right)^{\frac{1}{2}}$$

$$\rightarrow \quad \grave{f}_n = 26.444$$

Chapter 8 :

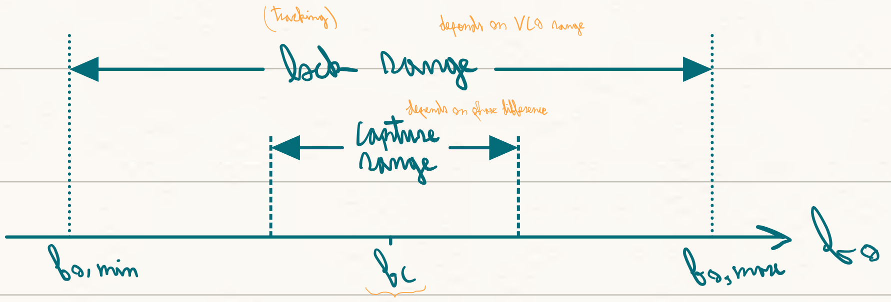
- phase - locked loop (PLL) is a negative feedback system in which the output frequency and phase are locked to those at the input.



- the voltage output of the phase detector is proportional to the difference in phase ($\theta_r - \theta_d$)
- the control voltage changes the output frequency until $f_d = f_r$. lock condition

if $\theta_d = \theta_r$, output of phase detector is zero

- at the locked state : $f_d = f_r$, $\theta_d \approx \theta_r$, $f_o = N \cdot f_r$
- $\theta_r - \theta_d \neq 0$ is required so that the control voltage keeps the PLL locked and allows the VCO to track changes in the input.



free - running frequency

frequency when the control input to the VCO is zero ($V_c = 0V$)

* Capture range : the range of frequencies over which the PLL can lock.

* lock range : the range of frequencies over which the PLL can maintain its locked state. equal or less than the range of the VCO

- the output frequency of the PLL can be increased by increasing the frequency allows PLL to be used as a frequency multiplier or synthesizer

divider ratio (N). $\therefore f_o = N f_r \rightarrow f_o \propto \overset{\text{integer}}{\widetilde{N}}$

- the output voltage of the phase detector is found from:

$$V_a = k_d \cdot (\theta_r - \theta_d)$$ k_d : phase detector gain factor in V/rad

- the VCO's output's deviation from its free-running frequency is:

$$\Delta\omega = (\underbrace{\omega_o}_{\text{output angular frequency}} - \underbrace{\omega_c}_{\text{free running angular frequency}}) = \underbrace{k_o}_{\text{gain factor of VCO}} \left(\frac{\text{rad/s}}{\text{V}}\right) V_c \rightarrow \omega_o = \omega_c + k_o V_c$$

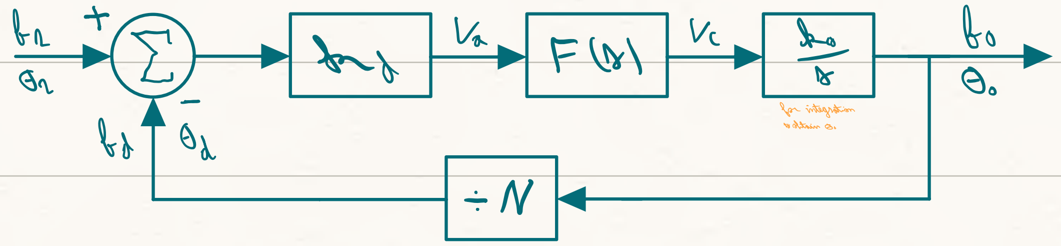
- the output phase is found from the output angular frequency as:

$$\omega_o = \frac{d\theta_o}{dt} \longrightarrow \theta_o = \int \omega_o \, dt$$

- at the output of the frequency divider, the phase and frequency are:

$$f_d = \frac{f_o}{N} \quad , \quad \theta_d = \frac{\theta_o}{N}$$

- therefore, the linearized PLL's block diagram becomes:



- the transfer function can be found as:

$$\frac{\theta_o(s)}{\theta_r(s)} = \frac{k_d \cdot F(s) \cdot \frac{k_o}{s}}{1 + k_d \cdot F(s) \cdot \frac{k_o}{s} \cdot \frac{1}{N}} = \frac{G(s)}{1 + \frac{G(s)}{N}}$$

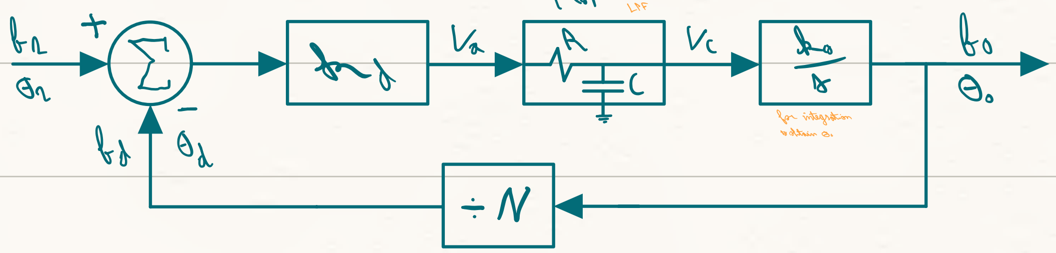
which gives four parameters that can be controlled: k_d, k_o, $F(s)$, and N

- a first order PLL has no loop filter, hence the transfer function becomes:

$$\therefore F(s) = 1 \rightarrow \frac{\theta_o(s)}{\theta_r(s)} = \frac{k_d k_o}{s + k_d k_o \cdot \frac{1}{N}} = \frac{N k_v}{\underbrace{s + k_v}_{\substack{\text{low pass filter} \\ \text{(PLL was linearized around output frequency)}}}} , \underbrace{k_v = \frac{k_d k_o}{N}}_{\substack{\text{bandwidth of filter} \\ \omega_n = k_v \text{ rad/s}}}$$

- If a first order low-pass filter is used as a loop filter, then the PLL is considered second order since it behaves as a second-order LPF.

LPF



$$\rightarrow \quad F(s) = \frac{1}{1+sRC} = \frac{\omega_L}{\omega_L + s} \quad , \quad \omega_L = \frac{1}{RC}$$

$$\rightarrow \quad \frac{\theta_o(s)}{\theta_r(s)} = \frac{k_d k_o \omega_L}{(\omega_L + s)s + k_d k_o \frac{\omega_L}{N}} = \frac{N k_v}{(\omega_L + s)\cdot\frac{s}{\omega_L} + k_v} = \frac{N}{\left(\frac{s}{\omega_n}\right)^2 + \left(\frac{2\zeta}{\omega_n}\right)s + 1}$$

$$\therefore \quad \overbrace{\omega_n}^{\text{natural frequency}} = \sqrt{k_v \omega_L} \quad , \quad 2\overbrace{\zeta}^{\text{damping ratio}} = \sqrt{\frac{\omega_L}{k_v}} \quad , \quad k_v = \frac{k_o k_d}{N}$$

- a butterworth filter has a damping ratio of $\frac{1}{\sqrt{2}}$ and its 3-dB bandwidth is equal to its natural frequency ($\omega_h = \omega_n$).

- The 3-dB bandwidth can be calculated from:

$$\omega_h = \omega_n \left[1 - 2\zeta^2 + (2 - 4\zeta^2 + 4\zeta^4)^{1/2}\right]^{1/2}$$

and the rise time is found from: $t_r \approx \frac{2.2}{\omega_{h\,(\text{rad/s})}}$ 10% to 90%

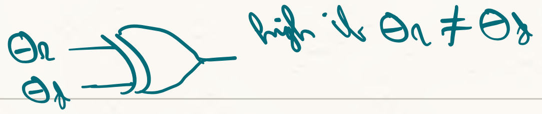
- therefore, a larger bandwidth gives a faster rise time but includes more noise.

+ There are three categories of phase detectors:

1- digital, 2- analog, 3- sampling.

+ digital phase detectors:

1- XOR phase detector:

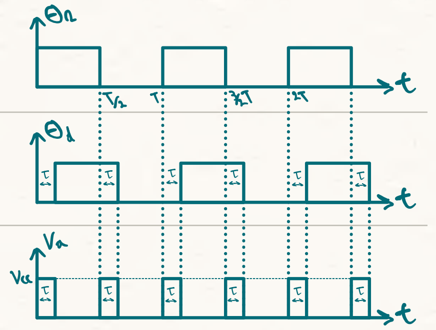


duty cycle must be 50%, if it is less then the V_{dc} - Φ_e plot tappers.

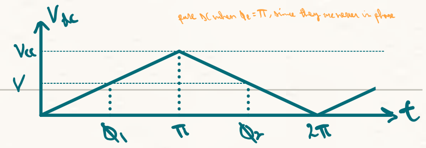
- the average DC value of V_a is: $V_{dc} = \frac{2\tau \cdot V_{cc}}{T}$

- the phase difference is found from:

$$\Phi_e = (\theta_r - \theta_d) = \frac{\tau}{T} \cdot 2\pi \rightarrow \tau = \frac{T}{2\pi} \cdot \Phi_e$$



- when V_{dc} is plotted as a function of Φ_e, a phase ambiguity is noticed. $V_{dc} = \frac{V_{cc}}{\pi} \Phi_e = \overbrace{k_d}^{\text{phase detector gain}} \Phi_e$



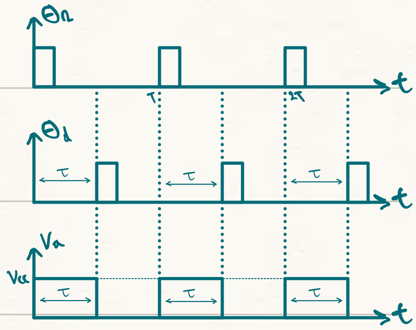
Φ_1 and Φ_2 both produce same V

2- set-reset (RS) flip-flop phase detector.

- solves phase ambiguity, but requires very narrow pulses. (so that they are not high simultaneously)

- the average dc voltage of V_a is: $V_{dc} = \frac{\tau \cdot V_{cc}}{T}$

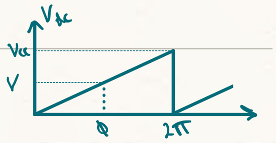
- Φ_e is found from the time delay as:



$$\Phi_e = (\theta_r - \theta_d) = \frac{\tau}{T} \cdot 2\pi \rightarrow \tau = \frac{T}{2\pi} \cdot \Phi_e$$, equivalent to the XOR

$$\rightarrow V_{dc} = \frac{V_{cc}}{2\pi} \cdot \Phi_e \rightarrow k_d = \frac{V_{cc}}{2\pi} , \quad k_{d,RS} = \frac{1}{2} \cdot k_{d,XOR}$$

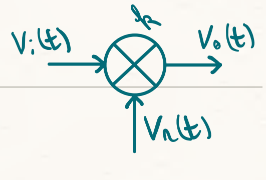
- if V_{dc} is plotted against Φ_e, it can be observed that there is no phase ambiguity, since there is only one pulse per period.



- the RS flip-flop phase detector has the disadvantages of requiring very narrow pulses and monostable vibrator inputs.

+ Analog phase detectors - mixer:

- If the inputs are sinusoidal, a mixer (multiplier) can be used as a phase detector.



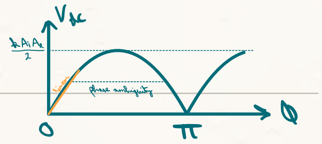
if $V_i(t) = A_i \cos(W_o t)$ & $V_r(t) = A_r \sin(W_o t + \phi)$

$$\therefore V_o(t) = \overset{\text{mixer gain}}{k} \cdot V_i(t) \cdot V_r(t) = \frac{kA_iA_o}{2} \cdot \sin(\phi) + \cancel{\frac{kA_iA_o}{2} \cdot \sin(2W_o t + \phi)}$$

LPF

$$\therefore V_{dc} = V_o(t)\big|_{LPF} = \frac{\overbrace{k\boxed{A_i}A_o}^{\text{constants}}}{2} \sin(\phi)$$

variable (amplifier and limiter used to make constant)



- this phase detector suffers from phase ambiguity, non-linearity for larger phase shifts, and V_{dc} depending on A_i (in addition to $\sin(\phi)$), which is variable.

- for small values of ϕ, the mixer phase detector is linearized as:

$$V_{dc} = \frac{kA_iA_r}{2} \underset{}{\overset{\text{radian}}{\phi}}$$

$\because \sin(\phi) \simeq \phi$ for small ϕ

- the performance of a PLL depends on the characteristics of the VCO.

- the VCO's tuning range determines the tracking range of the PLL.

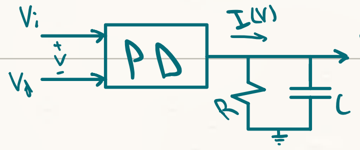
- the VCO's gain factor (k_o) should be high.

modulation sensitivity $k_o = \frac{f_{o,max} - f_{o,min}}{V_{t,max} - V_{t,min}}$ $\left(\frac{Hz}{V}\right)$ or $k_o = \frac{W_{o,max} - W_{o,min}}{V_{t,max} - V_{t,min}}$ $\left(\frac{rad/s}{V}\right)$

- the VCO must have high frequency stability (precise), fast response time, high linearity (operates in large linear region), and high spectral purity (purely sinusoidal).

- the loop filter of the PLL also removes harmonics produced by the phase detector and gives the order of the PLL.

- passive charge pump:

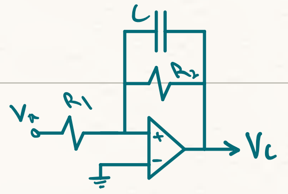


R // C

$$V_c = I(V) \cdot \frac{R}{sRC + 1}$$

cutoff: $\omega_L = \frac{1}{RC}$

- active charge pump:

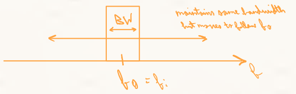


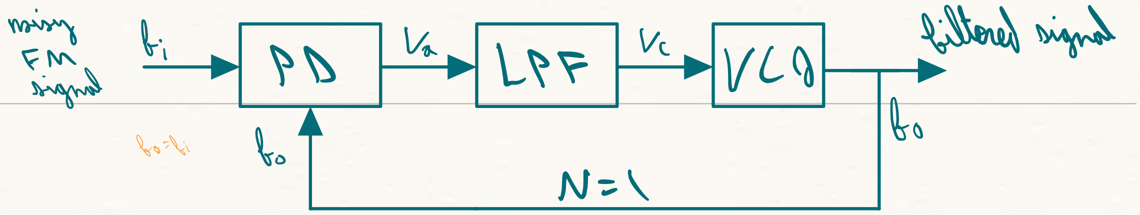
$R_2 // C \times \frac{1}{R_1}$

$$V_c = -\overset{\text{gain}}{\boxed{\frac{R_2}{R_1}}} \cdot \frac{1}{sR_2C + 1} \cdot V_a$$

+ PLL applications:

1- Tracking filter:

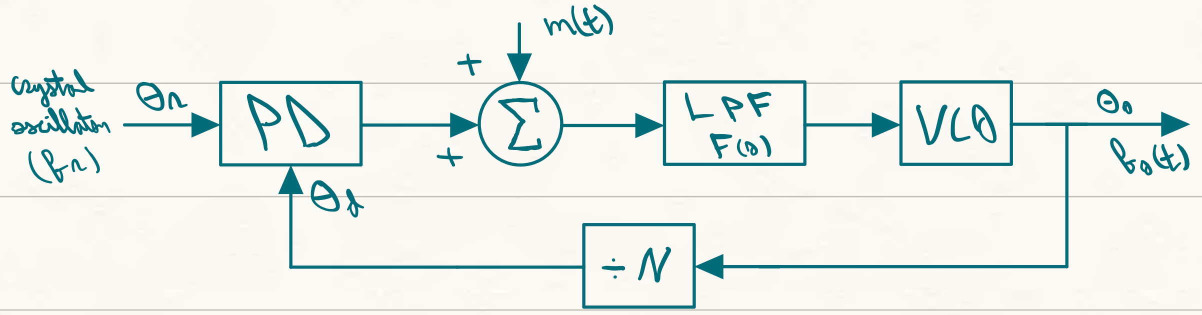




- functions as a filter that remains centered at f_i despite variations.

2- Phase and frequency modulation:

- using a VCO alone for FM modulation may not be reliable since its frequency stability depends on an LC circuit.
- instead, a PLL with a crystal oscillator input can be used as a stable frequency or phase modulator.



- the output is found from:

$$\therefore \Theta_o(s) = \frac{\Theta_r(s) \cdot \left[k_o k_d \frac{F(s)}{s}\right]}{1 + k_o k_d F(s)/Ns} + \frac{M(s) \cdot \left[k_o \frac{F(s)}{s}\right]}{1 + k_o k_d F(s)/Ns}$$

assuming $k_o k_d \frac{F(s)}{Ns} >> 1$ and $\Theta_r(s) = 2\pi f_r$

$$\rightarrow \Theta_o(s) = N \cdot \Theta_r(s) + \frac{N M(s)}{k_d}$$

$$\rightarrow f_o = \frac{1}{2\pi}\frac{d\theta_o(t)}{dt} = \overbrace{N f_{fr}}^{\text{carrier frequency}} + \frac{N}{2\pi k_d}\cdot\frac{dm(t)}{dt}$$ (phase modulation)

- phase modulation: phase is linearly proportional to m(t).

- frequency modulation: frequency is linearly proportional to m(t).

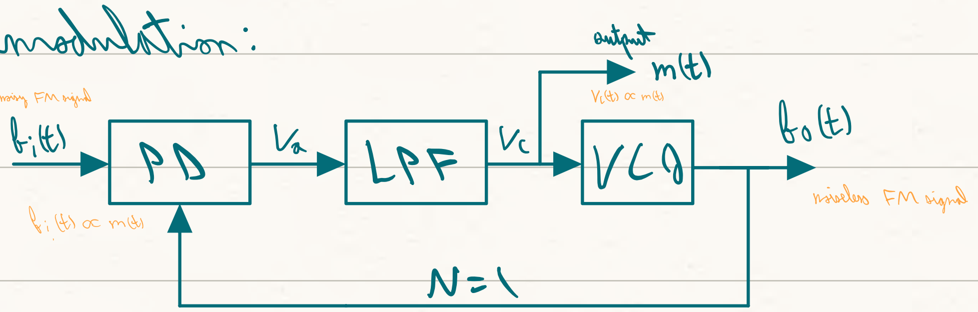
  - a phase modulation circuit can be transformed to a frequency modulation circuit by inserting m(t) into an integrator at first. (and vice versa)

if $m(t) = A_m \sin(w_m t)$:

$$f_o(t) = \overbrace{f_c}^{N f_{fr}} + \boxed{\frac{\overbrace{N A_m w_m}^{\text{only variable}}}{2\pi k_o k_d}}\cdot\cos(w_m t)$$

$$\therefore f_o(t) = f_c + \overbrace{\Delta f}\cos(w_m t) \quad \text{s.t., } 2\Delta f \le \text{lock range} \;\; \forall f$$

3- frequency demodulation:



If the input is AM, the output of the VCO will be purely sinusoidal with a fixed frequency.

- since the output of the VCO is the frequency modulated signal, then its input must be the message signal

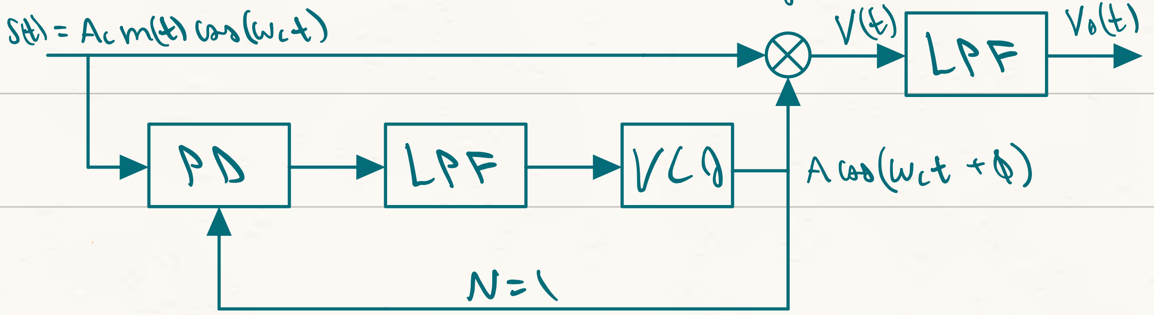
$\therefore N = 1 \rightarrow f_o(t) = f_i(t) \;\wedge\; f_o(t) \propto V_c(t) \;\therefore V_c(t) \propto m(t)$

- the tracking range of the PLL as a demodulator must be greater than or equal to the frequency deviation of the input FM signal.

$$TR = 2(\Delta w)_{max} = 2\overbrace{k_o}^{\text{VCO gain}}\underbrace{k_a}_{\text{LPF gain}}V_{c,max}$$

4- Carrier recovery (synchronization):

- in coherent demodulation, the local oscillator must have the same frequency and phase of the received signal.



$$\therefore \; V(t) = \frac{AA_c}{2} m(t) \cos(\phi) + \frac{AA_c}{2} m(t) \cos(2W_c t + \phi)$$

- when the PLL is locked $\phi \approx 0$, therefore:

$$V_o(t) = \frac{AA_c}{2} \cdot m(t)$$

- integrated circuits (ICs) reduce system size, power consumption, noise, delay, etc. as compared to discrete components.

- MC14046B is a CMOS PLL IC whose pin diagram is below:

$$\rightarrow f_{min} = \frac{1}{R_2(C + 32pF)}$$

$$\rightarrow f_{max} = f_{min} + \frac{1}{R_1(C + 32pF)}$$

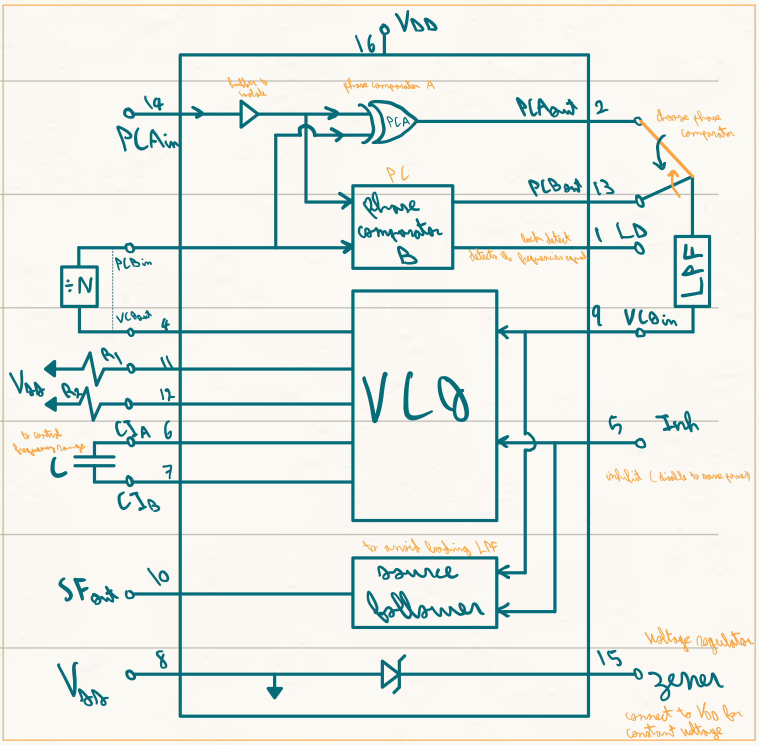
$$\rightarrow k_{VCO} = k_o = \frac{2\pi \Delta f_{VCO}}{V_{DD} - 2}$$

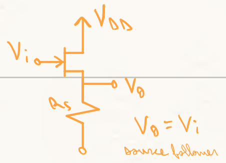
fmax − fmin

(Rad/s)

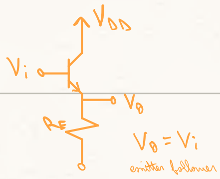
$$\forall \; 10k\Omega < R_1, R_2 < 1M\Omega$$

$$\wedge \; 100pF < C < 0.01 uF$$





source follower



emitter follower

example 8.1:

1st order → $TF = \frac{N k_v}{s + k_v}$, $k_v = \frac{k_o k_\phi}{N}$

∵ $f_o = 1MHz \rightarrow N = \frac{1M}{25k} = 40$

& $k_v = \frac{200\pi \cdot 2}{40} = 10\pi$ ∴ $TF = \frac{400\pi}{s + 10\pi}$

BW : at 0 rad/s TF = 40 , at w_h $TF = \frac{40}{2} \rightarrow \frac{400\pi}{w_h + 10\pi} = 20$

∴ BW = 10π rad/s

example 8.2:

∵ $\zeta = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{\frac{w_L}{k_v}} \rightarrow w_L = 2k_v = 20\pi$ rad/s

larger bandwidth than 1st order PLL

∵ butterworth → $W_n = \sqrt{W_L \cdot k_v} \rightarrow 10\pi \cdot \sqrt{2} = 14.14\pi$ rad/s

& $t_r = \frac{2.2}{14.14\pi} = 49.5$ ms

example 8.6:

∵ $f_r = 2.5kHz$, $f_o = 100kHz \rightarrow N = 40$

& second order PLL butterworth filter → $\zeta = \frac{1}{\sqrt{2}}$

∵ $2\zeta = \sqrt{\frac{W_L}{k_v}}$, $W_L = \frac{1}{RC}$, $k_v = \frac{k_o k_\phi}{N}$

assuming f_{min} = 40 kHz and f_{max} = 180 kHz , C = 1nF

> 10kΩ limit   ∵ $f_{max} = 2 \cdot f_{min}$

→ $R_2 = 10.77$ kΩ & $R_1 = 10.77$ kΩ $= R_2$

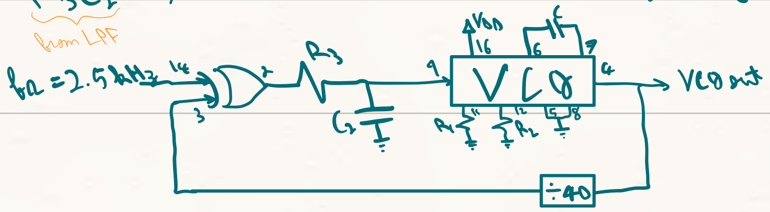
& $k_{VCO} = k_o = \frac{2\pi \cdot \Delta f_{VCO}}{V_{DD} - 2} = 80.783 \times 10^3$ rad/s/V

for XOR: $k_\phi = \frac{V_{CC}}{\pi} = \frac{V_{DD}}{\pi} = \frac{9}{\pi} = 2.864$ V/rad

$\rightarrow f_v = 5.786 \times 10^3$ Hz $\rightarrow W_L = 11572$ rad/s

$\rightarrow W_n = 8182.6$ rad/s $= 2604.6\pi$ rad/s $= W_n = BW$

$\therefore W_L = \dfrac{1}{\underbrace{R_3 C_2}_{\text{from LPF}}}$, if $C_2 = 1nF \rightarrow R_3 = 86.42\ k\Omega$



MOHAMMAD SANAD ALTAHER 130806

EE524: homework #3

8.1: ∵ $f_r = 50$ kHz ∧ $f_o = 1$ MHz → $N = 20$

∵ $k_d = 2$ V/rad , $k_o = 100$ Hz/V $= 200\pi$ rad/s/V

$$\rightarrow k_v = \frac{400\pi \text{ Hz}}{20} = 20\pi \text{ Hz}$$

$$\therefore \text{transfer function} = \frac{400\pi}{s + 20\pi}$$

∵ $t_r \approx \frac{2.2}{\omega_h}$ ∧ $\omega_h = 20\pi$ → $\boxed{t_r = 35 \text{ ms}}$

if $f_o = 1.2$ MHz → $N = 24$ → $k_v = 52.36$ Hz

→ $\boxed{t_r = 42 \text{ ms}}$

∵ $f_r = 50$ kHz ∧ frequency range = 400 kHz

→ $\frac{400}{50} = \boxed{8}$ different frequencies possible.

8.2: a) $\therefore$ $W_h = W_n$ $\wedge$ $W_n = \sqrt{k_v W_L}$ $\wedge$ $2\zeta = \sqrt{\frac{W_L}{k_v}}$ → $W_L = 2k_v$

$\therefore$ $W_n = 28.28\pi$ rad/s $= W_h$ → $\boxed{BW = 14.14 \text{ Hz}}$

b) $\therefore$ $k_v = 52.36$ Hz → $W_n = W_h = 74.05$ rad/s

→ $\boxed{BW = 11.79 \text{ Hz}}$

8.14: $\therefore$ $\Delta f_{VCO} = 2$ kHz, assuming $V_{DD} = 9V$ → $k_o = 1795.2$ rad/s

$\therefore$ demodulator → $TR = 2\Delta W_{max} = 2k_o k\, V_{C,max}$ $\wedge$ XOR → $k_d = \frac{V_{DD}}{\pi}$

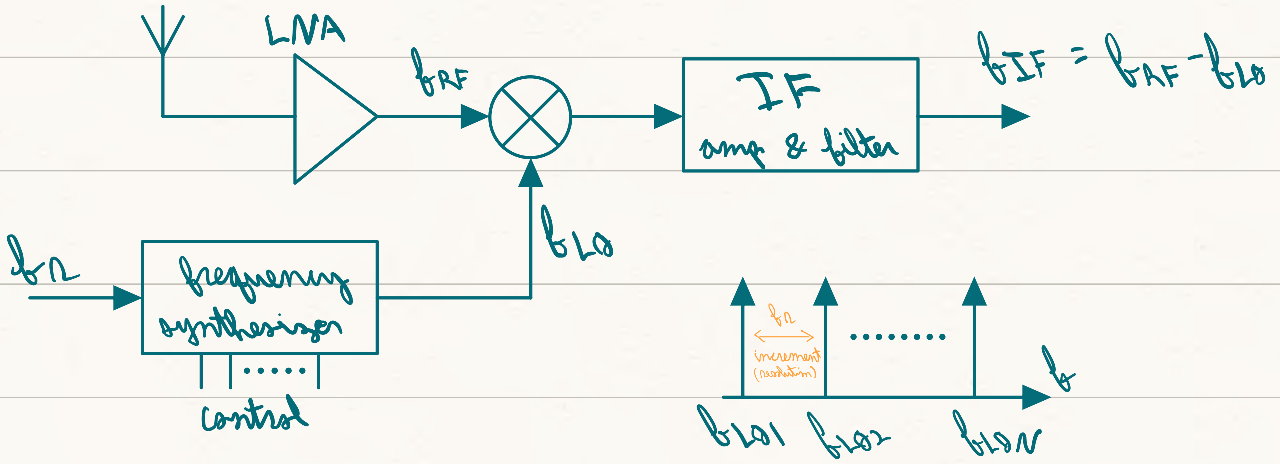
assuming $C = 0.5$ nF → $\boxed{R_2 = 19.0 \text{ k}\Omega}$ $\wedge$ $\boxed{R_1 = 18.6 \text{ k}\Omega}$

if $BW = 2$ kHz and $\zeta = \frac{1}{\sqrt{2}}$ → $\boxed{R_{LPF} = 48.6 \text{ k}\Omega}$ if $\boxed{C_{LPF} = 2\text{nF}}$

Chapter 10:

* frequency synthesizer: device that generates many precise frequencies from one or more reference frequencies.

- frequency synthesizers are often used as variable local oscillators for transmitters and receivers to select a specific station, or otherwise.



- frequency synthesizers are characterized by their respective frequency range ($f_{LON} - f_{LO1}$) and frequency resolution ($f_{LO(i+1)} - f_{LOi}$). (increment or step)

+ main types of frequency synthesizers:

  1- direct frequency synthesizer.

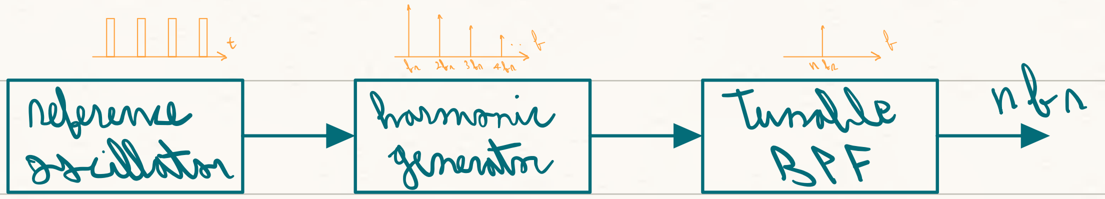
  2- phase-locked loop frequency synthesizer.

  3- direct digital frequency synthesizer. will not be discussed

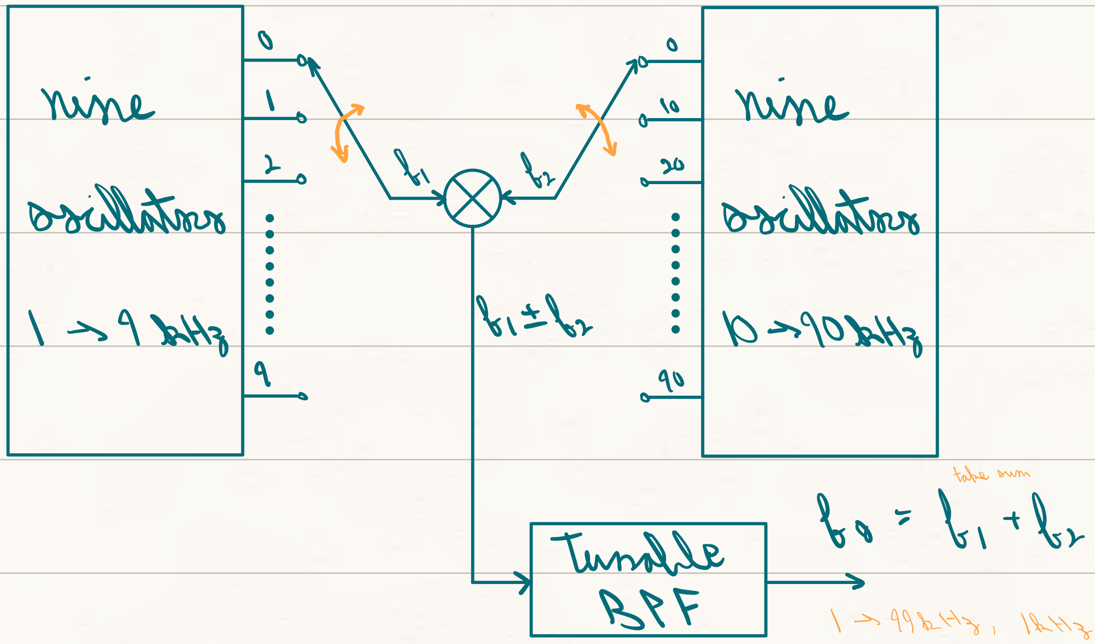
1- direct frequency synthesis:

- direct frequency synthesis is an old method that uses harmonic generators, filters, dividers, and mixers.
- in this technique, a reference oscillator with narrow pulses excites a harmonic generator, then a tunable BPF selects the required harmonic.

$n_{max} \approx 10$, since amplitude decreases



- the two decade direct frequency synthesizer shown below can generate 99 frequencies from 18 reference oscillators.



take sum

$1 \to 99$ kHz, 1kHz increment

- designing a tunable BPF with a narrow passband to select one of the 11 frequencies is challenging.

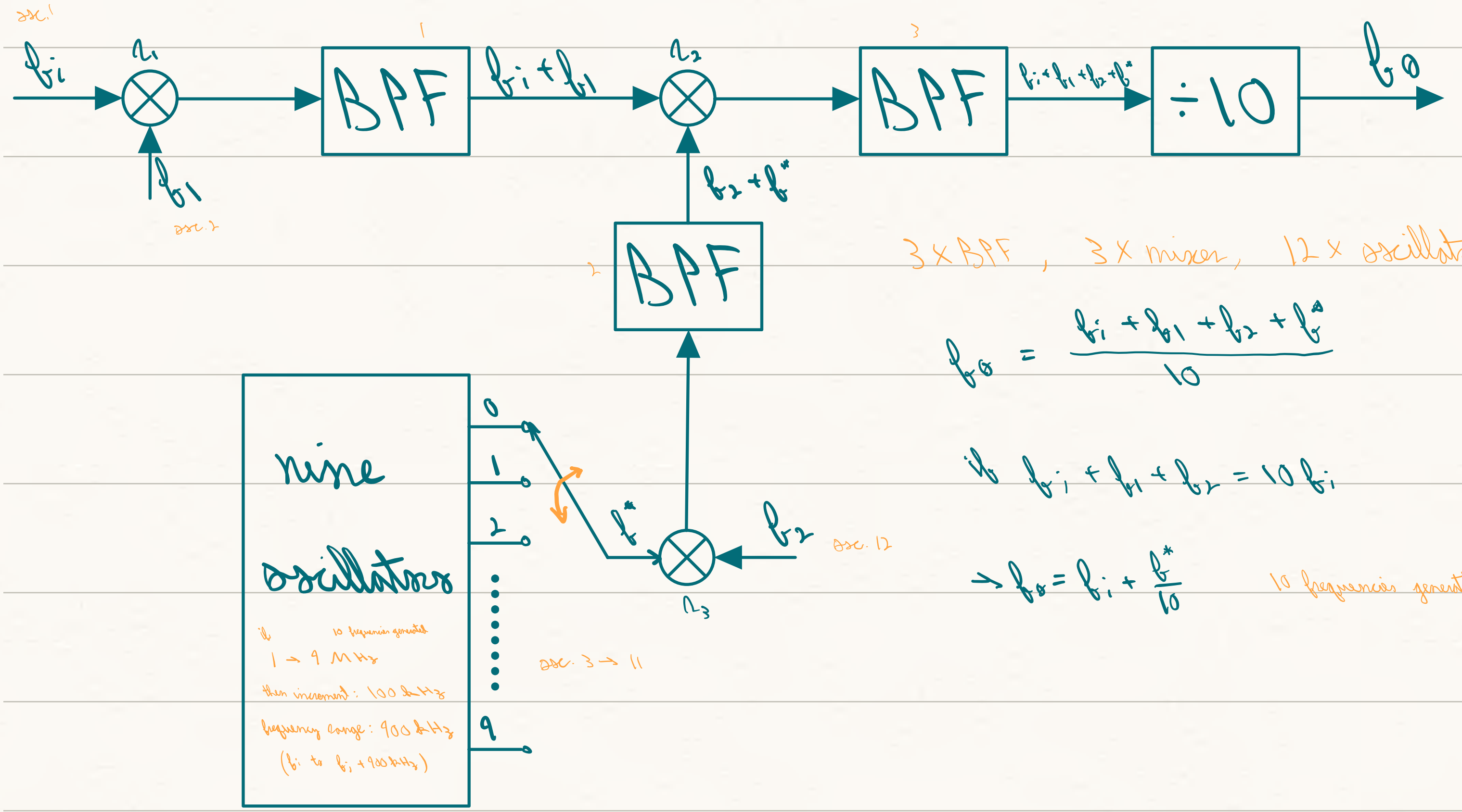
- the complexity of a BPF depends on the mixing ratio:

$$\Omega = \frac{f_1}{f_2}$$ larger / smaller $\Omega\uparrow \rightarrow$ complexity $\uparrow$

- a small mixing ratio is desirable because the spacing between frequencies will be large, which allows easy design of the filter.

+ double-mix-divide direct frequency synthesis:

  - offset frequencies are used to reduce filter complexity, as below:



3 × BPF, 3 × mixer, 12 × oscillator

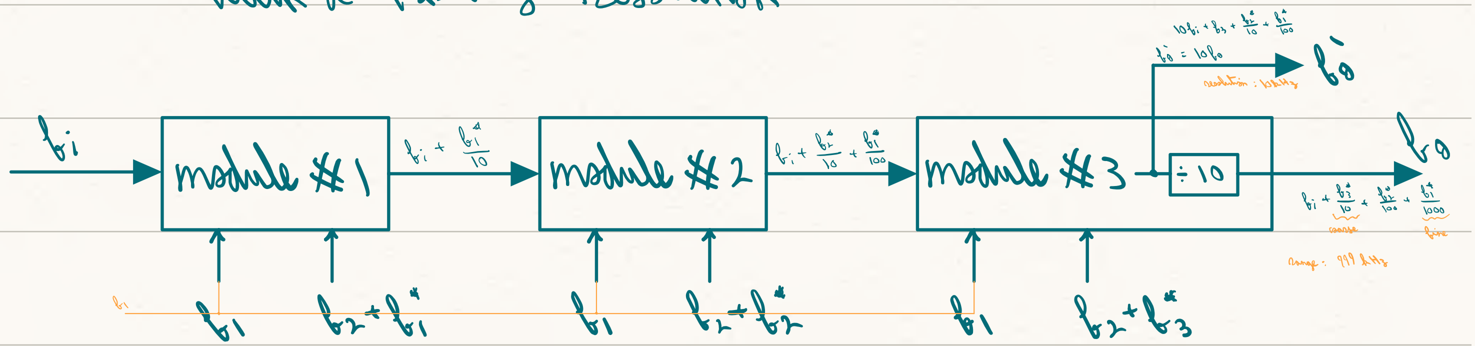
$$f_o = \frac{f_i + f_1 + f_2 + f^*}{10}$$

if $f_i + f_1 + f_2 = 10 f_i$

$$\rightarrow f_o = f_i + \frac{f^*}{10}$$ 10 frequencies generated

- f_i, f_1, and f_2 are chosen such that the mixing ratios of the two mixers are close to one.

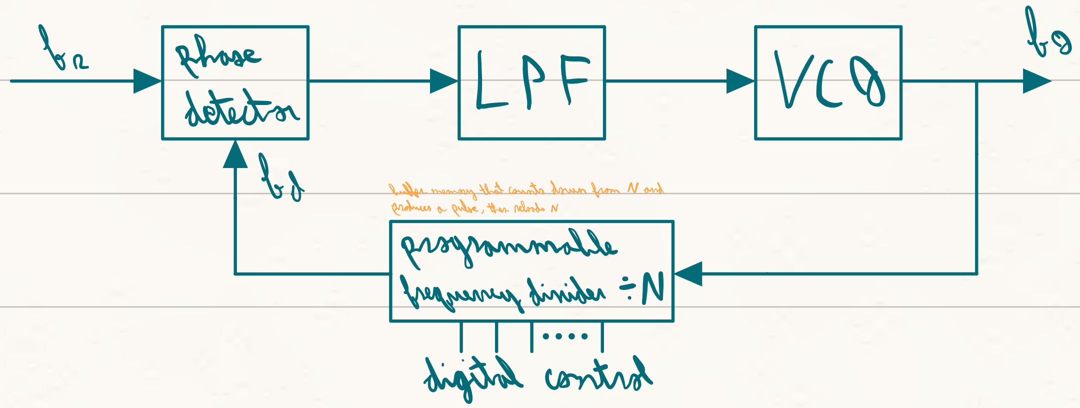
- two cascaded double-mix divide modules can generate 100 frequencies with a 10 kHz resolution. Three cascaded modules can generate 1000 frequencies with a 1 kHz increment.
- The three cascaded double-mix-divide modules shown below only need 12 reference oscillators to generate 1000 frequencies with a 1 kHz resolution.



2- PLL frequency synthesisers :

- PLL frequency synthesisers are low cost and only require a single reference frequency to implement any resolution or range.

1- simple PLL synthesiser :



- when locked: $f_r = f_d = \frac{f_o}{N} \rightarrow f_o = N f_r$

- the resolution is f_r and the range is $f_{o,min}$ to $f_{o,max}$:

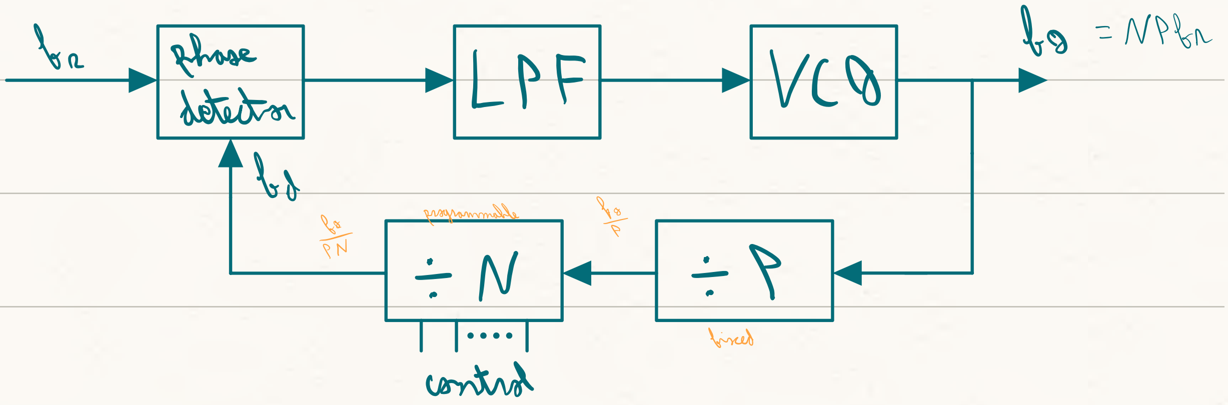
$$N_{min} = \frac{f_{o,min}}{f_r} \wedge N_{max} = \frac{f_{o,max}}{f_r}$$

+ This simple PLL synthesizer has two main problems:

1- programmable frequency dividers are slow devices. because of counting process

2- the switching time (t_s) of the PLL is inversely proportional to the reference frequency (f_r): $t_s \approx 25\, t_r$ $t_s \approx \frac{25}{f_r}$

2- PLL with fixed-modulus divider (prescaler):



- this solves the first problem since the prescaler drops the frequency to be manageable by the programmable divider.

$\therefore f_{o,n} = NPf_r$, $f_{o,n+1} = (N+1)Pf_r$

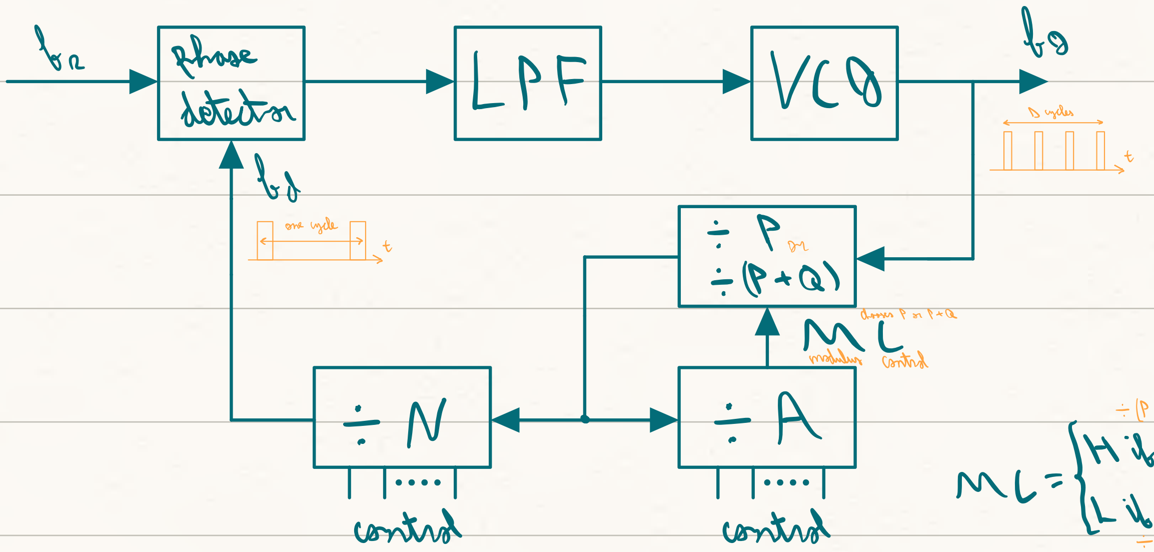
$\rightarrow$ resolution increment: Pf_r $\quad \frac{f_{o,max}}{P} \leqslant 5\text{ MHz}$ to be compatible with programmable divider

$$\therefore N_{min} = \frac{f_{o,min}}{Pf_r} , \quad N_{max} = \frac{f_{o,max}}{Pf_r}$$

- this type of PLL synthesizer solves the first problem, but worsens the second since the switching time becomes larger.

to achieve same resolution f_r for this PLL synthesize must be smaller than the previous one ($P f_{r,new} = f_{r,old} \rightarrow f_{r,new} = \frac{f_{r,old}}{P}$). Therefore, $t_{s,new} = P \cdot t_{s,old}$

3- PLL with dual-modulus prescalar synthesizer:



$$ML = \begin{cases} \overset{\div(P+Q)}{H} \text{ if } A>0 \\ \underset{\div P}{L} \text{ if } A=0 \end{cases}$$

- the prescalar here makes the resolution f_r instead of $P f_r$

- the modulus control is high for A cycles and low for N-A cycles.

prescalar is (P+Q)   prescalar is P

- the output frequency is found as:

$$f_o = D f_r = [(N-A)P + A(P+Q)] \cdot f_r$$

$$\rightarrow f_o = PN f_r + QA f_r$$

- if $Q < P$, then the resolution is $Q f_r$. Q is usually chosen as 1:

resolution = f_r

$A<N$, $A \overset{\text{never equal}}{<} P$ or signals will overlap

$$f_o = PN f_r + A f_r = P\left(N + \frac{A}{P}\right) \cdot f_r$$

- the ranges of N and A are found as:

$$D_{min} = P N_{min} + A_{min} = \frac{f_{o,min}}{f_r}$$

$$D_{max} = P N_{max} + A_{max} = \frac{f_{o,max}}{f_r}$$

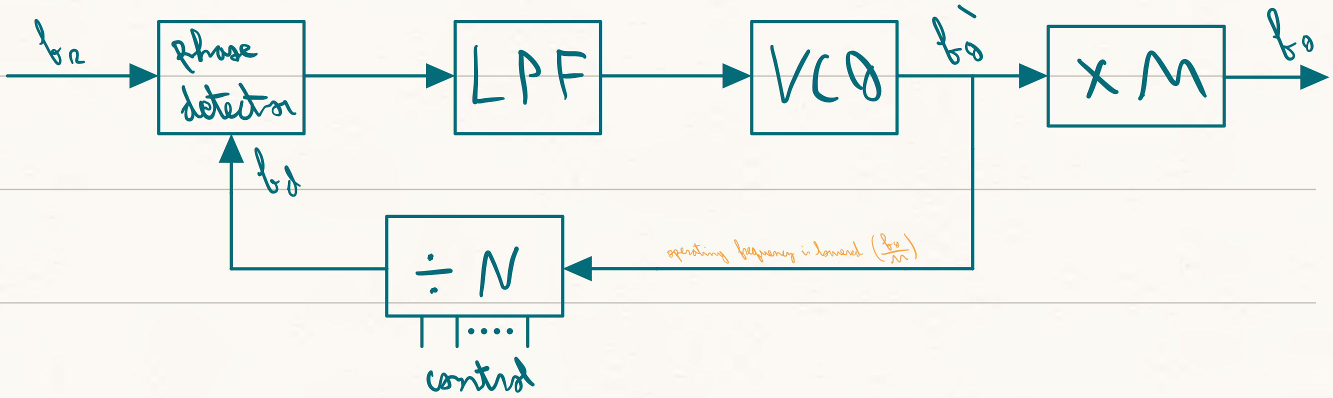
P=64, Q=1

- the prescalers are written in the form P/(P+Q) (e.g., 64/65).

- many PLL FS ICs exist, some have frequency hopping spread spectrum capabilities, which help against jamming

randomly hops to a different carrier

4- PLL with post-multiplier:



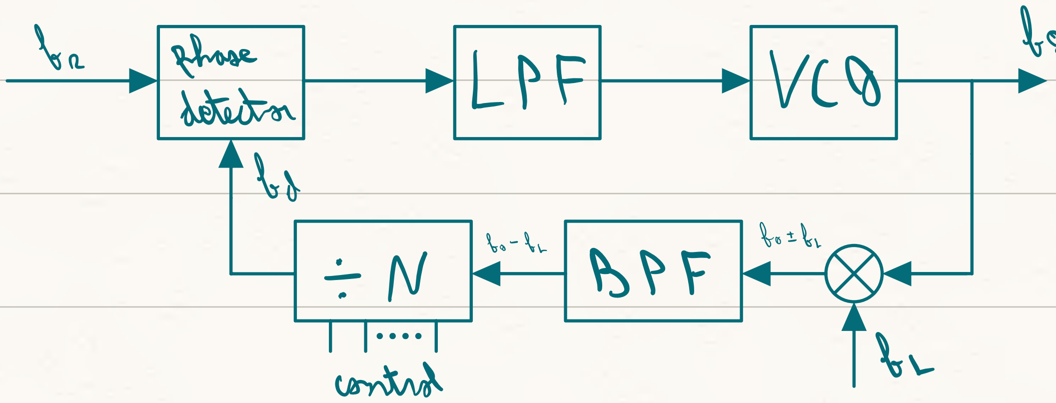
at lock: $f_r = f_d = \frac{f_o'}{N} \rightarrow f_o' = N f_r$

$$\therefore \quad f_o = N M f_r$$

N is variable

→ resolution: $M f_r$

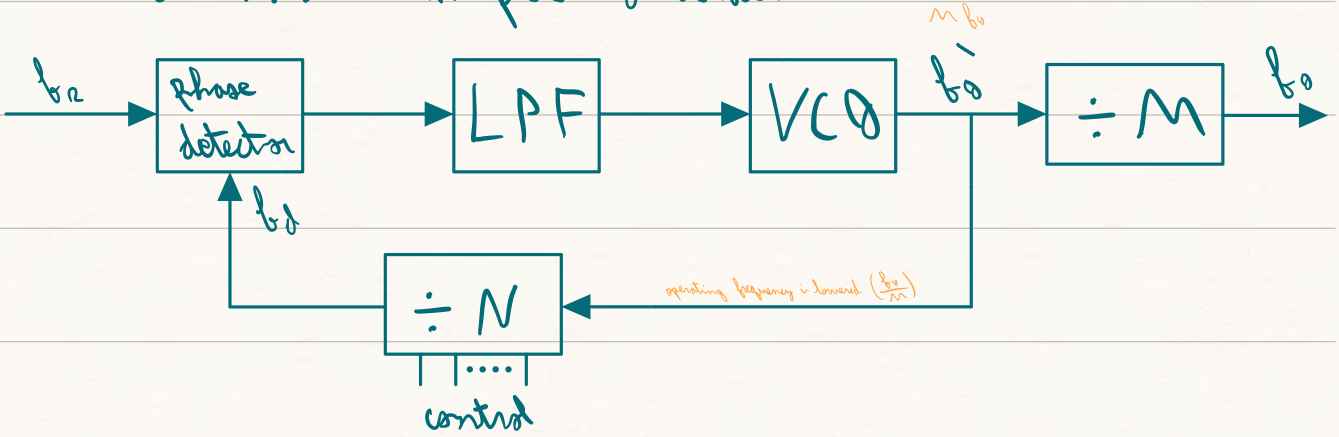
5- PLL with down conversion:



$$\because\ f_r = f_f = \frac{f_o - f_L}{N} \rightarrow f_o = f_L + N f_r$$

- Therefore, the resolution is f_r and the programmable divider operates at $f_o - f_L$

6- PLL with post-divider



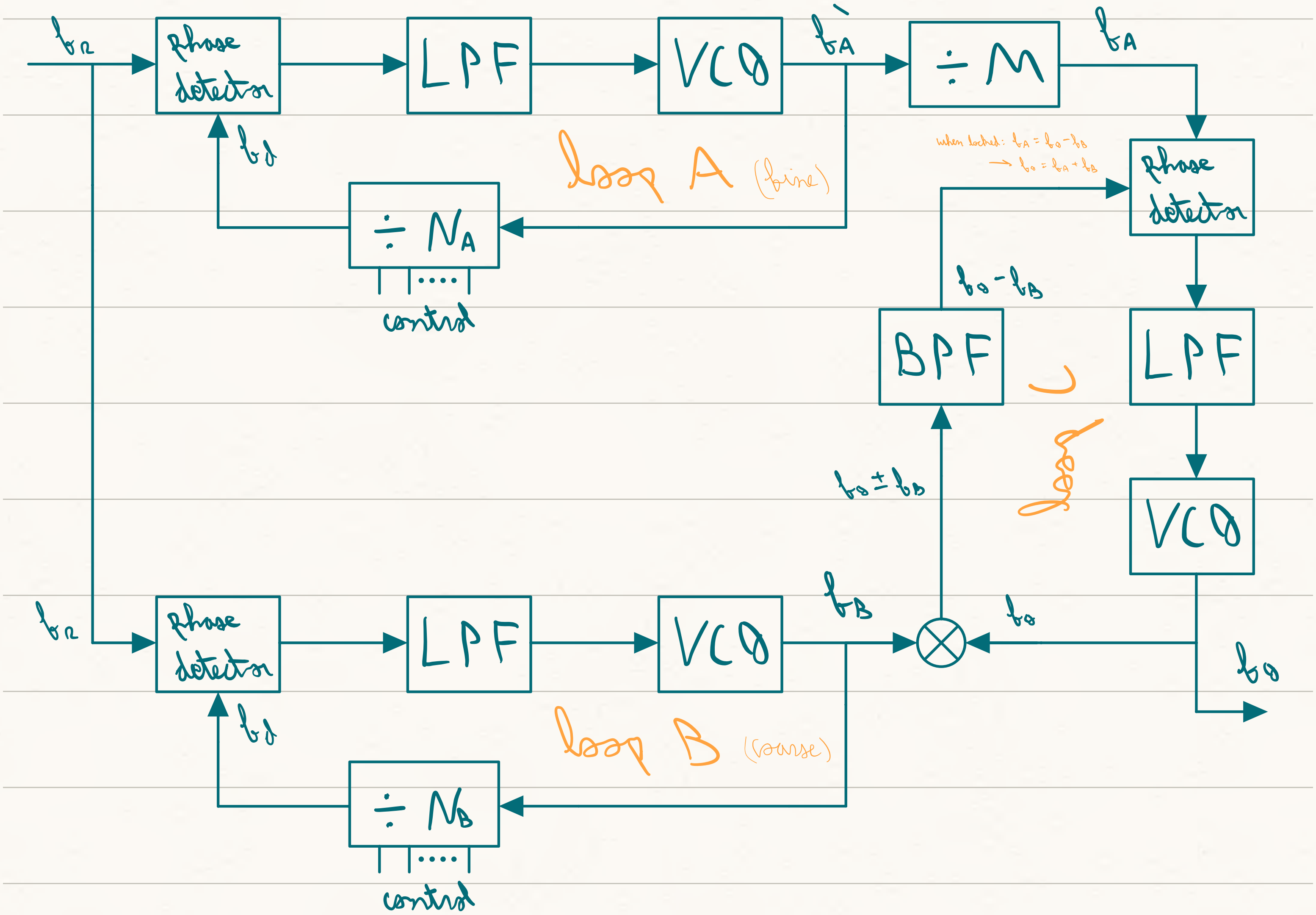
$$\because\ f_r = f_f = \frac{f_o'}{N}, \quad f_o' = N f_r, \quad f_o' = M f_o$$

$$\therefore\ f_o = \frac{N f_r}{M} \rightarrow \text{resolution}: \frac{f_r}{M}$$

- the resolution is low, which implies high switching speed, but the operating frequency of the divider is high.

7- multiple-loop frequency synthesizers:

- a compromise between high switching speed and low operating frequency of programmable dividers can be achieved using this type of PLL frequency synthesizers.



- in this configuration, the PLL in loop C to obtain $f_o = f_A + f_B$ is used instead of a mixer and BPF because the mixing ratio will make a BPF complex when f_B is fine.

example 10.2:

10 → 19.99 MHz , 10 kHz resolution implied 1000 frequencies

therefore, 3 cascaded double-mix-divide required

$f_o' = 10 f_i + f_3^* + \frac{f_2^*}{10} + \frac{f_1^*}{100}$, $f'_{o,min} = 10 f_i \rightarrow f_i = 1MHz$

but $10 f_i = f_i + f_1 + f_2 \rightarrow f_1 + f_2 = 9 MHz$

maximize mixing ratios: choose $f_1 = 3MHz$, $f_2 = 6MHz$

$$\rightarrow n_1 = \frac{f_1}{f_i} = 3 \wedge n_2 = \frac{f_2 + \overset{9MHz}{f_{3,max}^*}}{f_1 + f_i} = \frac{15}{4} = 3.75$$

worst case (highest n)

if f_o' is 14.34 MHz $\rightarrow f_3^* = 5 MHz$,

$f_2^* = 3 MHz$, $f_1^* = 4 MHz$

example 10.3:

must find N_{min}, N_{max}, A_{min}, A_{max}

105 → 127 in 1MHz increments : 23 frequencies

$P = 10$, $P + Q = 11 \rightarrow Q = 1$

$$D_{max} = \frac{f_{o,max}}{f_r} = \frac{127 MHz}{1MHz} = 127 = PN_{max} + A_{max}$$

$\therefore 127 = 10 N_{max} + A_{max} \rightarrow N_{max} = 12, A_{max} = 7$

$$\wedge D_{min} = \frac{f_{o,min}}{f_r} = \frac{105}{1} = 105 \rightarrow N_{min} = 10, A_{min} = 5$$

$\therefore$

N	A			
10	5 6 7 8 9		5	23
11	0 1 2 3 4	5 6 7 8 9	10	
12	0 1 2 3 4	5 6 7	8	

example 10.4: find R, f_R, number of hops, N_{min}, N_{max}, A_{min}, A_{max}, hopping freq.

$f_{o,min} = 180.4$, $f_{o,max} = 185.6$, frequencies ≥ 50, $f_R \geq 20$ kHz

$P = 64$, $Q = 1$, ROM $= 2^7 \times 16 \rightarrow 2^7 = 128$ hops

and $N + A = 16$

$\because$ oscillator $= 2$ MHz, $f_R = \dfrac{\text{oscillator}}{R} \geq 20$ kHz

$\rightarrow R \leq 100 \rightarrow RA_2\,RA_1\,RA_0 = 0\;0\;1 \rightarrow R = 64$

$\therefore f_R = \dfrac{2M}{64} = 31.25$ kHz

with such a resolution, the number of hops is : $\dfrac{(185.6 - 180.4)\text{ MHz}}{31.25\text{ kHz}}$

round down always

$\rightarrow 166.4 = 166$ hops $>$ required 50

$A_{max} \leq P-1 \;\forall A$

round up for min

$\because D_{min} = \dfrac{180.4\,M}{31.25k} = 5772.8 = PN_{min} + A_{min}$

N	A			
	13	:	60	51
	14	:	61	
	15	:	62	
	16	|	63	
90	17	|		
	18	|		
	19	|		
	20	|		
	0	|	60	64
	1	|	61	
	2	|	62	
	3	|	63	
91	4	|		
	5	|		
	6			
	7	|		
	0	|	49	52
	1	|	50	
92	2		51	
	|	|		
	|	|		

$\therefore$ 167 combinations

$\rightarrow 5773 = 64 \cdot N_{min} + A_{min} \rightarrow N_{min} = 90,\ A_{min} = 13$

round down for max

$\wedge\ D_{max} = \dfrac{185.6\,M}{31.25\,k} = \lfloor 5939.2 \rfloor = 64 \cdot N_{max} + A_{max}$

$\rightarrow N_{max} = 92,\ A_{max} = 51$

The hopping frequency gives how quickly the output frequency can change. This (hop time) is greater than or equal to the PLLs switching time

$\rightarrow t_s \approx \dfrac{25}{f_R} = 8 \times 10^{-4}$ s $\rightarrow t_H \geq 8 \times 10^{-4}$ s

$\therefore f_H \leq 1250$ Hz

example 10.5:

$f_r = 100$ kHz, resolution = 1 kHz

$\therefore$ resolution = $\frac{f_r}{M} \rightarrow M = 100$

required operating frequency of divider: $f_{o,max}' = 1010$ MHz

example 10.6:

$f_{o,min} = 34.4$ MHz, $f_{o,max} = 40$ MHz, increment: 1 kHz

$f_r = 100$ kHz, loop A: fine, loop B: coarse

f_A controls the response time of loop C, hence if f_A is higher response time of loop C will be lower. Therefore, offset f_A by 300k

$\rightarrow f_A = [300k, 399$ kHz$]$

loop A is responsible for the 1 kHz and 10 kHz increments and loop B is responsible for 0.1 MHz and 1 MHz increments

$\therefore f_A' = M f_A$, $f_A' = N_A f_r$

$\therefore$ resolution = $\frac{f_r}{M}$ $\wedge$ resolution = 1 kHz $\rightarrow M = 100$

$\rightarrow f_A' = [30$ MHz$, 39.9$ MHz$]$

$\rightarrow N_{A,min} = \frac{30M}{100k} = 300$, $N_{A,max} = 399$

$\therefore 300 \leq N_A \leq 399$

for loop B: since f_A was offset by $+300$ kHz, loop B should be offset by $-300\,kHz$

$\rightarrow f_B = [34.1\,MHz, 39.7\,MHz]$

$\therefore N_{B,min} = 341 \;,\; N_{B,max} = 397$

$\wedge \; f_o = f_A + f_B$

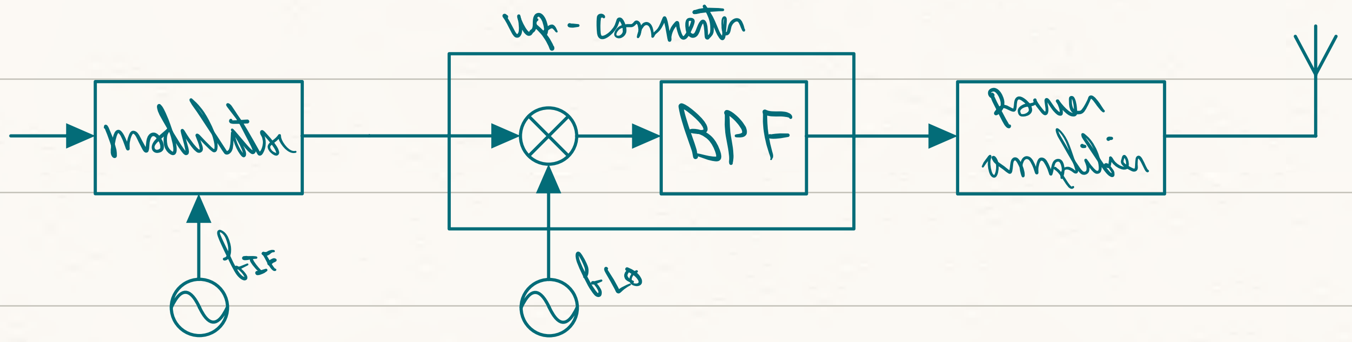
- If $f_o = 37.568\,MHz = (f_A + \overset{\text{offset}}{300\,kHz}) + f_B$

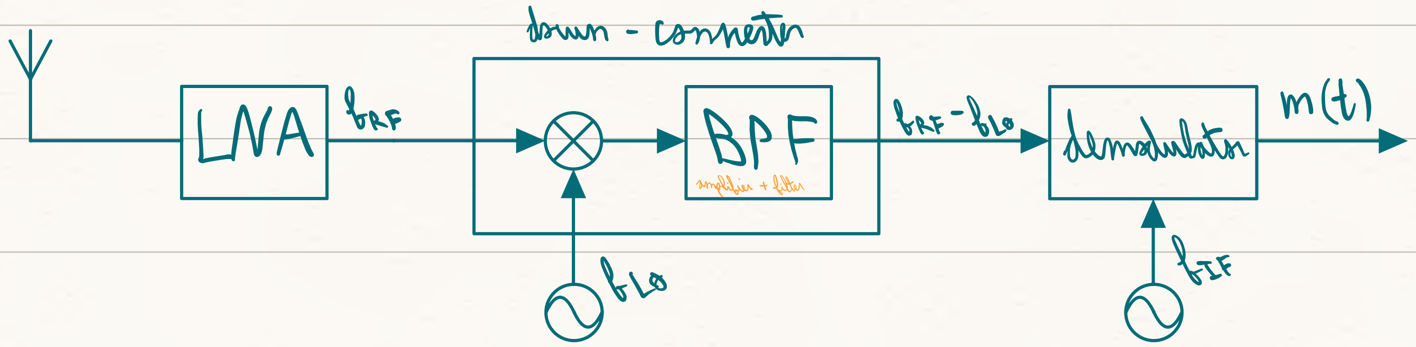
$\rightarrow f_B = 37.2\,MHz \rightarrow N_B = 372$

$\wedge \; f_A = 368\,kHz \rightarrow N_A = 368 \;\therefore\; f_A' = 36.8\,MHz$

Chapter 12:

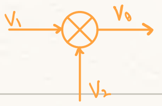
- modulators, demodulators, and frequency converters (mixers) are fundamental devices in communication systems.





+ frequency mixers:

- most used for frequency conversion, modulation, and demodulation
- mixers are four-quadrant multipliers that multiply two input signals.



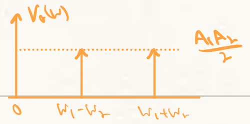
- if the two input signals are sinusoidal, then:

$$V_1(t) = A_1 \sin(w_1 t) \quad , \quad V_2(t) = A_2 \sin(w_2 t)$$

$$\rightarrow V_0(t) = V_1(t)\, V_2(t) = A_1 A_2 \sin(w_1 t) \sin(w_2 t)$$

$$\therefore V_0(t) = \frac{A_1 A_2}{2} \left[\cos[(w_1 - w_2) t] - \cos[(w_1 + w_2) t] \right]$$

this gives two frequency components:



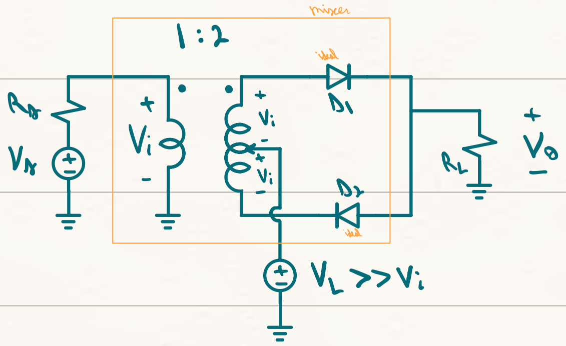
+ mixers are classified as:

1- active or passive.

2- switching-type or nonlinear.

opposite direction

1- simple two-diode switching-type mixer:



KVL

if $V_L > 0$: D_1 on, D_2 off , $-V_L - V_i + V_0 = 0$

$$\rightarrow V_0 = V_L + V_i$$

if $V_L < 0$: D_1 off, D_2 on, $-V_L + V_i + V_o = 0$

$$\rightarrow V_o = \overset{\text{negative}}{V_L} - V_i$$

hence, $V_o = V_L + P(t)\, V_i$, such that $P(t) = \begin{cases} +1, & V_L > 0 \\ -1, & V_L < 0 \end{cases}$

- given that $P(t)$ is a square wave, it can be expanded using the fourier series:

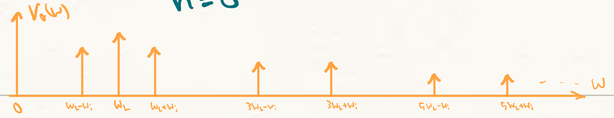
$$P(t) = \frac{4}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\sin[(2n+1)W_L t]}{2n+1}$$

then $V_o(t) =$

$$V_L \sin(W_L t) + V_i \sin(W_i t) \cdot \frac{4}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\sin[(2n+1)W_L t]}{2n+1}$$

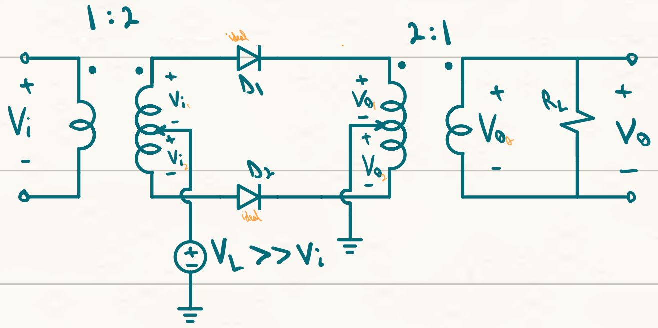
$\rightarrow V_o(t) =$

$$V_L \sin(W_L t) + \frac{2V_i}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\cos[(2n+1)W_L t - W_i t] - \cos[(2n+1)W_L t + W_i t]}{2n+1}$$



same direction

2- simple two-diode switching-type mixer:



assume $V_{o_1} + V_{o_2} = V_{ot} = 2V_{os}$

if $V_L > 0$:

$-V_L - V_{i_1} + V_{o_1} = 0 \rightarrow V_{o_1} = V_{i_1} + V_L$

" $-V_L + V_{i_2} - V_{o_2} = 0 \rightarrow V_{o_2} = V_{i_2} - V_L$

$\therefore V_{ot} = V_{i_1} + V_{i_2} + V_L - V_L = 2V_i \rightarrow V_{os} = V_i$

if $V_L > 0$: both D_1 and D_2 on , $V_o = V_i$

if $V_L < 0$: both D_1 and D_2 off , $V_o = 0$

$\rightarrow V_o = P(t)\, V_i$, such that $P(t) = \begin{cases} 1, & V_L > 0 \\ 0, & V_L < 0 \end{cases}$

- P(t) here is also a square wave that can be expanded as:

$$P(t) = \frac{1}{2} + \frac{2}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\sin[(2n+1) w_L t]}{2n+1}$$

$$\rightarrow V_o = \frac{V_i}{2} \sin(w_i t) + \frac{2 V_i \sin(w_i t)}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\sin[(2n+1) w_L t]}{2n+1}$$

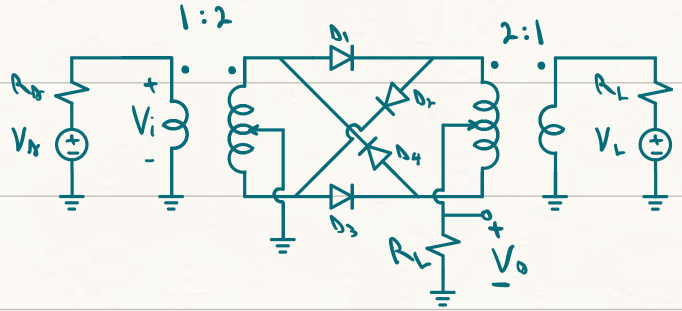
$$\therefore V_o(t) =$$

$$\frac{V_i}{2} \sin(w_i t) + \frac{V_i}{\pi} \cdot \sum \frac{\cos[(2n+1) w_L t - w_i t] + \cos[(2n+1) w_L t + w_i t]}{2n+1}$$

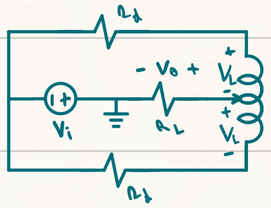
ring

3- four-diode switching-type mixer:

- This is a double-balanced mixer in which neither the local oscillator signal nor the input signal appear at the output.



- if $V_L > 0$: D_2 and D_3 on, D_1 and D_4 off
- if $V_L < 0$: D_1 and D_4 on, D_2 and D_3 off
- for nonideal diodes, the following is the equivalent circuit:



$\therefore$ $r_d \parallel r_d$ → V_o found as: (voltage division)

for $V_L > 0$:

$$V_o = -\frac{R_L}{R_L+\frac{r_d}{2}}\cdot V_i$$

for $V_L < 0$:

$$V_o = \frac{R_L}{R_L+\frac{r_d}{2}}\cdot V_i$$

- the output voltage can be expressed as:

$$V_o(t) = \frac{R_L}{R_L+\frac{r_d}{2}} ,$$

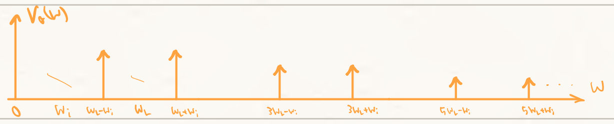
such that: $p(t) = \begin{cases} -1, & V_L > 0 \\ +1, & V_L < 0 \end{cases}$

again, $p(t)$ can be expanded as:

$$p(t) = \frac{4}{\pi}\cdot\sum_{n=0}^{\infty}\frac{\sin[(2n+1)\omega_L t]}{2n+1}$$

→ $V_o(t) =$

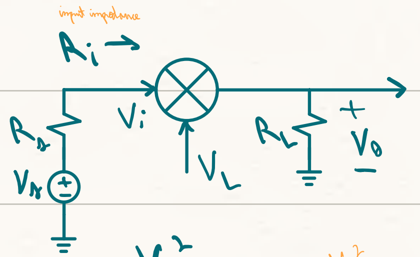
$$\frac{R_L}{R_L+\frac{r_d}{2}}\cdot\frac{2V_i}{\pi}\cdot\sum_{n=0}^{\infty}\frac{\cos[(2n+1)\omega_L t-\omega_i t]-\cos[(2n+1)\omega_L t+\omega_i t]}{2n+1}$$



+ conversion loss of mixers:

* conversion loss: ratio of output power in one sideband to input power of the signal.

- maximum power transfer occurs when matched:



$$\therefore R_s = R_i \approx R_L$$

$$\& \; R_i = R_L + \frac{R_s}{2}$$

$$\therefore V_i = \frac{V_s}{2} \rightarrow P_i = \frac{V_s^2}{4R_L} = \frac{V_i^2}{R_L}$$

four-diode switching-type (ring) mixer

- for the double-balanced ring mixer:

$$V_o\Big|_{\omega_L \pm \omega_i} = \frac{2V_i}{\pi} = \frac{V_s}{\pi}$$

$$\therefore P_o = \frac{V_s^2}{\pi^2 R_L}$$

- the conversion power gain of the mixer is found as:

$$G = \frac{P_o}{P_i} = \frac{4}{\pi^2} \rightarrow \overset{10\log_{10}(G)}{G(dB)} = \overset{loss}{-3.92 \text{ dB}}$$

- hence, the conversion loss is:

$$L = \frac{1}{G} = \frac{P_i}{P_o} \rightarrow L(dB) = 3.92 \;(dB)$$

simple two-diode switching-type

- for the single-balanced two-diode (opposite) mixer: $V_o\Big|_{\omega_i \pm \omega_L} = \frac{2V_i}{\pi}$

$$\rightarrow G = -3.92 \text{ dB}, \text{ same as ring.}$$

- for the single-balanced two-diode (same direction) mixer: $V_o\Big|_{\omega_i \pm \omega_L} = \frac{V_i}{\pi} = \frac{V_s}{2\pi}$

$$\therefore P_o = \frac{V_s^2}{4\pi^2 R_L}$$

$$\rightarrow L = \frac{P_i}{P_o} = \pi^2 \rightarrow L = 9.94 \text{ dB}$$

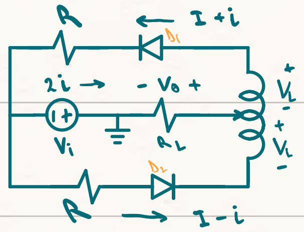
+ intermodulation distortion in mixers:

- as mentioned previously intermodulation distortion arises from nonlinear receivers.
- for diode mixers, the current flowing in the diode is a nonlinear function of the voltage drop across it, such that:

at room temperature

$$i_D = I_s\, e^{V_D/V_T} \quad , \quad V_T = \frac{kT}{q} \approx 0.026V$$

- after adding linearizing resistors, the following is the equivalent circuit of a double-balanced ring mixer:



$\therefore\ -V_i - V_o \cancel{-V_L} + V_{D1} + \cancel{IR} + iR = 0$

$\wedge\ V_i + \cancel{IR} - iR + V_{D2} \cancel{-V_L} + V_o = 0$

$\rightarrow -2V_i - 2V_o = -2iR + V_{D2} - V_{D1}$

$\therefore\ \frac{i_D}{I_s} = e^{V_D/V_t} \rightarrow V_D = V_t \cdot \ln\left(\frac{i_D}{I_s}\right)$

$\wedge\ i_{D1} = I + i,\ i_{D2} = I - i$

$\rightarrow V_{D2} - V_{D1} = V_T\left[\ln\left(\frac{i_{D2}}{I_s}\right) - \ln\left(\frac{i_{D1}}{I_s}\right)\right]$

$= V_T \cdot \ln\left(\frac{i_{D2}}{\cancel{I_s}} \cdot \frac{\cancel{I_s}}{i_{D1}}\right) = V_T \ln\left(\frac{I-i}{I+i}\right)$

$\therefore\ -2V_i - 2V_o = -2iR + V_T \ln\left(\frac{I-i}{I+i}\right)$

$\rightarrow V_i + V_o = iR + \frac{V_T}{2}\ln\left(\frac{I+i}{I-i}\right)$

$\therefore\ V_o = -2iR_L$

$\rightarrow V_i = i(2R_L + R) + \frac{V_T}{2}\ln\left(\frac{I+i}{I-i}\right)$

diode DC current

$$\therefore\ V_i = i(2R_L + R) + \frac{V_T}{2}\ln\left(\frac{I+i}{I-i}\right)$$

if $i << I$, the ln term is expanded to give:

$$V_i \approx \left(2R_L + R + \frac{V_T}{I}\right)\cdot i + \frac{1}{3}\left(\frac{i}{I}\right)^3 + \ldots$$

which can be inverted to find i as:

$$i \approx \frac{V_i}{2R_L + R} - \frac{V_T}{3} \cdot \frac{V_i^3}{(2R_L+R)^4 I^3}$$

- Therefore, $V_o = 2iR_L$ is found to be:

$$V_o \approx \underbrace{\boxed{\frac{2R_L}{2R_L + R}}}_{a_1} \cdot V_i - \underbrace{\boxed{\frac{V_T}{3} \cdot \frac{2R_L}{(2R_L+R)^4 I^3}}}_{a_3} \cdot V_i^3$$

or $V_o \approx k_1 V_i - k_3 V_i^3$

power

- hence, the third-order intermodulation distortion is proportional to k_3^2. Then, the goal is to minimize k_3.

∴ power

- it can be seen that k_3 is inversely related to the linearizing resistors and the diode DC current.

R

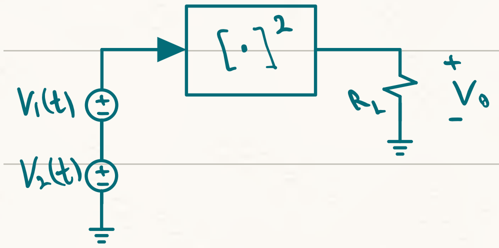
I

- therefore, increasing the linearizing resistors' values will decrease IMD.

intermodulation distortion

nonlinear

+ Square-law mixers:



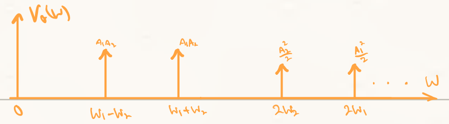
- diodes and transistors can be made to operate nonlinearly in order to be used as square-law mixers.

- the two signals are added then squared:

$$V_o(t) = [V_1(t) + V_2(t)]^2 = [A_1 \sin(w_1 t) + A_2 \sin(w_2 t)]^2$$

$$\rightarrow V_o(t) = \frac{A_1^2}{2}[1 - \cos(2w_1 t)] + \frac{A_2^2}{2}[1 - \cos(2w_2 t)] +$$

$$A_1 A_2 [\cos(w_1 t - w_2 t) - \cos(w_1 t + w_2 t)]$$



- diodes may be used as square-law mixers, but they introduce large conversion losses.

- for the diode to operate in its square-law region,

$V_i << V_{DC}$

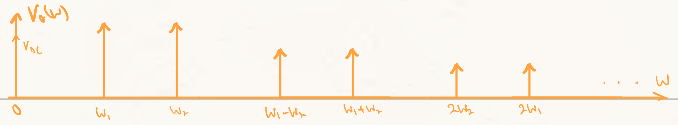
$\rightarrow V_o(t) = V_{DC} + k_1 V_i + k_2 V_i^2 + \dots$

if $V_i = A_1 \sin(w_1 t) + A_2 \sin(w_2 t)$

$\rightarrow V_o(t) = V_{DC} + k_1 \cdot [A_1 \sin(w_1 t) + \dots$

$A_2 \sin(w_2 t)] + k_2 \cdot [\dots$

$A_1 \sin(w_1 t) + A_2 \sin(w_2 t)]^2$



+ Amplitude modulation and demodulation:

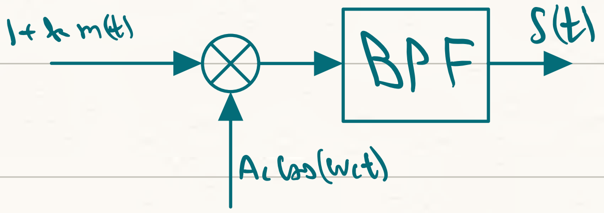
1- DSB-LC amplitude modulation:

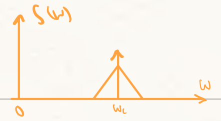
full-wave

$S(t) = A_c[1 + k\, m(t)] \cos(w_c t) \rightarrow |k\, m(t)|_{max} \leq 1$

product modulator

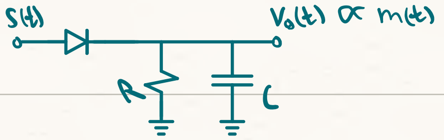
- the modulator is a mixer followed by a BPF:





- the demodulator is an envelope detector:

$RC = \frac{1}{\sqrt{W_m W_c}}$

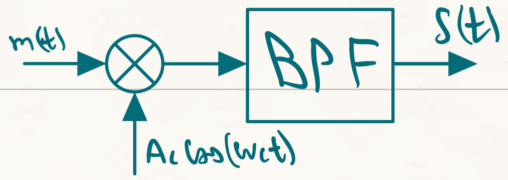


2- DSB-SC AM:

product modulator

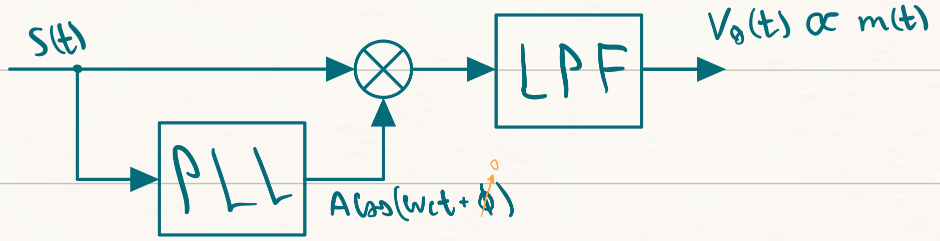
- similarly, the modulator is a mixer and BPF:

$S(t) = A_c\, m(t) \cos(w_c t)$



coherent detector

- the demodulator consists of a product modulator with a PLL producing a synchronized local oscillator.

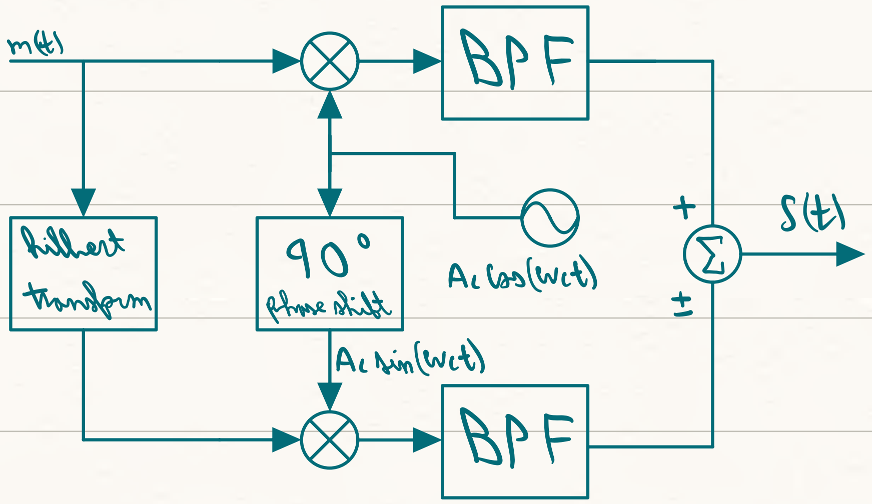


3- SSB-SC AM:

$$s(t) = A_c\, m(t)\cos(w_c t) \pm A_c\, \overset{\text{hilbert transform}}{\hat{m}}(t)\sin(w_c t)$$

- the modulator requires two mixers, hilbert transformer, BPFs, adder, and phase shifter (90°). whereas the demodulator is exactly the same as for DSB-SC (coherent demodulator).

SSB modulator



ASK

4- Amplitude Shift Keying:

- this is the digital version of AM. hence, standard AM modulators (product modulator) can be used, but the message signal must be a unipolar digital signal

- an envelope detector can be used as a demodulator.

+ phase and frequency modulation and demodulation:

- in angle modulation, the modulated signal is represented as:

$$S(t) = A_c \cos[W_c t + \theta(t)]$$

frequency modulation → $f(t) \propto m(t)$ i.e., $\frac{d\theta(t)}{dt} \propto m(t)$

phase modulation → $\theta(t) \propto m(t)$

- since $f(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$, then a frequency modulator can be

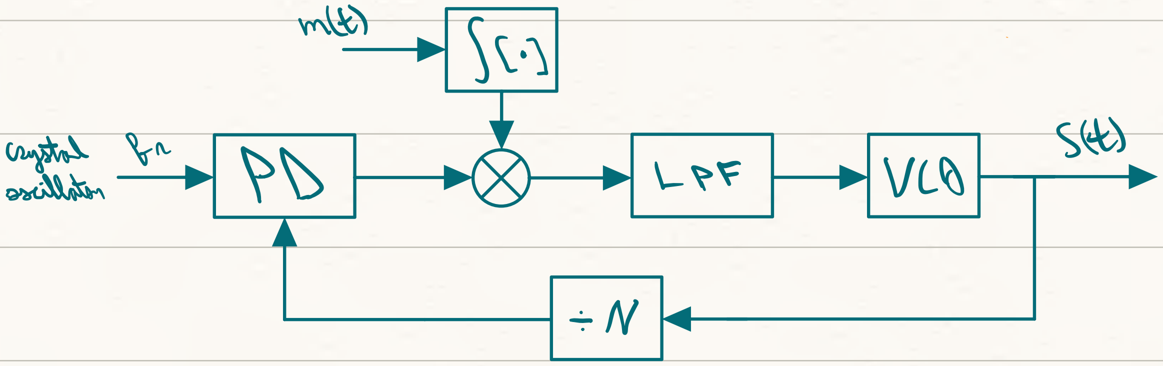
integrate m(t) then apply to a phase modulator to get frequency modulation

converted to a phase modulator by differentiating m(t) first.

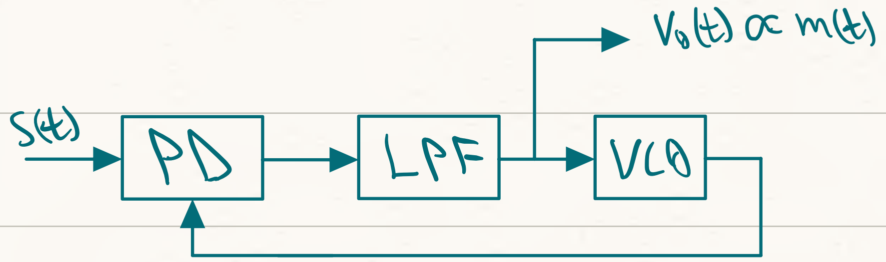
1- Analog frequency modulation:

the VCO produces a signal whose frequency changes with the voltage of the input message signal

- a signal can be modulated by simply inputting it to a VCO or a PLL for higher frequency stability.



- the demodulator can be a PLL or a balanced frequency discriminator.



2- Analog phase modulation:

- the same modulator and demodulator as FM can be used here, but without the integrator

3- frequency shift keying:

- same modulator and demodulator can be used here,

unipolar or polar

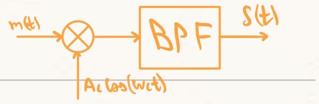
but the message signal has to be in digital form.

4- phase shift keying:

- the modulator here is the same as DSB-SC, but the

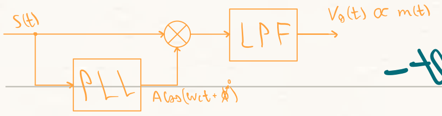
digital

message signal must be in polar form.



coherent detector

- the demodulator is the same as DSB-SC and SSB.



final practice:

Chapter 8 quiz:

$f_{min} = 200$ kHz , $f_{max} = 300$ kHz

increment = 10 kHz , LPF: $R_0 = 10k\Omega$, $C_0 = 1nF$

$V_{cc} = 10V$ XOR PD

a) $\because f_{min} = 200 kHz = \frac{1}{R_2(C+32pF)}$

$\wedge$ $100pF < C < 0.01\mu F$, $10k\Omega < R_1, R_2 < 1M\Omega$

try $C = 1nF \rightarrow R_2 = 4845\ \Omega$ invalid

try $C = 240pF \rightarrow R_2 = 17.73\ k\Omega$ valid

$\because f_{max} = 300 kHz = f_{min} + \frac{1}{R_1(C+32pF)}$

$\rightarrow R_1 = 35.46\ k\Omega$ valid

b) $\because$ XOR $k_d = \frac{V_{cc}}{\pi} = \frac{10}{\pi}$, $k_o = \frac{2\pi\,\Delta f}{V_{cc}-2}$

$\rightarrow k_o = 24000\pi\ \frac{rad/s}{V}$

$\because f_{co} = 240 kHz \rightarrow N = \frac{240k}{10k} = 24$

$\therefore k_v = 10$ kHz

$\because \omega_L = \frac{1}{RC} = 100$ kHz

$\rightarrow \zeta = \frac{1}{2}\cdot\sqrt{\frac{\omega_L}{k_v}} = 1.5811$

$\rightarrow \omega_n = 31622.8$ rad/s

$\wedge\ \omega_h = \omega_n \left[1 - 2\zeta^2 + (2 - 4\zeta^2 + 4\zeta^4)^{1/2}\right]^{1/2}$

$\rightarrow \omega_h = 11095.69$ rad/s $= 1766$ Hz

Chapter 8 homework:

8.1: k_d: phase detector gain, k_o: VCO gain

$\because f_r = 50$ kHz, $f_o = 1$ MHz, $k_d = 2$ V/rad

$k_o = \overset{200\pi \text{ rad/s/V}}{100 \text{ Hz/V}}$, $t_r = \frac{2.2}{\omega_h}$, $N = 20$

$\because \omega_h = k_v = \frac{k_d k_o}{N} = 20\pi^{\text{Hz}} = \omega_h^{\text{rad/s}}$

$\rightarrow t_r \approx 35$ ms

for $f_o = 1.2$ MHz $\rightarrow N = 24 \rightarrow \omega_h = 52.36^{\text{rad/s}}$

$\rightarrow t_r = 42.02$ ms

$\because f_{\text{free running}} = 1$ MHz $\rightarrow f_{\min} = 800$ kHz

$\wedge\ f_{\max} = 1.2$ MHz $\rightarrow N_{\min} = 16 \wedge N_{\max} = 24$

$\therefore$ range $= 8$ frequencies or $\frac{200 \text{ kHz} \times 2}{50 \text{ kHz}} = 8$

8.2: a) $\because f_o = 1$ MHz $\rightarrow N = 20 \rightarrow k_v = 20\pi$

$\because \zeta = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{\frac{\omega_L}{k_v}} \rightarrow \overset{\text{bandwidth of LPF}}{\omega_L = 40\pi \text{ rad/s}}$

b) $\overset{\text{bandwidth of LPF}}{\omega_L = 104.72 \text{ rad/s}}$

8.14: $\because TR = 4$ kHz $= 2 k_o k V_{c,\max}$

final 12/2018 :

Q1: a) $f_{min} = 21$ MHz , $f_{max} = 26$ MHz , res : 1kHz

∴ $\Delta f = 5$ MHz & number of freq. = 5000

output $= f_o' = 10 f_i + f_4^* + \frac{f_3^*}{10} + \frac{f_2^*}{100} + \frac{f_1^*}{1000}$

four modules required.

→ $f_i = 2$ MHz & $10 f_i = f_i + f_1 + f_2$

→ 18 MHz $= f_1 + f_2$ take $f_1 = 6$ MHz , $f_2 = 12$ MHz

→ $n_1 = \frac{6\text{MHz}}{2\text{MHz}} = 3$, $n_2 = \frac{12 + f^*_{max}}{6+2}$

→ $n_2 = \frac{12+9}{6+2} = 2.625$

If $f_o = 23.456$ → $f_i = 2$ MHz , $f_4^* = 3$ MHz

$f_3^* = 4$ MHz , $f_2^* = 5$ MHz , $f_1^* = 6$ MHz

b) $f_{min} = 935$ MHz , $f_{max} = 960$ MHz , res = 200kHz

P = 64 , Q = 1 , crystal = 10.24 MHz

∴ ×5 post multiplier → inc = 200k/5 = 40kHz

∴ $\frac{\text{crystal}}{40\text{kHz}} \leq R$ → R = 256 (011)

$f'_{min} = 187$ MHz , $f'_{max} = 192$ MHz

$D_{min} = P N_{min} + A_{min} = \frac{f'_{min}}{f_r'} = 4675$

→ $N_{min} = 73$, $A_{min} = 3$

$(A_{max}) = P - 1 = 63$

∧ $D_{max} = PN_{max} + A_{max} = \frac{f'_{max}}{f_r} = 4800$

$\rightarrow N_{max} = 75 \;, \; A_{max} = 0$

N	A		
73	3 - 63	→	61
74	0 - 63	→	64
75	0 - 0	→	1

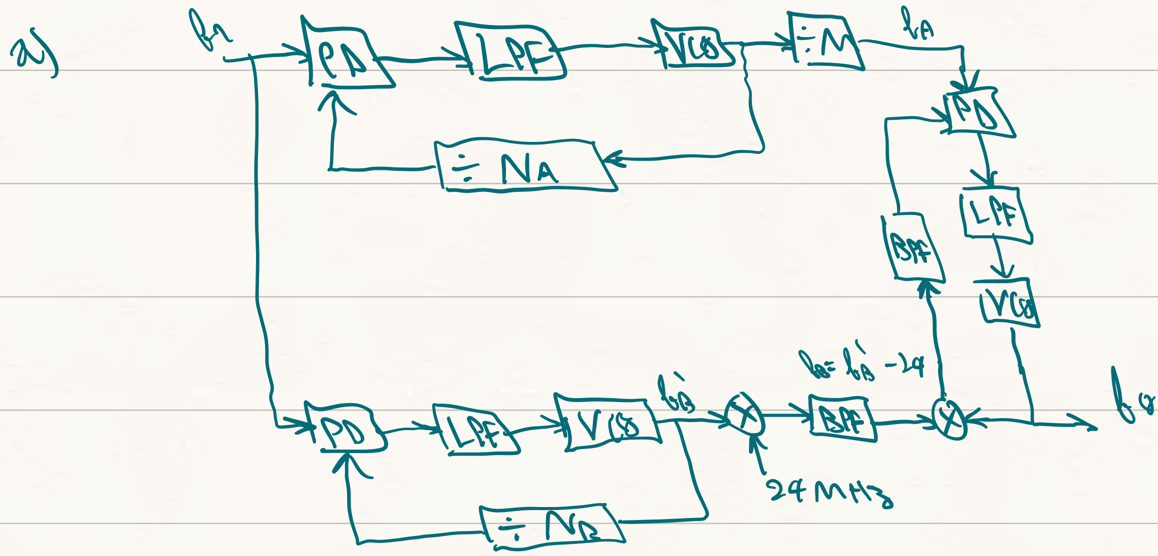
} 126 frequencies

$f_{max,\,dividers} = \frac{f_{max}}{P} = \frac{192M}{64} = 3MHz$

Q2: $f_{min} = 25MHz$, $f_{max} = 29MHz$, $f_{res} = 1kHz$

$f_r = 100k$

a)



for $f_{res} = 1kHz$ from $f_r = 100kHz \rightarrow M = 100$

$f_A = 0kHz \rightarrow 99kHz$ offset 10 Hz

$\rightarrow f_A = 500kHz \rightarrow 599kHz$

$f_B = 24.5MHz \rightarrow 28.5MHz$

max input to programmable divider

$\rightarrow f_A' = 50\ \text{MHz} \rightarrow 59.9\ \text{MHz}$

max input to prog. divider

$\sim f_B' = 48.5\ \text{MHz} \rightarrow 52.5\ \text{MHz}$

$\therefore N_{A,min} = \frac{50m}{100k} = 500 , \quad N_{A,max} = 599$

$\sim N_{B,min} = 485 , \quad N_{B,max} = 525$

for $f_o = 26.543 \rightarrow f_B = 26.0\ \text{MHz}$

$\rightarrow N_B = \frac{f_B + 24\ \text{MHz}}{f_r} = 500$

$\sim f_A = 543\ \text{kHz} \rightarrow f_A' = 54.3\ \text{MHz}$

$\rightarrow N_A = 543$

Q3: a) for two opposite diodes:

$L = 4\ \text{dB} , \quad A = \frac{2V_i}{\pi} = \frac{2}{5\pi}$

for two same direction diodes:

$L = 10\ \text{dB} , \quad A = \frac{1}{5\pi}$

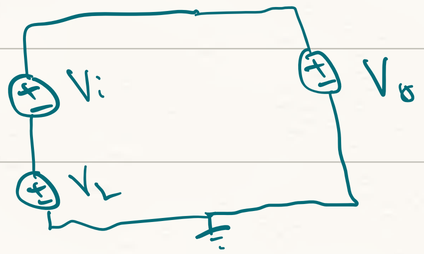
for four diodes:

$L = 4\ \text{dB} , \quad A \approx \frac{2}{5\pi}$

- the two opposite diodes can be used for DSB-LC
- both same direction diodes and four diodes can be used for DSB-SC
- V_L is chosen much larger than V_i so that it controls

the switching

h) if D2 is damaged, the equivalent circuit is:



→ $V_o = V_L + V_i$ when $V_L > 0$

if $V_L < 0$, D1 is off, $V_o = 0$

$$\therefore \quad V_o = \begin{cases} V_L + V_i, & V_L > 0 \\ 0, & V_L < 0 \end{cases}$$

$$\rightarrow V_o = p(t) \cdot [V_L + V_i]$$

$$p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{\sin[(2n+1)w_L t]}{2n+1}$$

$$\rightarrow V_o = \frac{1}{2}[V_L \sin(w_L t) + V_i \sin(w_i t)] + \frac{2}{\pi} V_L \sin(w_L t) \cdot \sum \cdots$$

$$+ \frac{2}{\pi} V_i \sin(w_i t) \cdot \sum \cdots$$

$$\rightarrow V_o = \frac{V_L}{2}\sin(w_L t) + \frac{V_i}{2}\sin(w_i t) + \frac{V_L}{\pi}\sum \frac{\cos((2n+1)w_L t - w_L t) - \cdots}{2n+1}$$

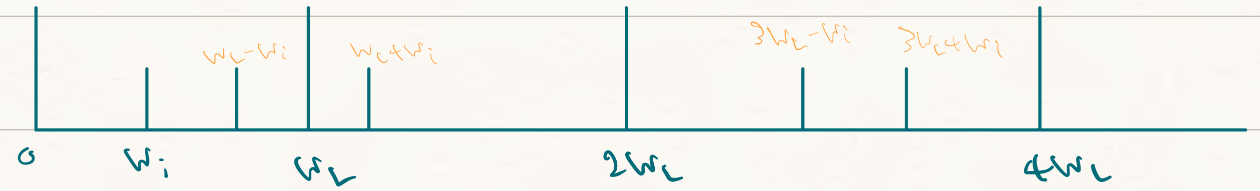
$$+ \frac{V_i}{\pi} \sum \frac{\cos((2n+1)w_L t - w_i t) - \cos((2n+1)w_L t + w_i t)}{2n+1}$$

→ at w_L: $\frac{V_L}{2}$, at w_i: $\frac{V_i}{2}$, at 0: $\frac{V_L}{\pi}$

at $2w_L$: $\frac{V_L}{\pi}$, at $2w_L$: $\frac{V_L}{3\pi}$, at $4w_L$: $\frac{V_L}{3\pi}$

at $w_L - w_i$: $\frac{V_i}{\pi}$, at $w_L + w_i$: $\frac{V_i}{\pi}$, at $3w_L - w_i$: $\frac{V_i}{3\pi}$

at $3w_L + w_i$: $\frac{V_i}{3\pi}$



Q4: a) $k_3 = \frac{I_{DSS} V_i V_L}{V_p^2} = \frac{0.1}{25} = 4 \times 10^{-3}$ A

conversion gain : $\left(\frac{k_3 \cdot R_L}{V_i}\right)^2 = 0$ dB

if $R_L = 500 \rightarrow V_o = 2V$

find 1/2020:

Q1: a) output res = 200 kHz → before 5X : 40 kHz

∵ crystal: 10.24 MHz → $R = \frac{10.24M}{40k} = 256$

→ code = 011

∵ $f_{min} = \frac{935M}{5} = 187$ MHz

→ $D_{min} = PN_{min} + A_{min} = \frac{187M}{40k} = 4695$

∴ $N_{min} = 73$, $A_m = 3$

∵ $f_{max} = \frac{960M}{5} = 192$ MHz

→ $D_{max} = PN_{max} + A_{max} = 4800$

→ $N_{max} = 75$, $A_{max} = 0$ $\quad |A_{max}| = 63$ $\quad P-1$

N	73	74	75	
A	3 → 63	0 → 63	0	
	61	64	1	→ 126

f_{max} at input is $\frac{f_{max}'}{P} = 3$ MHz

b) 21 to 25 M → 4 MHz → 400 frequencies

$\therefore$ 3 modules required → $f_0' = 10 f_i + f_3^* + \frac{f_2^*}{10} + \frac{f_1^*}{100}$

$f_i = \overset{2.1}{2}$ MHz & $10 f_i = f_1 + f_2 + f_i$

→ $\overset{18.9}{18}$ MHz $= f_1 + f_2$, take $f_1 = \overset{6.3}{6}$ MHz

and $f_2 = \overset{12.6}{12}$ MHz

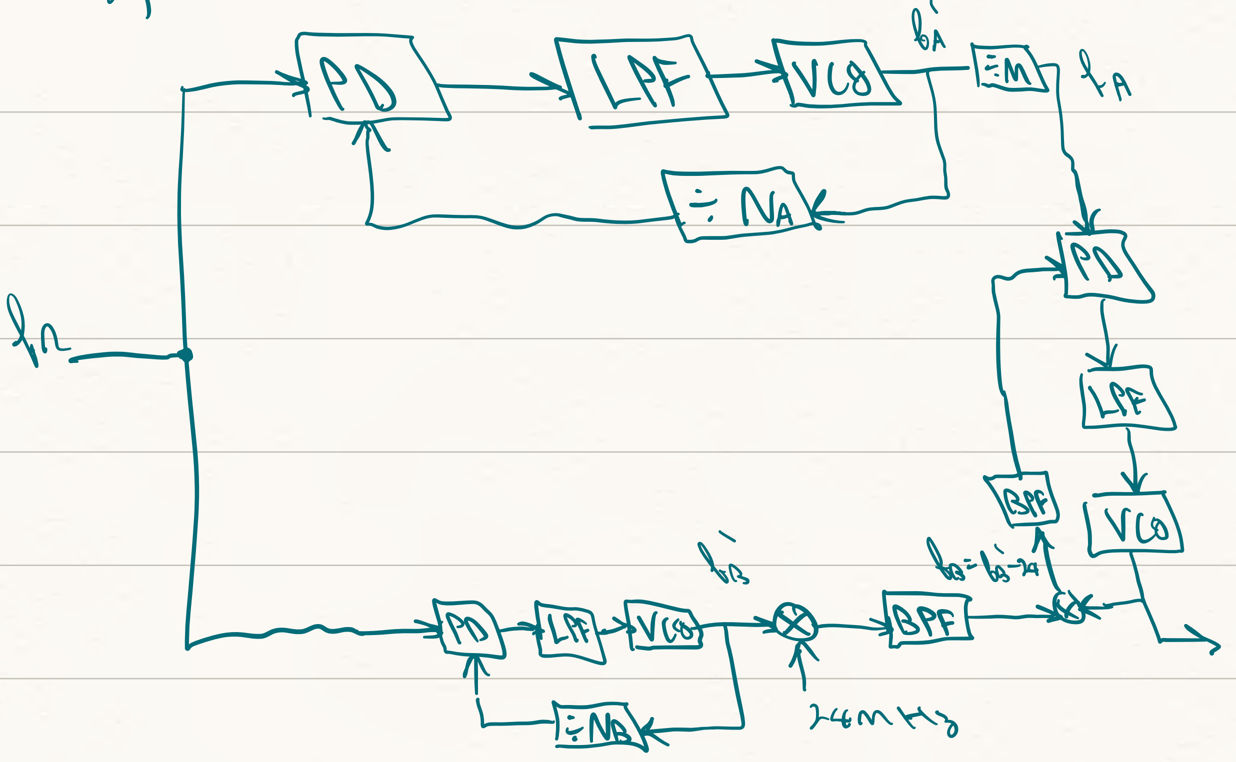
→ $N_1 = \frac{f_1}{f_i} = 3$, $N_2 = \frac{f_2 + f_1^*}{f_1 + f_i}$

$\therefore N_2 = \frac{\overset{12.6}{12} + 9}{\underset{8.4}{8}} = \overset{2.67}{2.625}$

if $f_0 = 23.45$ → $\overset{2.45}{3.45} = f_3^* + \frac{f_2^*}{10} + \frac{f_1^*}{100}$

→ $f_3^* = \overset{2}{3}$ MHz $f_2^* = 4$ MHz, $f_1^* = 5$ MHz

Q2: a)



for Resolution = 1kHz from 100 kHz f_R → $m = 100$

to keep adder loop operating at high speed, offset f_A by 500kH

→ $f_A = 500$ kHz → 599 kHz

∴ $f_A' = 50$ MHz → 59.9 MHz

∴ $N_A = 500$ → 599

& $f_B = 24.5$ MHz → 28.5 MHz

→ $f_B' = 48.5$ MHz → 52.5 MHz

∴ $N_B = 485$ → 525

(c) at divider A, $f_{max} = 59.9$ MHz

at divider B, $f_{max} = 52.5$ MHz

& $f_o = 27.643$M → $f_A = 543$ kHz

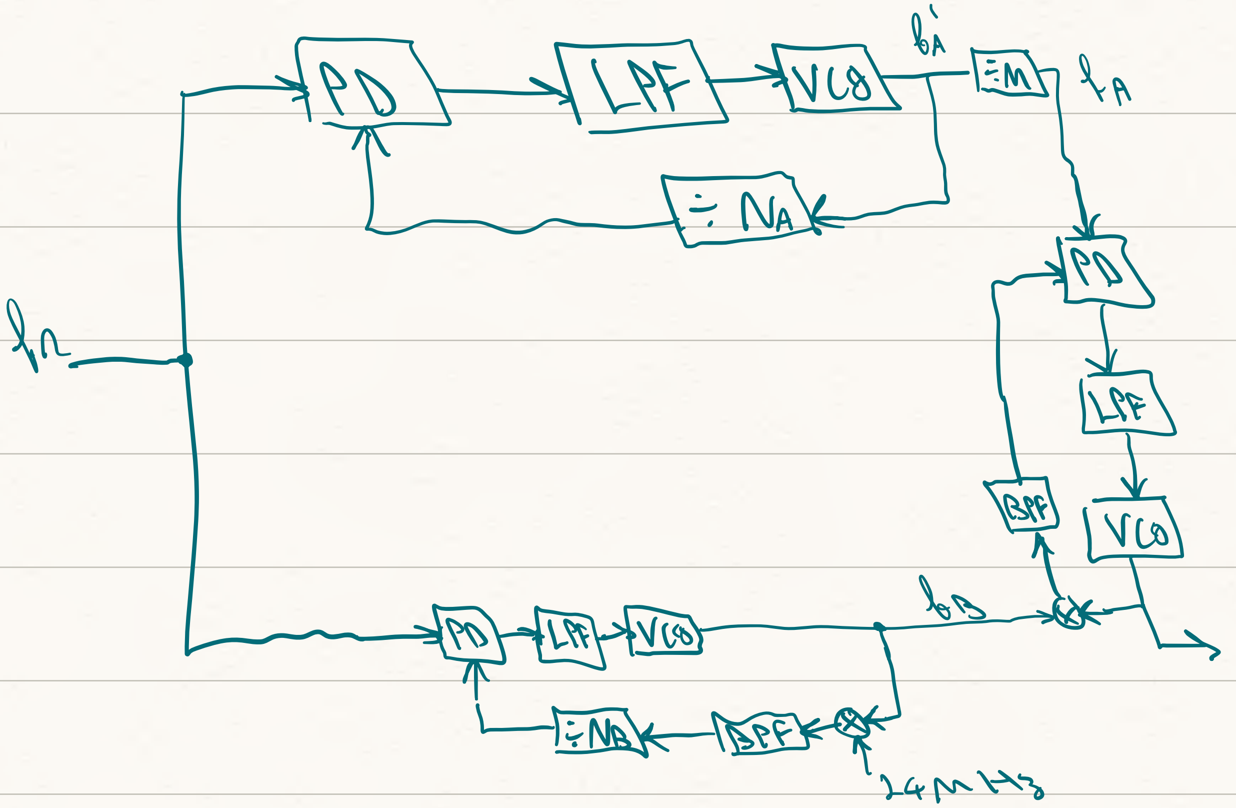
& $f_B = 29.1$ MHz

∴ $N_A = 543$ & $N_B = 511$

downconverter in the loop!

→ $N_B = 5$ → 45 if f_A offset by 500kH

for $f_o = 29.643$ → $N_B = 31$



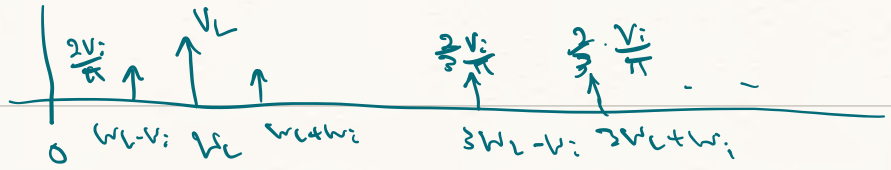
If f_A offset by 300 kHz → $N_A = 300 \rightarrow 399$

〃 $N_B = 7 \rightarrow 49$

for $f_o = 27.693$ MHz → $N_A = 343$, $N_B = 33$

Q3: a) 1- two-diodes in opposite directions:

$$V_o(t) = V_L \sin(\omega_L t) + \frac{2V_i}{\pi} \sum \frac{\cos[(2n+1)\omega_L - \omega_i]t - \cos[(2n+1)\omega_L - \omega_i]t}{2n+1}$$

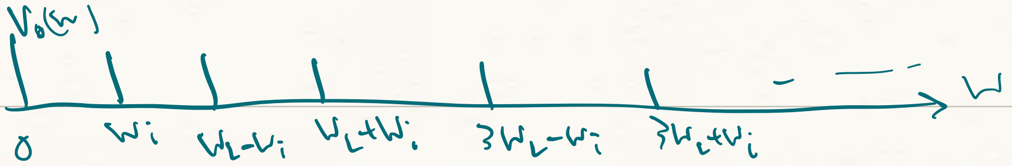


$L = \left(\frac{0.2}{0.4/\pi}\right)^2 = \left(\frac{\pi}{2}\right)^2 \approx 4$ dB

amplitude of $\omega_L - \omega_i$ component: $\frac{2V_i}{\pi} = \frac{0.4}{\pi}$

2- two-diodes in same direction:

$$V_o(t) = \frac{V_i}{2}\sin(\omega_i t) + \frac{V_i}{\pi} \sum \frac{\cos(2n+1)(\omega_L - \omega_i)t - \cos(2n+1)\omega_L t}{2n+1}$$



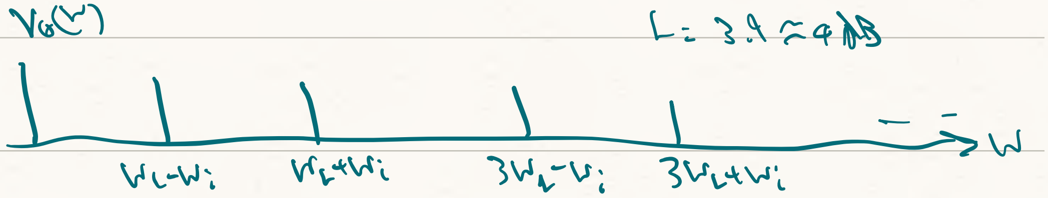
$L = \pi^2 = 9.94$ dB

$\omega_L - \omega_i$ component : $\frac{V_i}{\pi} = \frac{0.2}{\pi}$

3- four-diodes :

$$V_o(t) = \frac{R_L}{R_L + \frac{r_d}{2}} \cdot \frac{2V_i}{\pi} \cdot \sum \frac{\cos[(2n+1)\omega_L - \omega_i]t - \cos[(2n+1)\omega_L+\omega_i]}{2n+1}$$

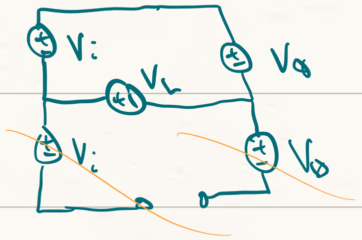
$L = 3.9 \simeq 4$ dB



at $\omega_L - \omega_i$ $A = \frac{R_L}{R_L + \frac{r_d}{2}} \cdot \frac{0.4}{\pi} \approx \frac{0.4}{\pi}$

- for DSB-LC, we need a component at ω_L, hence only two opposite diodes can be used
- for DSB-SC, no component should be at ω_L, hence both two-diodes in same direction and four diodes work.
- V_L should be much larger than V_i so that it controls the switching → switching speed will be that of V_L

b) the equivalent circuit will be :



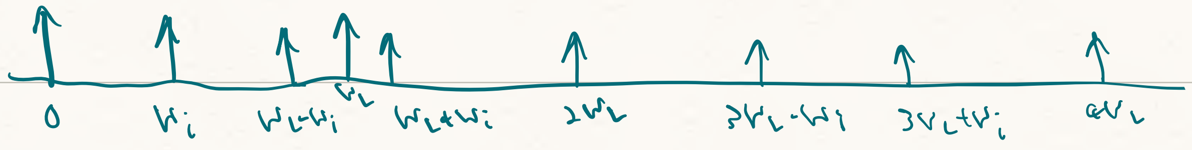
$\therefore$ $V_o = V_L + V_i$ for $V_L > 0$

& $V_o = 0$ for $V_L < 0$

→ $V_o(t) = p(t) \cdot (V_L + V_i)$ s.t., $p(t) = \begin{cases} 1, & V_L > 0 \\ 0, & V_L < 0 \end{cases}$

$$\rightarrow p(t) = \frac{1}{2} + \frac{2}{\pi}\sum \frac{\sin[(2n+1)\omega_L t]}{2n+1}$$

$$\rightarrow V_o(t) = \frac{V_L}{2}\sin(\omega_L t) + \frac{V_i}{2}\sin(\omega_i t) + \frac{V_L}{\pi}\cdot\sum\frac{\cos[(2n+1)\omega_L-\omega_i]t - \cos[(2n+1)\omega_L+\omega_i]t}{2n+1} + \frac{V_i}{\pi}\cdot\sum\frac{\cos[(2n+1)\omega_L-\omega_i]t - \cos[(2n+1)\omega_L+\omega_i]t}{2n+1}$$

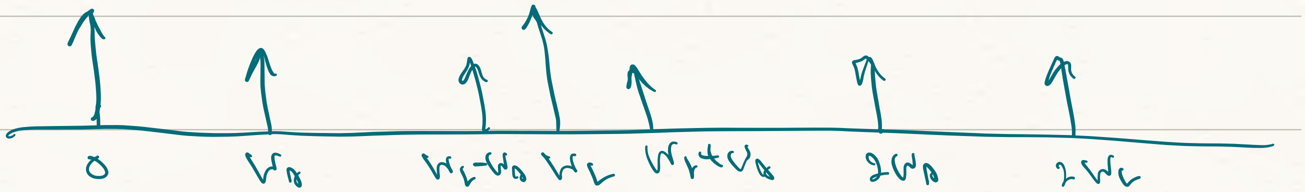


Q4: a) $V_i = 2\sin(\omega_s t) + 3\sin(\omega_L t)$

$$\rightarrow V_o(t) = 1.8\sin(\omega_s t) + 2.7\sin(\omega_L t) + \frac{2\cdot 2\cdot 3\cdot 0.1}{2}\cdot\left[\cos[(\omega_L-\omega_s)t] - \cos[(\omega_L+\omega_s)t]\right] + \frac{[2\sin(\omega_s t)]^2}{20} + 0.1[3\sin(\omega_L t)]^2$$

$$\rightarrow V_o(t) = 1.8\sin(\omega_s t) + 2.7\sin(\omega_L t) + 0.6\cos[(\omega_L-\omega_s)t] - 0.6\cos[(\omega_L+\omega_s)t] + 0.2[1-\cos(2\omega_s t)] + 0.45\cdot[1-\cos(2\omega_L t)]$$

$\therefore$



b) for a phase modulator PLL: $\theta_o(s) = \dfrac{k_{vo}V_m(s)}{s + k_o k_d F(s)/N}$

$$\rightarrow \theta_o(t) = N\theta_R(t) + \frac{N\, m(t)}{k_d}$$

$$\sim f_o(t) = \frac{1}{2\pi}\frac{d\theta_o(t)}{dt} = Nf_R + \frac{N}{2\pi k_d}\cdot\frac{dm(t)}{dt}$$

$$\sim s(t) = A_c \cos[\omega_c t + \theta(t)]$$

$$\because \theta_R(t) = 2\pi\int f_R\, dt$$

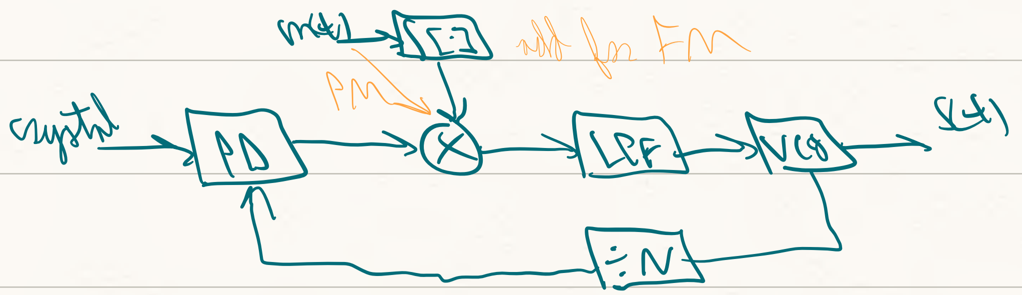
$$\rightarrow \theta_o(t) = 50\cdot 4\pi t\times10^6 + 50\cdot\sin(\omega_0 t)$$

$$\rightarrow f_o(t) = 100\times10^6 + \frac{50}{4\pi}\cdot 4\pi\times10^3\cdot\cos(2\pi\times10^3 t)$$

$$\rightarrow f_o(t) = 100\times10^6 + \underbrace{50\times10^3}_{\Delta f}\cos(2\pi\times10^3 t)$$

$$\rightarrow s(t) = A_c\cos[2\pi\times10^8 t + 50\sin(2\pi\times10^3 t)]$$

$\rightarrow f_c = 100$ MHz , TR $\geq 2\times50\times10^3 = 100$ kHz



- add integrator to go from PM to FM

final 1/2021:

Q1: a) $f_{min} = 250k = \frac{1}{R_2(C+32p)}$, take $\underline{C = 250pF}$

$\rightarrow \underline{R_2 = 14.1844\ k\Omega}$

$f_{max} = f_{min} + \frac{1}{R_1(C+32p)} \rightarrow \underline{R_1 = 11.82\ k\Omega}$

b) $\because f_o = 300\ kHz \rightarrow N = 30$, for XOR $k_\phi = \frac{V_{DD}}{\pi}$

$\wedge\ k_{vo} = \frac{2\pi \Delta f}{V_{DD}-2} \rightarrow k_v = \frac{600k \cdot 10/8}{30} = 25\ kHz$

$\because \zeta = \frac{1}{\sqrt{2}} \wedge 2\zeta = \sqrt{\frac{\omega_L}{k_v}} \rightarrow \omega_L = 50\ krad/s$

$\wedge\ \omega_L = \frac{1}{R_3 C_2}$, take $C_2 = 1nF \rightarrow R_3 = 20\ k\Omega$

loop bandwidth when $\zeta = \frac{1}{\sqrt{2}} \rightarrow \omega_n = \omega_h$

$\wedge\ \omega_n = \sqrt{k_v \omega_L} = 35.35\ krad/s$

Q2: 4 modules required $\because$ 9000 frequencies

$f_o' = 10f_i + f_4^* + \frac{f_3^*}{10} + \frac{f_2^*}{100} + \frac{f_1^*}{1000}$

$f_{min} = 10f_i = 21\ MHz \rightarrow f_i = 2.1\ MHz$

$\because 10f_i = f_i + f_1 + f_2 \rightarrow 18.9MHz = f_1 + f_2$

$\rightarrow$ take $f_1 = 6.3MHz$, $f_2 = 12.6\ MHz$

$N_1 = \frac{f_1}{f_i} = 3$, $N_2 = \frac{f_2 + f_1^*}{f_1 + f_i} = \frac{12.6+4}{6.3+2.1} = 2.57$

Q3: before $\times 5$ $\rightarrow f_{ref} = 40kHz \rightarrow R = \frac{2048}{40k} = 512$ (100)

$\because f'_{min} = \frac{925}{5} = 185\ MHz$, $P = 60$, $Q = 1$

$\rightarrow D_{min} = \frac{185M}{40k} = P N_{min} + A_{min} = 4625$

$\rightarrow N_{min} = 72$, $A_{min} = 17$ $|A_{max}| = P-1 = 63$

$f'_{max} = \frac{460}{5} = 192 MHz \rightarrow D_{max} = \frac{192M}{40k} = P N_{max} + A_{max}$

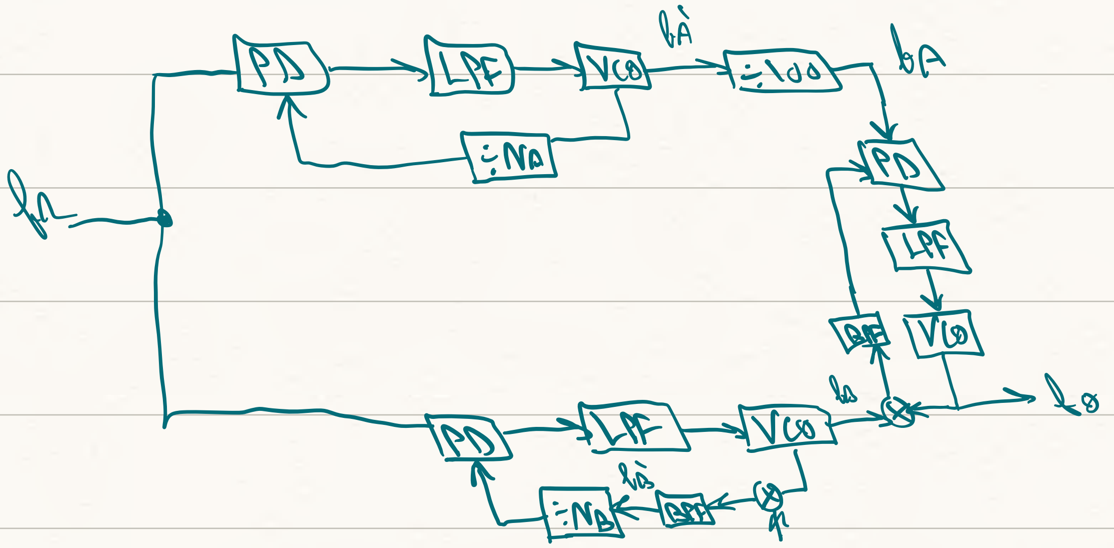
$\rightarrow N_{max} = 75$, $A_{max} = 0$

N	72	73	74	75	
A	17 → 63	0 → 63	0 → 63	0	number of freq.
	47	64	64	1	176

f_{max} at input = 192MHz / 64 = 3 MHz

Q4: ∵ $f_R = 100$ kHz, $f_{res} = 1$kHz → M = 100

a)



offset f_A by 500 kHz → f_A = 600 → 599 kHz

f_B = 21.5 MHz → 25.5 MHz

∴ f'_A = 60 MHz → 59.9 MHz → N_A = 600 → 599

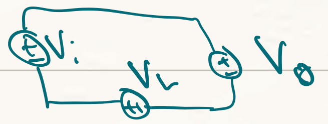
& f'_B = 0.5 MHz → 4.5 MHz → N_B = 5 → 45

b) max B = 4.4 MHz , max A = 59.4 MHz

for $f_o = 24.563$ MHz $\rightarrow f_A = 563$ kHz $\rightarrow N_A = 563$

$f_B = 24 \rightarrow f_B' = 3$ MHz $\rightarrow N_B = 30$

Q5: a)



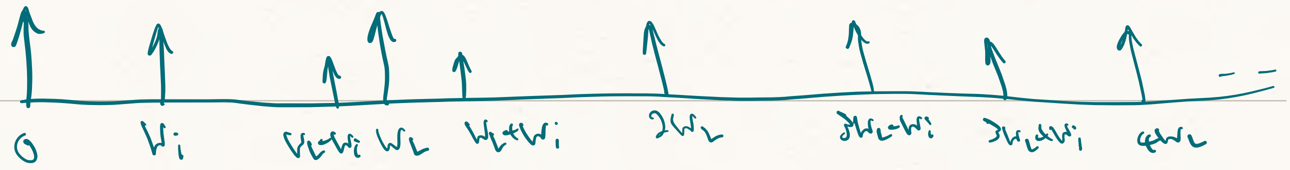
$\rightarrow V_o = V_L + V_i$ for $V_L > 0$

& $V_o = 0$ for $V_L < 0$

$$\rightarrow P(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{\sin[(2n+1)w_L t]}{2n+1}$$

$$\therefore V_o(t) = \frac{V_L}{2}\sin(w_L t) + \frac{V_i}{2}\sin(w_i t) + \frac{2V_L}{\pi} \cdot \left[\sum \frac{\cos[(2n+1)w_L t - w_L t] - \cos[(2n+1)w_L t + w_L t]}{2n+1}\right]$$

$$+ \frac{2V_i}{\pi} \cdot \sum \frac{\cos[(2n+1)w_L - w_i]t - \cos[(2n+1)w_L + w_i]t}{2n+1}$$



- $V_L >> V_i$ so that V_L controls switching and switching speed will be w_L

b) $\theta_o(t) = N\theta_n(t) + \frac{N}{k_f} m(t)$

& $\theta_n(t) = 2\pi \int f_n \, dt$

$\rightarrow \theta_o(t) = 200\pi \cdot 10^6 t + 250 \sin(2\pi \times 10^3 t)$

& $f_o(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$

$\rightarrow f_o(t) = \overbrace{100\cdot10^6}^{f_c} + \overbrace{250\times10^3}^{\Delta f}\cos(2\pi\times10^3 t)$

$\rightarrow S(t) = A_c\cos[\theta_o(t)]$

$= A_c\cos[200\pi\cdot10^6 t + 250\sin(2\pi\times10^3 t)]$

$\therefore f_c = 100\,\text{MHz}$, $TR \geq 2\Delta f = 500\,\text{kHz}$

- integrate the message signal to go from PM to FM

final 1/2023:

Q1: a) $f_{min} = 200\,kHz$, $f_{max} = 600\,kHz$, $f_r = 20\,kHz$

at $f_o = 400\,kHz \rightarrow N = \frac{f_o}{f_r} = 20$

conditions for VCO : $100pF < C < 0.01\,\mu F$

$10k\Omega < R_1, R_2 < 1M\Omega$

for $f_{min} = 200\,kHz = \frac{1}{R_2(C+32pF)}$, try $\boxed{C = 150pF}$

$\rightarrow \boxed{R_2 = 27.5\,k\Omega}$

for $f_{max} = 600\,k\Omega = f_{min} + \frac{1}{R_1(C+32pF)} \rightarrow \boxed{R_1 = 13.7\,k\Omega}$

to find R_3 and C_2 , find W_L ∴ $W_L = \frac{1}{R_3 C_2}$

& $2\zeta = \sqrt{\frac{W_L}{k_V}} \rightarrow W_L = 2k_V$ ∴ $\zeta = \frac{1}{\sqrt{2}}$

$k_V = \frac{k_o k_d}{N}$, $N = 20$, $k_o = \frac{2\pi\,\Delta f}{V_{DD} - 2} = 100k\pi$

& k_d for XOR $= \frac{V_{DD}}{\pi} \rightarrow k_V = \frac{\pi \times 10^6}{20} = 50k\pi$

$\rightarrow W_L = 100k\pi$ rad/s , take $\boxed{C_2 = 200pF}$

$\rightarrow \boxed{R_3 = 15.9\,k\Omega}$

$W_n = W_h$ for $\zeta = \frac{1}{\sqrt{2}} = \sqrt{k_V W_L} = 70711\pi$ rad/s

or $\boxed{BW = 35.36\,kHz}$

b) ∴ $\theta_o(s) = N\theta_r(s) + \frac{N\,M(s)}{k_d}$

$\rightarrow \theta_o(t) = N\theta_r(t) + \frac{N}{k_d} m(t)$

$\because \; \theta_n(t) = 2\pi \int f_n \, dt$

$\rightarrow \theta_o(t) = 200\pi \times 10^6 \; t \; + \; 75 \sin(2\pi \times 10^3 \, t)$

$\because \; f_o(t) = \frac{1}{2\pi} \frac{d\theta_o(t)}{dt}$

$\rightarrow f_o(t) = 100 \times 10^6 \; + \; 75 \times 10^3 \cos(2\pi \times 10^3 \, t)$

$\therefore \; f_c = 100$ MHz , $\Delta f = 75 \times 10^3$ Hz

$TR \geqslant 2\Delta f = 150$ kHz

Q2: a) taken before decade divider → f_o'

$\because$ number of frequencies > 1k but < 10k → four module

$\therefore \; f_o' = 10 f_i + f_4^* + \frac{f_3^*}{10} + \frac{f_2^*}{100} + \frac{f_1^*}{1000}$

$f_i = \frac{f'_{o,min}}{10} = 3$ MHz

$\wedge \; 10 f_i = f_i + f_1 + f_2 \rightarrow f_1 + f_2 = 27$ MHz

take $f_1 = 9$ MHz , $f_2 = 18$ MHz

$\rightarrow n_1 = \frac{f_1}{f_i} = 3$, $n_2 = \frac{f_2 + f^*_{1,max}}{f_1 + f_i} = \frac{18+9}{9} = 3$

if $f_x = f_o' = 34.567$ MHz

$\rightarrow f_i = 3$ MHz, $f_4^* = 4$ MHz, $f_3^* = 5$ MHz, $f_2^* = 6$ MH

$\wedge \; f_1^* = 7$ MHz

at least 4 (4×2)+1 at least 9

only 12 oscillators , 12 BPF , and 12 mixers

b) $f_o = f_n \cdot N \cdot P \rightarrow$ resolution $= P f_n$

$\therefore$ 200kHz $= 25 f_r \rightarrow f_r = 8$ kHz

$t_s \approx \frac{25}{f_r} = 3.125$ ms

$\therefore f_{o,max} = N_{max} \cdot P f_r \rightarrow N_{max} = 550$

& $f_{o,min} = N_{min} \cdot P f_r \rightarrow N_{min} = 450$

max frequency at programmable divider: 4.4 MHz

Q3: $\therefore f_{or}$(after $\times 4$) $= 200$ kHz $\rightarrow f_r = 50$ kHz

$R = \frac{\text{crystal}}{f_r} = 128$ (010)

$f'_o = \frac{f_o}{4} \rightarrow f'_{o,min} = 231.25$ MHz

$\rightarrow D_{min} = P N_{min} + A_{min} = \frac{f'_{o,min}}{f_r}$, $P = 64$, $Q = 1$

$\rightarrow N_{min} = 72$, $A_{min} = 17$

& $f'_{o,max} = 237.5$ MHz $\rightarrow D_{max} = 4750$

$\rightarrow N_{max} = 74$, $A_{max} = 14$, $|A_{max}| = \overset{63}{P-1}$

N	72	73	74	
A	17 → 63	0 → 63	0 → 14	frequencies
	47	64	15	126

frequency range of VCO = $f'_{o,min} \rightarrow f'_{o,max}$

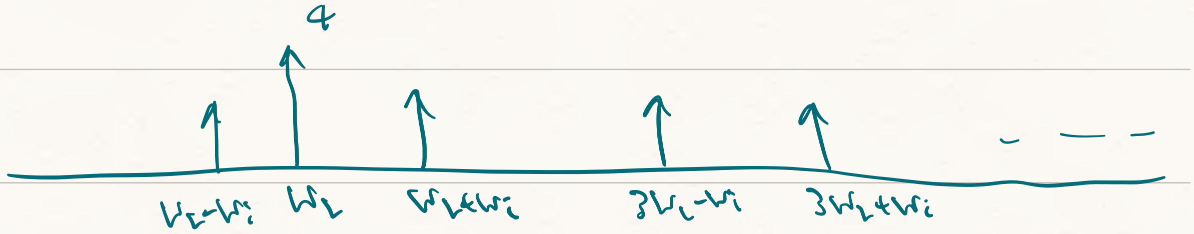
= 231.25 MHz → 237.5 MHz

fmax at programmable dividers: $\frac{f'_{o,max}}{P} = 3.71$ MHz if $f_0 = 930$ MHz → $f_0' = 232.5$ MHz

→ $D = 4650$ → $N = 72$, $A = 41$

Q4: a) 1- two opposite diodes:

$$V_o(t) = 4\sin(\omega_L t) + \frac{0.4}{\pi}\sum\frac{\cos[(2n+1)\omega_L t - \omega_i t] - \cos[(2n+1)\omega_L t + \omega_i t]}{2n+1}$$



$A|_{\omega_i} = 0$ $\quad$ $A|_{\omega_L} = 4$

$A|_{\omega_L-\omega_i} = \frac{0.4}{\pi}$ $\quad$ $A|_{\omega_L+\omega_i} = \frac{0.4}{\pi}$

2- two same direction diodes:

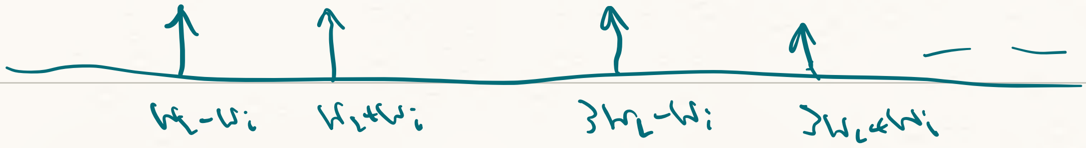
$$V_o(t) = 0.1\sin(\omega_i t) + \frac{0.2}{\pi}\cdot\sum\frac{\cos[(2n+1)\omega_L t - \omega_i t] - \cos[(2n+1)\omega_L t + \omega_i t]}{2n+1}$$

$A|_{\omega_i} = 0.1$, $A|_{\omega_L} = 0$

$A|_{\omega_L-\omega_i} = \frac{0.2}{\pi}$, $A|_{\omega_L+\omega_i} = \frac{0.2}{\pi}$

3- four-diode ring mixer: $\frac{R_L}{R_L+\frac{R_d}{2}} \approx 1$

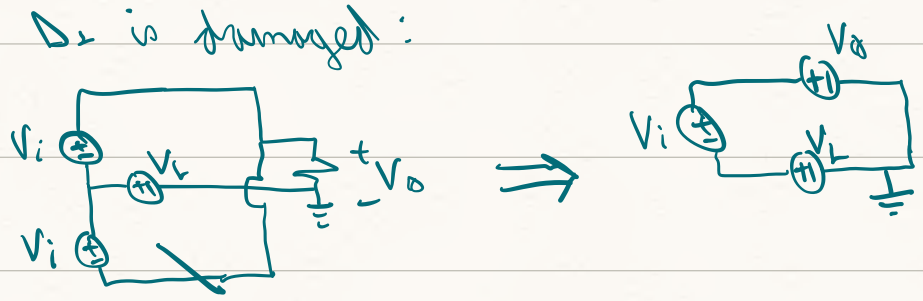
$$V_o(t) = \frac{0.4}{\pi}\cdot\sum\frac{\cos[(2n+1)\omega_L t - \omega_i t] - \cos[(2n+1)\omega_L t - \omega_i t]}{2n+1}$$



$$A\Big|_{\omega_i} = 0 \,, \quad A\Big|_{\omega_L} = 0$$

$$A\Big|_{\omega_L-\omega_i} = \frac{0.8}{\pi} \,, \quad A\Big|_{\omega_L+\omega_i} = \frac{0.4}{\pi}$$

h) when D_2 is damaged:



when $V_L > 0 \rightarrow V_o = V_L + V_i$

when $V_L < 0 \rightarrow V_o = 0$

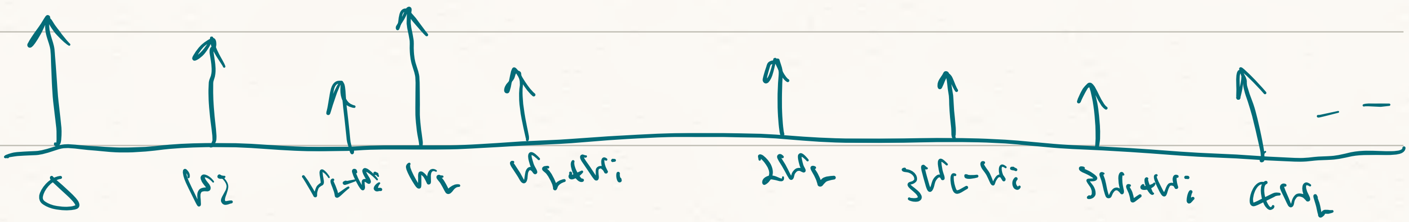
$V_o(t) = p(t)\left[V_L + V_i\right]$ s.t., $p(t) = \begin{cases} 1 , & V_L > 0 \\ 0 , & V_L < 0 \end{cases}$

$$\rightarrow p(t) = \frac{1}{2} + \frac{2}{\pi} \cdot \sum \frac{\sin[(2n+1)\omega_L t]}{2n+1}$$

$$\therefore V_o(t) = \frac{V_L}{2} \sin(\omega_L t) + \frac{V_i}{2} \sin(\omega_i t) + - -$$

$$\frac{V_L}{\pi} \cdot \sum \frac{\cos[(2n+1)\omega_L t - \omega_L t] - \cos[(2n+1)\omega_L t + \omega_L t]}{2n+1} + \cdots$$

$$\frac{V_i}{\pi} \cdot \sum \frac{\cos[(2n+1)\omega_L t - \omega_i t] - \cos[(2n+1)\omega_L t + \omega_i t]}{2n+1}$$



final 6/2021:

Q1: a) $\because$ $f_{min} = 200\ kHz = \frac{1}{R_2(C+32pF)}$, $C = 200pF$

$\rightarrow R_2 = 21.55\ k\Omega$

and $f_{max} = f_{min} + \frac{1}{R_1(C+32pF)} \rightarrow R_1 = 10.78\ k\Omega$

b) $\because$ $f_o = 400\ kHz$ and $f_r = 10\ kHz \rightarrow N = 40$

k_d for XOR $= \frac{V_{DD}}{\pi}$, $k_o = \frac{2\pi \Delta f}{V_{DD}-2} = \frac{800k\pi}{8}$

$\rightarrow k_v = \frac{k_d k_o}{N} = 20k\ Hz$

$\because$ $\zeta = \frac{1}{\sqrt{2}}$ and $2\zeta = \sqrt{\frac{w_L}{k_v}} \rightarrow w_L = 40k\ rad/s$

and $w_L = \frac{1}{R_3 C_2}$, let $C_2 = 1nF \rightarrow R_3 = 25\ k\Omega$

turn VCO off to conserve energy when no input

c) zener: voltage regulation, inhibit: disable

LD: shows if f_o and f_r locked

source follower: isolates input and reduces loading

Q2: a) $f_o = 10 f_i + f_4^a + \frac{f_3^a}{10} + \frac{f_2^a}{100} + \frac{f_1^a}{1000}$

$\rightarrow f_i = 2.2\ MHz$, $19.8\ MHz = f_1 + f_2$

take $f_1 = 6.6\ MHz$, $f_2 = 13.2\ MHz$

$n_1 = \frac{f_1}{f_i} = 3$, $n_2 = \frac{f_2 + f^a_{1,max}}{f_i + f_1} = 2.523$

b) 12 oscillators, at least 9 mixers $^{2\times4+1}$

four modules and mixer connected to oscillators

Q3: ∵ $f_r = 40\text{kHz} \rightarrow R = \frac{10.24M}{40k} = 256$ (011)

∵ $f_{o,min}' = 185\text{MHz}$, $P = 64$, $Q = 1$

$\rightarrow D_{min} = PN_{min} + QA_{min} = \frac{185M}{40k}$

$\rightarrow N_{min} = 72$, $A_{min} = 17$ $|A_{max}| = P-1 = 63$

& $f_{o,max}' = 190\text{MHz} \rightarrow D_{max} = 4750$

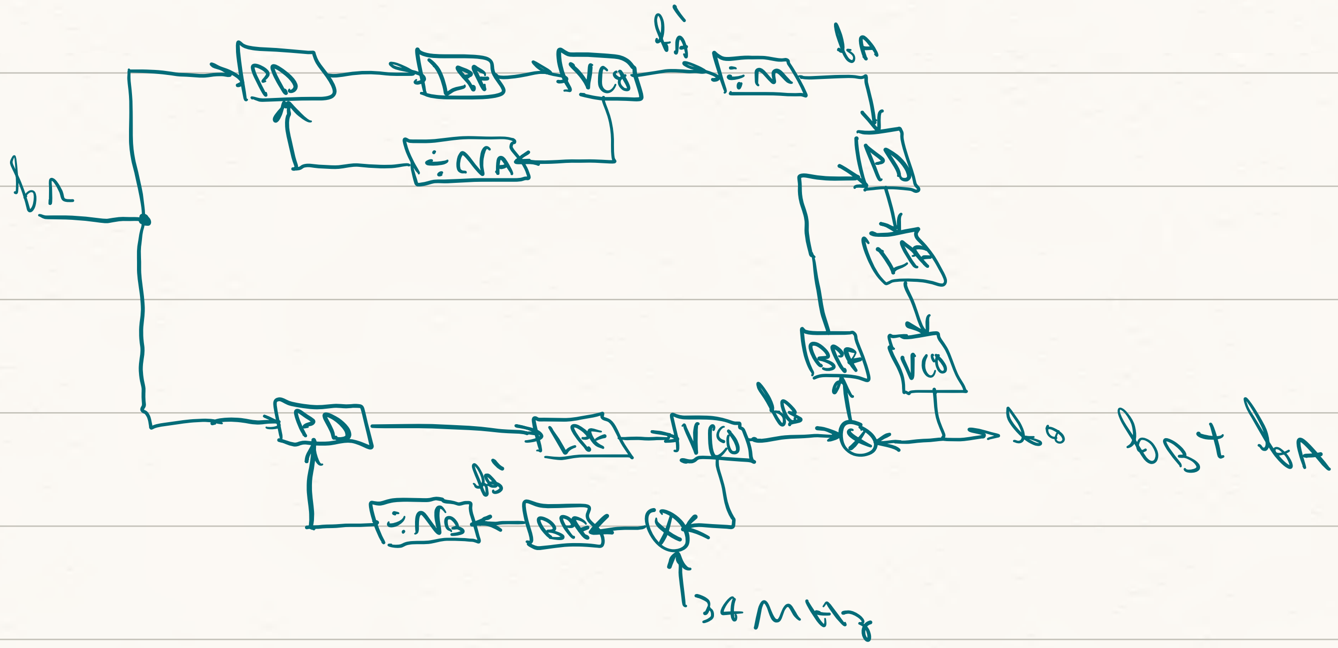
$\rightarrow N_{max} = 74$, $A_{max} = 14$

N	72	73	74	
A	17 → 63	0 → 63	0 → 14	
	47	64	15	126

tuning range of VCO: 185 → 190 MHz

f_{max} at programmable dividers: 2.969 MHz

Q4: ∵ $f_r = 100\text{kHz}$ & $f_{res} = 1\text{kHz} \rightarrow M = 100$



a) Offset f_A by 600 kHz to avoid aliasing

down after loop → $f_A = 600\text{kHz} \rightarrow 599\text{ kHz}$

$N_A = 600 \rightarrow 599 \quad (300 \rightarrow 399)$

$\rightarrow f_A' = 60\text{ MHz} \rightarrow 59.9\text{ MHz}$

$\rightarrow f_B = 34.6\text{ MHz} \rightarrow 37.5\text{ MHz} - 1\text{kHz}$

$\therefore f_B' = 600\text{kHz} \rightarrow 3.6\text{ MHz} - 1\text{kHz}$

$7 \rightarrow 37 - 1\ (36)$

$\rightarrow N_B = 6 \rightarrow 36 - 1\ (34)$

39.9

b) $f_{max,A} = 59.9\text{ MHz}$, $f_{max,B} \approx 3.6\text{ MHz}$

ii) $f_0 = 36.678 \rightarrow f_B = 36 \rightarrow N_B = \overset{22}{20}$

$N_A = \overset{378}{678}$

Q5: a) $v_o(t) = p(t)[v_i(t) + v_L(t)]$ s.t., $p(t) = \begin{cases} 1, & v_L > 0 \\ 0, & v_L < 0 \end{cases}$

b) $f_o(t) = N f_R + \frac{1}{2\pi}\frac{dm(t)}{dt} \cdot \frac{N}{k_d}$

$= 80\text{ MHz} + 2.667 \times 10^4 \cos(2\pi \times 10^3 t)$

$\rightarrow f_c = 80\text{ MHz}, \ \Delta f = 26.7\text{ kHz}$

$TR \geq 2\Delta f = 53.3\text{ kHz}$

c) 10 dB for same direction, 4 dB for opposite and rising