Subject Chapter 9 : maxwell's equations Date 21/7/2020 No.

- faraday's law : induced emf (in a closed circuit) is equal to the time rate of change of the magnetic flux linkage by the circuit

$$\rightarrow \quad V_{emf} = -N \frac{d\phi}{dt}$$

where N: number of turns in the circuit | ϕ: flux through each turn.

- the negative sign in faraday's law is a result of Lenz's law which states that the induced emf acts in such a way to oppose the flux inducing it. (the magnetic field produced by the induced current will oppose the magnetic field inducing the current)
- if a circuit has one turn $(N=1) \rightarrow V_{emf} = -\frac{d\phi}{dt}$

$\because V_{emf} = \oint_L \bar{E} \cdot d\bar{l} \quad \wedge \quad \phi = \int_s \bar{B} \cdot d\bar{s}$

$$\rightarrow \boxed{V_{emf} = \frac{-d}{dt} \left[\int_s \bar{B} \cdot d\bar{s} \right]}$$

- Reminder : Stokes's theorem : $\oint_L \bar{A} \cdot d\bar{l} = \int_s (\nabla \times \bar{A}) \, d\bar{s}$

divergence theorem : $\oint_s \bar{A} \cdot d\bar{s} = \int_v \nabla \cdot \bar{A} \, dv$

$\nabla \times \bar{A}$: curl of A | $\nabla \cdot \bar{A}$: divergence of $\bar{A}$

$\because \nabla \times \bar{E} = \frac{-d}{dt} \bar{B} \quad \therefore \oint_L \bar{E} \cdot d\bar{l} = \int_s (\nabla \times \bar{E}) \, d\bar{s} = \frac{-d}{dt} \int_s \bar{B} \, d\bar{s}$

- if the curl of a vector $\neq 0$ then the vector is <u>not</u> conservative
- moving loop in a static $\bar{B}$ field : a conducting loop is assumed to be a number of free electrons.

$\because$ force on a charge moving through a magnetic field $\bar{B}$ with a constant velocity u is : $F_m = Q\bar{u} \times \bar{B}$

$\rightarrow \frac{F_m}{Q} = \bar{u} \times \bar{B}$ take equal to $\bar{E}_m$ (motional electric field)

$\therefore V_{emf} = \oint_L \bar{E}_m \cdot d\bar{l} = \oint_L (\bar{u} \times \bar{B}) \cdot d\bar{l}$

$\xrightarrow{\text{stokes's theorem}} V_{emf} = \int_s (\nabla \times \bar{E}_m) \, d\bar{s} = \int_s \nabla \times (\bar{u} \times \bar{B}) \, d\bar{s}$

$$\therefore \boxed{\nabla \times \bar{E}_m = \nabla \times (\bar{u} \times \bar{B})}$$

* a moving loop in a Time-varying field:

the sum of equations for a stationary loop in a time-varying field and a moving loop in a static field yield the equation for a moving loop in a time varying field

$\because V_{emf} = \oint_L \bar{E} \cdot d\bar{l} = -\frac{d}{dt}\int_S \bar{B}\, d\bar{S} + \oint_L (\bar{u} \times \bar{B})\, d\bar{l}$

$\rightarrow V_{emf} = \int_S (\nabla \times \bar{E}) d\bar{S} = \int_S \nabla \times (\bar{u} \times \bar{B})\, d\bar{S} - \int_S \frac{d\bar{B}}{dt} d\bar{S}$

$\therefore \boxed{\nabla \times \bar{E} = \nabla \times (\bar{u} \times \bar{B}) - \frac{d\bar{B}}{dt}}$

example 9.1:

a) $\because$ stationary loop and time-varying field $\rightarrow V_{emf} = -\int_S \frac{dB}{dt}\, d\bar{S}$

$\rightarrow V_{emf} = -\frac{d}{dt}\int_0^y \int_0^x 4\cos(10^6 t) \times 10^{-3}\, dx\, dy$

bar is at $y = 8$ cm → effective length = 8 cm & width = 6 cm (example)

$\therefore V_{emf} = -\frac{d}{dt}\int_0^{8cm}\int_0^{6cm} 4\times10^{-3}\cos(10^6 t)\, dx\, dy$

$\rightarrow V_{emf} = -\frac{d}{dt}[(8cm)\cdot(6cm)\cdot(4m)\cdot\cos(10^6 t)]$ V

$\rightarrow V_{emf} = 19.2\sin(10^6 t)$ V

b) $\because$ moving loop in static field $\rightarrow V_{emf} = -uBl = -4.8$ mV

c) $\because V_{emf} = -\frac{d\Phi}{dt}$ & $\Phi = \int_S \bar{B} \cdot d\bar{S}$ & y is changing

$\rightarrow \Phi = \int_0^y \int_0^{6cm} 4m\cos(10^6 t - y)\, dx\, dy = [0.24\sin(10^6 t) - 0.24\sin(10^6 t - y)]$ mWb

$\because y = ut = 20t \quad \therefore V_{emf} = -\frac{d}{dt}[0.24m\sin(10^6 t) - 0.24m\sin(10^6 t - 20t)]$

$\therefore V_{emf} = \frac{d}{dt}(0.24m\sin(10^6 t - 20t)) - \frac{d}{dt}(0.24m\sin(10^6 t))$

$\rightarrow V_{emf} = 239.9952\cos((10^6 - 20)t) - 240\cos(10^6 t)$ V

practice exercise 9.1: $\because$ moving loop in static field: $V_{emf} = -uBl$

a) $V_{emf} =$ ~~-0.4V~~ 0.4V b) 20 mA $(\frac{V}{R})$ d) $\frac{(0.4)^2}{20} = 8$ mW example 9.2 : $\because$ moving loop in static field : $\int_L (\bar{u} \times \bar{B}) d\bar{l}$ & $\bar{u} = \rho \omega \bar{a}_\phi$

$\therefore \bar{u} = 0.04 \cdot 2\pi \cdot 50 \cdot \bar{a}_\phi = 4\pi \bar{a}_\phi$ $\therefore \bar{u} = -\sin\phi \cdot 4\pi \bar{a}_x + 4\pi \cos\phi \bar{a}_y$

$\therefore \bar{u} \times \bar{B} = -50 \cdot 4\pi \cdot \cos\phi \cdot \bar{a}_z = -200\pi \cos\phi \cdot \bar{a}_z$ m

$\rightarrow -0.2\pi \cos\phi \cdot \bar{a}_z$ & $d\bar{l} = dz$

$\therefore V_{emf} = -\int_0^{3cm} 0.2\pi \cos\phi \, dz = 6m \cdot \pi \cdot \cos\phi$ V

$\because \phi = \int_t \omega \, dt \rightarrow$ at $t = 1s$ $\phi = \int_0^1 100\pi \, dt = 100\pi$ X

$\therefore V_{emf} = 18.85$ mV X

$\because d\phi = \omega \cdot dt$ & at $t = 0$, $\phi = \frac{\pi}{2}$ rad ($\because$ in yz-plane)

& $\phi = \omega t + C_0 \rightarrow C_0 = \frac{\pi}{2}$ $\therefore$ at $t = 1ms$ $\phi = \frac{3}{5}\pi$

$\therefore V_{emf} = -6\pi \cdot \cos(\frac{3}{5}\pi) = 5.825$ V

b) $\because V_{emf} = -6\pi \cdot \cos(\phi)$m and at 3ms, $\phi = \frac{4}{5}\pi$ rad

$\rightarrow V_{emf} = 15.25$mV & $R = 0.1\Omega$ $\therefore I = 0.1525$ A

practice exercise 9.2 :

a) $\because \bar{B} = 50 \bar{a}_x$ mWb/m² & $\bar{u} = -4\pi \sin\phi \bar{a}_x + 4\pi \cos\phi \bar{a}_y$

$\therefore \bar{u} \times \bar{B} = -200m\pi \sin\phi \cdot \bar{a}_z$ $\therefore V_{emf} = 6\pi \sin\phi$ mV

$\rightarrow$ at $t = 1ms$, $V_{emf} = -17.93$mV

$\rightarrow$ at $t = 3ms$, $V_{emf} = -11.08$mV $\rightarrow I = 0.111$ A

b) ~~$\because V_{emf} = \oint_L (\bar{u} \times \bar{B}) d\bar{l} - \frac{d}{dt} \int_S \bar{B} \, d\bar{S}$~~

~~& $-\int \frac{\partial \bar{B}}{\partial t} d\bar{S}$ & $\bar{B} = \cos\phi \cdot 0.02t \cdot \bar{a}_\rho + \sin\phi \cdot 0.02t \, \bar{a}_\phi$~~

~~$\rightarrow -\int_{\frac{\pi}{2}}^{\phi} \int_0^{3cm} [0.02 \cos\phi +$~~

$\because V_{emf} = \frac{-d\Phi}{dt}$ & $\Phi = \int_S \bar{B} \cdot d\bar{S}$ & $d\bar{S} = \rho \cdot d\phi \cdot dz$

$\rightarrow \Phi = \int_0^{3cm} \int_{\frac{\pi}{2}}^{\phi} [0.02t \cdot \cos\phi] \cdot d\phi \cdot dz \cdot \rho = 0.8mt \cdot \int_0^{3cm} \int_{\frac{\pi}{2}}^{\phi} \cos\phi \, d\phi \, dz$

$\rightarrow \Phi = 0.03 \cdot 0.8m \cdot t \cdot [\sin\phi - \sin\frac{\pi}{2}] = 24 \cdot t \cdot \sin\phi - 24t$

$\therefore V_{emf} = 24 - 24 \sin(100\pi \cdot t) - 24 \cdot 100\pi \cdot t \cdot \cos(100\pi t)$

$\because V_{emf} = \oint_L (\bar{u}\times\bar{B})\, d\bar{l} - \frac{d}{dt}\int_S \bar{B}\, d\bar{s} = \frac{-d\Phi}{dt}$

$\wedge\ \Phi = \int_S \bar{B}\, d\bar{s} \quad \wedge \quad \bar{B} = 0.02t\, \bar{a}_x = 0.02t\cos\phi\, \bar{a}_\rho - 0.02t\sin\phi\, \bar{a}_\phi$

$\therefore \Phi = -\int_0^{0.03}\int_0^{0.04} 0.02t\sin\phi \cdot d\rho\, dz = -24\mu\, t\sin\phi$

$\because \phi = 100\pi t + \frac{\pi}{2} \rightarrow \Phi = -24\mu\cdot t\cdot\cos(100\pi t)$

$\rightarrow V_{emf} = 24\mu\cdot\frac{d}{dt}[t\cos(100\pi t)] = 24\mu\cdot[\cos(100\pi\cdot t) - 100\pi\cdot t\cdot\sin(100\pi\cdot t)]$

$\therefore$ at $t = 1ms$, $V_{emf} = 20.495\ \mu V$

$\wedge$ at $t = 3ms$, $V_{emf} = -4.193\ mV \rightarrow I = -41.93\ mA$

example 9.3: (recall machines):

$\because \tilde{F} = Ni \quad \wedge \quad \hat{R} = \frac{l}{\mu A} \rightarrow \Phi = \frac{Ni\cdot\mu A}{l}$

$\wedge\ V_2 = -N_2\cdot\frac{d\Phi}{dt} \rightarrow V_2 = -N_2\cdot\frac{N_1\cdot\mu\cdot A}{l}\cdot\frac{di}{dt}$

$\therefore V_2 = -0.02\cdot 3\cdot 100\pi\cdot\sin(100\pi t) = -6\pi\cos(100\pi t)$

9.4: displacement current: 22/2/2021

$\because \nabla\times\bar{H} = \bar{J} + \bar{J}_d \quad \wedge \quad \nabla\cdot(\nabla\times\bar{H}) = 0$

$\rightarrow \nabla\cdot\bar{J}_d = -\nabla\cdot\bar{J}$

$\because \nabla\cdot\bar{J} = \frac{\partial\rho_v}{\partial t} \quad \wedge \quad \rho_v = \nabla\cdot\bar{D}$

$\therefore \nabla\cdot\bar{J}_d = \nabla\cdot\frac{\partial\bar{D}}{\partial t} \qquad \therefore \boxed{\bar{J}_d = \frac{\partial\bar{D}}{\partial t}}$

$\wedge\ \boxed{\nabla\times\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t}}$ where $\bar{J}_d$ is the displacement current density

example 9.4:

$\because D = \varepsilon E \quad \wedge \quad E = \frac{V}{d} \rightarrow D = 2\varepsilon_0\cdot\frac{50\sin(10^3 t)}{3\times10^{-3}}$

$\rightarrow J_d = \frac{\partial D}{\partial t} = \frac{100\varepsilon_0}{3\times10^{-3}}\cdot\frac{\partial\sin(10^3 t)}{\partial t}$ $\quad 10^3\cos(10^3 t)$

$\rightarrow J_d = \frac{100}{3}\cdot\varepsilon_0\cdot\cos(10^3 t)\ A/m^2$

$\therefore I = J_d\cdot A = \frac{100}{3}\cdot\frac{10^{-9}}{36\pi}\cdot\cos(10^3 t)\cdot 5\times10^{-4}$

$\rightarrow I = 147.4\cdot\cos(10^3 t)\ fA$

practice exercise 9.4:

a) $\because D = \varepsilon E \rightarrow D = \frac{10^{-9}}{36\pi}\cdot 20\cos(\omega t - 50x)\,\bar{a}_y$

$\therefore \bar{J}_d = \frac{\partial \bar{D}}{\partial t} = \frac{-10^{-9}}{36\pi} \cdot W \cdot 20 \sin(Wt - 50x)\, \bar{a}_y = -20W\epsilon_0 \sin(Wt - 50x)$ A/m^2

b) $\because \nabla \times \bar{E} = \frac{-\partial B}{\partial t} \rightarrow \bar{B} = -\int_t (\nabla \times \bar{E})\, dt)$

~~$\rightarrow \bar{B} = -\int_t \frac{\partial}{\partial x}(20\cos(wt - 50x))\, dt$~~

~~$\rightarrow \bar{B} = 1000\int_t \sin(wt - 50x)\, dt = \frac{1000}{W} \cdot \cos(Wt - 50x)\, \bar{a}_z$~~

~~$\nabla \times \bar{E} = \frac{\partial}{\partial x} \bar{E}\, \bar{a}_z - \frac{\partial}{\partial z} \bar{E}\, \bar{a}_x$~~

$\because \nabla \times \bar{H} = J_d \rightarrow \begin{vmatrix} \bar{a}_x & \bar{a}_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\bar{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\bar{a}_y$

$\therefore \left(\overset{0}{\cancel{\frac{\partial H_x}{\partial z}}} - \frac{\partial H_z}{\partial x}\right)\bar{a}_y = -20W\epsilon_0 \sin(Wt - 50x)\bar{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\bar{a}_z$

$\rightarrow H_z\, \bar{a}_y = \epsilon_0 20W \int \sin(Wt - 50x)\, dx\, \bar{a}_y$

$\rightarrow \bar{H} = 20W \cdot \epsilon_0 \cdot \frac{-1}{50} \cdot -\cos(Wt - 50x)\, \bar{a}_z$

$\rightarrow \bar{H} = 0.4 \cdot W \cdot \epsilon_0 \cdot \cos(Wt - 50x)\, \bar{a}_z$

c) $\because \nabla \times \bar{E} = \frac{-\partial B}{\partial t} = -\mu_0 \cdot \frac{\partial \bar{H}}{\partial t}$

$\nabla \times \bar{E} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix} = \left(-\frac{\partial E_y}{\partial z}\right)\bar{a}_x + \left(\frac{\partial E_y}{\partial x}\right)\bar{a}_z$

$\therefore \frac{\partial E_y}{\partial x} \cancel{\bar{a}_z} = -\mu_0 \frac{\partial H_z}{\partial t} \cancel{\bar{a}_z}$

$\rightarrow -1000 \cdot (-\sin(Wt - 50x)) = -\mu_0 \cdot 0.4 \cdot W^2 \cdot \epsilon_0 \cdot (-\sin(Wt + 50x))$

$\rightarrow 1000 \cdot \cancel{\sin(Wt - 50x)} = 0.4 \cdot W^2 \cdot \epsilon_0 \cdot \mu_0 \cdot \cancel{\sin(Wt - 50x)}$

$\rightarrow W^2 = \frac{2500}{\epsilon_0 \cdot \mu_0} \rightarrow W \cong 1.5 \times 10^{10}$ Rad s^{-1}

9.5 : maxwell's equations in final forms

maxwell's equations:

Gauss's law: $\nabla \cdot \bar{D} = \rho_V$ $\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho_V \, dV$

nonexistance of isolated magnetic charge: $\nabla \cdot \bar{B} = 0$ $\oint_S \bar{B} \cdot d\bar{s} = 0$

faraday's law: $\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}$ $\oint_L \bar{E} \cdot d\bar{l} = -\frac{\partial}{\partial t} \int_S \bar{B} \cdot d\bar{s}$

Ampere's circuit law: $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$ $\oint \bar{H} \cdot d\bar{l} = \int_S (\bar{J} + \frac{\partial \bar{D}}{\partial t}) \cdot d\bar{s}$

+ lorentz force equation: $\bar{F} = Q(\bar{E} + \bar{u} \times \bar{B})$

+ equation of continuity: $\nabla \cdot \bar{J} = -\frac{\partial \rho_V}{\partial t}$

9.6 : Time - Varying potentials 23/2/2021

$\because \bar{B} = \nabla \times \bar{A} \quad \wedge \quad \nabla \times \bar{E} = -\frac{\partial B}{\partial t}$

$\rightarrow \nabla \times \bar{E} = -\nabla \times \frac{\partial \bar{A}}{\partial t} \rightarrow \nabla \times (\bar{E} + \frac{\partial \bar{A}}{\partial t}) = 0$

$\because \bar{E} = -\nabla \cdot V \quad \wedge \quad \nabla \times (\nabla \cdot V) = 0$

$\therefore \nabla \times (\bar{E} + \frac{\partial \bar{A}}{\partial t}) = \nabla \times (\nabla \cdot V) \rightarrow \boxed{\bar{E} + \frac{\partial \bar{A}}{\partial t} = -\nabla \cdot V}$

$\because \bar{D} = \varepsilon \bar{E} \quad \wedge \quad \nabla \cdot \bar{D} = \rho_V$

$\rightarrow \nabla \cdot \bar{E} = \nabla(-\nabla \cdot V - \frac{\partial \bar{A}}{\partial t})$

$\therefore \nabla \cdot \bar{E} = \boxed{\frac{\rho_V}{\varepsilon} = -\nabla^2 \cdot V - \frac{\partial}{\partial t}(\nabla \cdot \bar{A})}$

lorenz condition for potentials

$\nabla \cdot \bar{A} = -\mu \varepsilon \frac{\partial V}{\partial t} \quad \therefore \boxed{\nabla^2 \bar{A} = \mu \varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} - \mu \bar{J}}$

$\wedge \quad \boxed{\nabla^2 \cdot V - \mu \varepsilon \frac{\partial^2 V}{\partial t^2} = \frac{-\rho_V}{\varepsilon}}$

Subject 9.7 : time - harmonic fields Date 26/2/2021 No.

- scalar laplacian : $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

- vector laplacian : $\nabla^2 \bar{A} = \nabla^2 A_x \, \bar{a}_x + \nabla^2 A_y \, \bar{a}_y + \nabla^2 A_z \, \bar{a}_z$

- hence the vector laplacian is the sum of the scalar laplacians of each component of a vector: $\nabla^2 A_x \bar{a}_x = \left[\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \right] \bar{a}_x$

& a Time - harmonic field is one that varies periodically or sinusoidally with time.

- phasor review : $z = x + jy = r \angle \phi$

$\rightarrow z = r e^{j\phi} = r(\cos\phi + j\sin\phi)$

$\wedge \; r = \sqrt{x^2 + y^2} \quad \wedge \; \phi = \tan^{-1}\left(\frac{y}{x}\right)$

- z, x, y, r, and ϕ are not coordinate variables.

example 9.5 :

a) $\frac{3j + 4j^2}{(-1+6j)(4+4j+j^2)} = \frac{3j-4}{(-1+6j)(3+4j)} = \frac{3j-4}{-3+14j+24j^2} = \frac{-4+3j}{-27+14j}$

$-27 + 14j = 5\sqrt{37} \angle -0.47835 \quad \wedge \quad -4+3j = 5 \angle -0.643501$

$\rightarrow \frac{1}{\sqrt{37}} \angle 0.165151 = 0.16216 + 0.02703 j$

b) $1+j = \sqrt{2} \angle \frac{\pi}{4} \quad \wedge \quad 4-8j = 4\sqrt{5} \angle -1.10715$

$\rightarrow \frac{1+j}{4-8j} = \frac{\sqrt{10}}{20} \angle 1.89255 \rightarrow \left[\frac{1+j}{4-8j}\right]^{1/2} = 0.397635 \angle 0.946275$

$= 0.2325 + 0.32258 j$

example 9.6 :

$\because \bar{A} = 10\cos(10^8 t - 10x + 60^\circ)\, \bar{a}_z \quad \wedge \quad \bar{A} = Re\left(A_0 e^{-\gamma R x} \bar{a}_z e^{j\omega t}\right)$

$\rightarrow \bar{A} = Re\left\{10 e^{j(10^8 t - 10x + 60^\circ)}\right\} = Re\left\{10 e^{j10^8 t} e^{j(-10x + 60^\circ)}\right\} \bar{a}_z$

$\therefore \bar{A}_s = 10 e^{j(60^\circ - 10x)} \bar{a}_z$

- instantaneous form : $I(t) = Re\left(I_s e^{j\omega t}\right)$

$\rightarrow B(t) = Re\left\{B_s \cdot e^{j\omega t}\right\} \rightarrow B(t) = Re\left\{(-20j\, \bar{a}_x + 10 e^{2\pi j x/3} \bar{a}_y) e^{j\omega t}\right\}$

$\rightarrow B(t) = 10\cos\left(\omega t + 2\pi \cdot \frac{x}{3}\right) \bar{a}_y$

- instantaneous form: $\because -20j = 20e^{-j\frac{\pi}{2}}$

$\rightarrow B_s = 20e^{-j\frac{\pi}{2}}\,\bar{a}_x + 10e^{j2\pi x/3}\,\bar{a}_y$

$\rightarrow B(t) = Re\{(20e^{-j\frac{\pi}{2}}\,\bar{a}_x + 10e^{j2\pi x/3}\,\bar{a}_y)\,e^{j\omega t}\}$

$\rightarrow B(t) = Re\{20e^{j(\omega t-\frac{\pi}{2})}\,\bar{a}_x + 10e^{j(2\pi x/3+\omega t)}\,\bar{a}_y\}$

$\rightarrow B(t) = 20\cos(\omega t-\frac{\pi}{2})\,\bar{a}_x + 10\cos(\omega t+\frac{2\pi x}{3})\,\bar{a}_y$

$\therefore B(t) = 20\sin(\omega t)\,\bar{a}_x + 10\cos(\omega t+\frac{2\pi x}{3})\,\bar{a}_y$

Practice exercise 9.6:

$\because P = Re\{P_s e^{j\omega t}\,a_y\} \rightarrow 2\cos(10t + x - \frac{\pi}{4} - \frac{\pi}{2})\,\bar{a}_y$

$\rightarrow P = Re\{2e^{j(10t+x-\frac{3}{4}\pi)}\,\bar{a}_y\} \rightarrow P_s = 2e^{j(x-\frac{3}{4}\pi)}\,\bar{a}_y$

$\because Q(t) = Re\{Q_s e^{j\omega t}\} \wedge Q_s = \sin(\pi y)e^{jx}\,\bar{a}_x - \sin(\pi y)e^{jx}\,\bar{a}_z$

$\rightarrow Q(t) = Re\{\sin(\pi y)\,e^{j(x+\omega t)}\,\bar{a}_x - \sin(\pi y)\,e^{j(x+\omega t)}\,\bar{a}_z\}$

$\rightarrow Q(t) = \sin(\pi y)\cdot\cos(x+\omega t)\,\bar{a}_x - \sin(\pi y)\cos(x+\omega t)\,\bar{a}_z$

$\rightarrow Q(t) = (\bar{a}_x - \bar{a}_z)\cdot\sin(\pi y)\cdot\cos(x+\omega t)$

example 9.9:

$\because \bar{E} = Re\{E_s e^{j\omega t}\} \rightarrow \bar{E}_s = \frac{50}{\rho}e^{j\beta z}\,\bar{a}_\phi$

$\wedge\ \bar{H}_s = \frac{H_0}{\rho}e^{j\beta z}\,\bar{a}_\rho$

$\because \nabla\cdot\bar{D} = 0 \wedge \nabla\cdot B = 0 \rightarrow \varepsilon_0\nabla\cdot\bar{E} = 0 \wedge \mu_0\nabla\cdot\bar{H} = 0$

$\therefore \nabla\cdot\bar{E}_s = 0 \wedge \nabla\cdot\bar{H}_s = 0$

$\rightarrow \frac{1}{\rho}\cdot\frac{\partial E_\phi}{\partial \phi} = \nabla E_s = \frac{50}{\rho^2}\cdot E_s = 0$ X

$\because \nabla\times\bar{E} = -\frac{\partial B}{\partial t} \wedge \nabla\times\bar{H} = J + \frac{\partial D}{\partial t}$

$\rightarrow \nabla\times\bar{E}_s = -\mu_0\frac{\partial \bar{H}_s}{\partial t} \wedge \nabla\times\bar{H}_s = \cancel{\bar{E}_s}^{0} + \varepsilon_0\frac{\partial \bar{E}_s}{\partial t}$

$\therefore \nabla\times\bar{E}_s = -\mu_0\cdot j\omega\,\bar{H}_s \wedge \nabla\times\bar{H}_s = \varepsilon_0\cdot j\omega\cdot\bar{E}_s$

$\wedge\ \nabla\times\bar{E}_s = \frac{1}{\rho}\left[\frac{\partial(\rho E_{s\phi})}{\partial\rho}\right]\bar{a}_z = 0$ X

$\wedge\ \nabla\times\bar{H}_s = \boxed{\frac{\partial A_\rho}{\partial z}\,a_\phi} \rightarrow \nabla\times\bar{H}_s = \frac{H_0}{\rho}\cdot j\beta\,e^{j\beta z}\cdot a_\phi$

$\because \nabla\times\bar{H}_s = \varepsilon_0\cdot \cancel{j}\omega\cdot\frac{50}{\cancel{\rho}}\cdot\cancel{e^{j\beta z}}\,\cancel{a_\phi} = \frac{H_0}{\cancel{\rho}}\cdot \cancel{j}\beta\cdot\cancel{e^{j\beta z}}\,\cancel{a_\phi}$

$\therefore \boxed{\varepsilon_0\cdot\omega\cdot 50 = H_0\cdot\beta}$

∧ $\nabla \times \bar{E}_s = \cancel{\frac{1}{\rho}} \cdot \frac{\partial E_{s\phi}}{\partial z} \bar{a}_\rho = \frac{-50}{\rho^2} \cdot j\beta \cdot e^{j\beta z} \bar{a}_\rho$

$\because \nabla \times \bar{E}_s = -\mu_0 \cdot j\omega \cdot \bar{H}_s$

$\rightarrow \frac{-50}{\cancel{\rho^2}} \cdot \cancel{j}\beta \cdot \cancel{e^{j\beta z}} \, \cancel{\bar{a}_\rho} = -\mu_0 \cdot j\omega \cdot \frac{H_0}{\rho} \cdot e^{j\beta z} \bar{a}_\rho$

$\rightarrow \frac{-50}{\cancel{\rho}} \cdot \beta = -\mu_0 \cdot \omega \cdot H_0 \rightarrow 50\beta = \mu_0 \rho \cdot \omega \cdot H_0$

$\because \beta = \frac{\varepsilon_0 \cdot \omega \cdot 50}{H_0} \rightarrow \frac{50\, \varepsilon_0 \cdot \omega \cdot 50}{H_0} = \mu_0 \cdot \rho \cdot \omega \cdot H_0$

$\rightarrow H_0^2 = 50^2 \cdot \frac{\varepsilon_0}{\mu_0} \rightarrow H_0 = 50 \cdot 2.65258 \times 10^{-3} = \boxed{0.1326}$

∧ $\beta =$

Practice exercise 9.7:

$\because E = \mathrm{Re}\{E_s \cdot e^{j\omega t}\} \rightarrow E_s = \frac{\sin\theta}{r} \cdot e^{-j\beta r} a_\phi$

∧ $\nabla \times \bar{E}_s = -\mu_0 \frac{\partial \bar{H}_s}{\partial t} = -\mu_0 \cdot j\omega \bar{H}_s$

∧ $\nabla \times \bar{E}_s = \frac{1}{r\sin\theta}\left[\frac{\partial (A_\phi \sin\theta)}{\partial \theta}\right] \bar{a}_r + \frac{1}{r}\left[\frac{\partial (r \cdot A_\phi)}{\partial r}\right] \bar{a}_\theta$

$\rightarrow \nabla \times \bar{E}_s = \frac{2}{r\sin\theta} \cdot \frac{\sin\theta \cdot \cos\theta}{r} \cdot e^{-j\beta r} \bar{a}_r + \frac{\sin\theta}{r} \cdot -j\beta \cdot e^{-j\beta r} \bar{a}_\theta$

$\rightarrow \nabla \times \bar{E}_s = \frac{2\cos\theta}{r^2} \cdot e^{-j\beta r} \bar{a}_r + \frac{\sin\theta}{r} \cdot -j\beta e^{-j\beta r} \bar{a}_\theta$

$\therefore \bar{H}_s = [\nabla \times \bar{E}_s] / (-\mu_0 j\omega)$

$\rightarrow \bar{H}_s = \frac{2j\cos\theta}{\mu_0 \cdot \omega r^2} \cdot e^{-j\beta r} \bar{a}_r + \frac{\sin\theta}{r\mu_0 \cdot \omega} \cdot \beta \cdot e^{-j\beta r} \bar{a}_\theta$

$\because \bar{H} = \mathrm{Re}\{H_s e^{j\omega t}\} \rightarrow$

$\because H_{sr} = \frac{2j\cos\theta}{r^2 \mu_0 \omega} \cdot e^{-j\beta r} = \frac{2\cos\theta}{\mu_0 \omega r} \cdot e^{-j\beta r} \cdot e^{-j\frac{\pi}{2}}$

$\rightarrow \bar{H} = \frac{2\cos\theta}{\mu_0 \cdot \omega r^2} \cdot \sin(\omega t - \beta r) \bar{a}_r + \frac{\sin\theta}{\mu_0 \omega} \cdot \beta \cdot \cos(\omega t - \beta r) \bar{a}_\theta$

$\because \beta = \frac{\omega}{c} = 0.2$ rad/m

$\rightarrow \bar{H} = \frac{1}{12\pi r^2} \cdot \cos\theta \cdot \sin(6\times10^7 t - 0.2r) \bar{a}_r$

$+ \frac{1}{120\pi r} \cdot \sin\theta \cdot \cos(6\times10^7 t - 0.2r) \bar{a}_\theta$

practice exercise 8.7 (Redone): $\boxed{\beta^2 = \omega^2 \cdot \mu \cdot \varepsilon}$

$\because \nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t} = -\mu_0 \cdot \frac{\partial \bar{H}}{\partial t}$

$\rightarrow \nabla \times \bar{E}_s = -\mu_0 \cdot \frac{\partial \bar{H}_s}{\partial t} = -\mu_0 \cdot j\omega \cdot \bar{H}_s$

$\because \bar{E} = \text{Re}\{\bar{E}_s \cdot e^{j\omega t}\}$ & $\bar{E}_s = \frac{\sin\theta}{r} \cdot e^{-j\beta r} \bar{a}_\theta$

$\rightarrow \nabla \times \bar{E}_s = \frac{1}{r\sin\theta}\left[\frac{\partial(A_\phi \sin\theta)}{\partial\theta}\right] a_r - \frac{1}{r}\cdot\left[\frac{\partial(r A_\phi)}{\partial r}\right] a_\theta$

$\rightarrow \nabla\times\bar{E}_s = \frac{2\cos\theta}{r^2}\cdot e^{-j\beta r}\,\bar{a}_r + \frac{\sin\theta}{r} j\beta e^{-j\beta r}\,\bar{a}_\theta$

$\rightarrow \nabla\times\bar{E}_s = \frac{2\cos\theta}{r^2} e^{-j\beta r}\,\bar{a}_r + \frac{\sin\theta}{r}\beta e^{j(\frac{\pi}{2}-\beta r)}\,\bar{a}_\theta$

$\therefore \frac{-2\cos\theta}{r^2\cdot j\omega\cdot\mu_0} e^{-j\beta r}\,\bar{a}_r + \frac{-\sin\theta}{r\cdot j\omega\cdot\mu_0}\beta e^{j(\frac{\pi}{2}-\beta r)}\,\bar{a}_\theta = \bar{H}_s$

$\rightarrow \bar{H}_s = -\frac{\cos\theta}{12\pi r^2}\cdot e^{-j(\beta r+\frac{\pi}{2})}\,\bar{a}_r - \frac{\sin\theta}{r\cdot 24\pi}\cdot\beta e^{j(\frac{\pi}{2}-\beta r-\frac{\pi}{2})}\,\bar{a}_\theta$

$\therefore \bar{H} = \frac{-\cos\theta}{12\pi r^2}\cos(6\times10^7 t - \beta r - \frac{\pi}{2})\,\bar{a}_r - \frac{\sin\theta}{r\cdot 24\pi}\cdot\beta\cdot\cos(6\times10^7 t - \beta r)\,\bar{a}_\theta$

$\because \nabla\times\bar{H}_s = \frac{\partial\bar{D}_s}{\partial t} = \varepsilon_0 j\omega\cdot\bar{E}_s$

$\nabla\times\bar{H}_s = \frac{1}{r}\left[\frac{\partial(r H_{s\theta})}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\bar{a}_\phi$

$\rightarrow \nabla\times\bar{H}_s = \left(\frac{-1}{r}\cdot\frac{\sin\theta}{24\pi}\cdot\beta\cdot(-\beta j)\cdot e^{-\beta j r} - \frac{\sin\theta}{12\pi r^2} e^{-j(\beta r+\frac{\pi}{2})}\right.$

$\rightarrow \left[\frac{\sin\theta}{24\pi r}\cdot\beta^2\cdot\cancel{j}\cdot\cancel{e^{-j\beta r}} - \frac{\sin\theta}{12\pi r^2} e^{-j(\beta r+\frac{\pi}{2})}\right] a_\phi = \varepsilon_0 j\omega\cdot\frac{\cancel{\sin\theta}}{\cancel{r}}$

$\rightarrow \frac{1}{24\pi}\cdot\beta^2 - \frac{1}{12\pi r^2 j}\cdot e^{-j\frac{\pi}{2}} = \varepsilon_0\omega$

$\rightarrow \frac{1}{24\pi}\beta^2 - \frac{1}{12\pi r^2}\left[\cos(\pi) + \cancelto{0}{j\sin(\pi)}\right] = \varepsilon_0\omega$

$\rightarrow \boxed{\frac{\beta^2}{24\pi} + \frac{1}{12\pi r^2} = \varepsilon_0\omega}$

$\because \nabla\cdot\bar{H}_s = 0$ $\nabla\cdot\bar{H}_s = +\frac{\cos\theta}{12\pi r^2}\cdot e^{-j\frac{\pi}{2}}\cdot j\beta e^{-j\beta r} - \frac{2\beta e^{-j\beta r}}{r^2\cdot 24\pi}\cdot\cos\theta$

Take $\frac{1}{12\pi r} = 0 \rightarrow \frac{\beta^2}{24\pi} = \varepsilon_0\omega \rightarrow \beta^2 = 0.04 \rightarrow \boxed{\beta = 0.2\,\text{rad}}$

$\therefore \boxed{\bar{H} = \frac{-\cos\theta}{12\pi r^2}\cdot\sin(6\times10^7 t - 0.2r)\,\bar{a}_r - \frac{\sin\theta}{120\pi r}\cos(6\times10^7 t - \beta r)\,\bar{a}_\theta}$

example 9.8:

∵ $\bar{E} = \text{Re}\{\bar{E}_s \cdot e^{jwt}\} \rightarrow \bar{E}_s = 20 e^{-j(\beta z + \frac{\pi}{2})} \bar{a}_y$

must satisfy maxwell's four equations:

$\nabla \cdot \bar{D} = 0 \quad \nabla \cdot \bar{B} = 0 \quad \nabla \times \bar{E} = -\frac{\partial B}{\partial t} \wedge \nabla \times \bar{H} = \bar{J} + \frac{\partial D}{\partial t}$

∵ $\nabla \times \bar{E}_s = -\mu_0 \cdot \frac{\partial \bar{H}_s}{\partial t}$

∧ $\nabla \times \bar{E}_s = -20 e^{-j\frac{\pi}{2}} \cdot (-\beta) \cdot e^{-j\beta z} \bar{a}_x = -\mu_0 j w \bar{H}_s$

$\rightarrow \bar{H}_s = \frac{-1}{\mu_0 w} \cdot \frac{1}{j^2} \cdot 20 \beta \cdot e^{-j\beta z} \bar{a}_x = \frac{20 \beta e^{-j\beta z}}{\mu_0 w} \bar{a}_x$

∵ $\nabla \times \bar{H}_s = 4\varepsilon_0 \cdot j \cdot w \cdot \bar{E}_s$

∧ $\nabla \times \bar{H}_s = \frac{-20\beta}{\mu_0 w} j\beta e^{-j\beta z} \bar{a}_y = 4\varepsilon_0 j w \, 20 e^{-j\beta z} e^{-j\frac{\pi}{2}}$

$\rightarrow \beta^2 = 4\varepsilon_0 \mu_0 w^2 \cancelto{1}{e^{-j\frac{\pi}{2}}} \quad \therefore \beta = 2w\sqrt{\varepsilon_0 \mu_0} = \frac{2}{3} \text{ rad}$

∴ $\bar{H}_s = \frac{40 e^{-j\beta z}}{3\mu_0 w} \bar{a}_x \rightarrow \bar{H} = \frac{40}{3\mu_0 w} \cdot \sin(10^8 t - \beta z) \bar{a}_x$

$\rightarrow \bar{H} = \frac{1}{3\pi} \sin(10^8 t - \frac{2z}{3}) \bar{a}_x$

∵ $\boxed{\bar{H} = \text{Im}\{\bar{H}_s \cdot e^{jwt}\}}$

practice exercise 9.8:

∵ $\bar{H} = \text{Im}\{\bar{H}_s \cdot e^{jwt}\} \rightarrow \bar{H}_s = 2 e^{j(\frac{\pi}{2} - 3y)} \bar{a}_z$

∵ $\nabla \times \bar{H}_s = \bar{J} + \frac{\partial \bar{D}}{\partial t}$ ∧ $\sigma = 0 \rightarrow \nabla \times \bar{H}_s = 5\varepsilon_0 \cdot jw \cdot \bar{E}_s$

∧ $\nabla \times \bar{H}_s = 2 e^{j\frac{\pi}{2}} \cdot (-3) \cdot e^{-3jy} \bar{a}_x = -6j e^{-3jy} = 5\varepsilon_0 jw \bar{E}_s$

$\rightarrow \bar{E}_s = \frac{-6}{5\varepsilon_0 \cdot w} \cdot e^{-j3y} \bar{a}_x$

∵ $\nabla \times \bar{E}_s = -\frac{\partial \bar{B}}{\partial t} = -2\mu_0 \cdot jw \cdot \bar{H}_s$

∧ $\nabla \times \bar{E}_s = -\left[\frac{-6}{5\varepsilon_0 w} \cdot (-j3) \cdot e^{-j3y}\right] \bar{a}_z = \frac{-18j}{5\varepsilon_0 w} e^{-j3y} \bar{a}_z$

∴ $\frac{9}{5} e^{-j3y} = \mu_0 \cdot \varepsilon_0 \cdot w^2 \cdot \bar{H}_s$

$\rightarrow w^2 = \frac{9 e^{-j3y}}{5\mu_0 \varepsilon_0} \cdot \frac{1}{2} e^{j(3y - \frac{\pi}{2})} \rightarrow w^2 = \frac{9}{10\mu_0 \varepsilon_0} \cancelto{1}{e^{-j\frac{\pi}{2}}}$

∴ $\boxed{w = 2.846 \times 10^8 \text{ rad/s}}$

∧ $\bar{E} = \frac{-6}{5\varepsilon_0 w} \cdot \cos(wt - 3y) \bar{a}_x = -476.86 \cos(2.846 \times 10^8 t - 3y) \bar{a}_x$

Subject Chapter 10: electromagnetic wave propagation Date 2/3/2021 No.

- waves are means of transporting energy or information.

+ different media:

	σ	ε	μ	condition
free space:	0	ε_0	μ_0	N/A
lossless dielectrics:	$\simeq 0$	$\varepsilon_r\varepsilon_0$	$\mu_r\mu_0$	$\sigma << \omega\varepsilon$
lossy dielectrics:	$\neq 0$	$\varepsilon_r\varepsilon_0$	$\mu_r\mu_0$	N/A
good conductors:	$\simeq \infty$	ε_0	$\mu_r\mu_0$	$\sigma >> \omega\varepsilon$

- waves are functions of both space and time.

× wave equations:

$$\because \nabla \times \bar{E} = -\mu \frac{\partial H}{\partial t} \quad \wedge \quad \nabla \times \bar{H} = \sigma \bar{E} + \varepsilon \frac{\partial E}{\partial t}$$

$$\rightarrow \nabla \times \nabla \times \bar{E} = -\mu \frac{\partial}{\partial t}\left[\nabla \times \bar{H}\right] = -\mu\sigma \frac{\partial \bar{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\because \nabla \times \nabla \times \bar{A} = \nabla \nabla \cdot \bar{A} - \nabla^2 \cdot A$$

$$\therefore \nabla \cdot \nabla \cdot \bar{A} - \nabla^2 \cdot \bar{E} = -\mu\sigma \frac{\partial \bar{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t}$$

- assuming a source-free simple medium $\rightarrow \rho_v = 0 \rightarrow \nabla \cdot \bar{D} = 0$

$$\therefore \nabla \cdot \nabla \cdot \bar{E} = 0 \rightarrow \boxed{\nabla^2 \bar{E} = \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}}$$

If E is converted to phasor form: $\nabla^2 \bar{E}_s = \mu\sigma \frac{\partial \bar{E}_s}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{E}_s}{\partial t^2}$

$$\because \frac{\partial \bar{E}_s}{\partial t} = j\omega \bar{E}_s \rightarrow \nabla^2 E_s = \mu\sigma j\omega \bar{E}_s + j^2 \mu\varepsilon\omega^2 \bar{E}_s$$

$$\text{If } \boxed{\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\rightarrow \boxed{\nabla^2 \cdot \bar{E}_s = \gamma^2 \bar{E}_s}$$

similarly: $\boxed{\nabla^2 \cdot \bar{H}_s = \gamma^2 \cdot H_s}$

- in free-space ∧ $\bar{E} = E_x \bar{a}_x$ ∧ dependent on z only

$\rightarrow \nabla^2 \cdot \bar{E} = \frac{\delta^2 E_x}{\delta z^2} = \mu \epsilon \frac{\delta^2 E_x}{\delta t^2}$

∵ $1/\sqrt{\mu\epsilon} = u$: velocity of wave → $\frac{\delta^2 E_x}{\delta z^2} \cdot u^2 - \frac{\delta^2 E_x}{\delta t^2} = 0$

- a uniform plane wave lies in a single plane and is constant over it (same direction, magnitude, and phase over the ∞ plane)
- if phasors are taken: $\frac{\delta^2 E_x}{\delta z^2} - \frac{1}{u^2} \cdot \frac{\delta^2 E_x}{\delta t^2} = 0 \rightarrow \frac{\delta^2 \bar{E}_s}{\delta z^2} - j^2 \cdot \omega^2 \cdot \frac{1}{u^2} \cdot \bar{E}_s = 0$

$\rightarrow \frac{\delta^2 E_s}{\delta z^2} + \frac{\omega^2}{u^2} \cdot \bar{E}_s$, ∵ $\beta^2 = \omega^2 \cdot \epsilon \cdot \mu$

$\therefore \frac{\delta^2 \bar{E}_s}{\delta z^2} + \beta^2 \cdot \bar{E}_s = 0$

- in sinusoidal form:

∵ $E = E_s e^{j\omega t}$ ∧ $\beta = \pm\sqrt{\beta^2} \rightarrow E = E^+ + E^-$

∧ $E^+ = E_0 \cdot e^{j(\omega t - \beta z)}$ ∧ $E^- = E_0 \cdot e^{j(\omega t + \beta z)}$ | E_0 : amplitude

$\therefore E_x^+(z,t) = \text{Re}\{E_x^+(z) e^{j\omega t}\} = \boxed{|E_0| \cos(\omega t - \beta z)}$

where β : phase constant (rad/m) ∧ ω : frequency (angular), $\omega = 2\pi f$

∧ $E_x^-(z,t) = |E_0| \cos(\omega t + \beta z)$

- if time is fixed, E_x^+ can be plotted against z which gives the wavelength, λ, as: $\lambda = u \cdot T = \frac{u}{f} = \frac{2\pi u}{\omega} = \boxed{\frac{2\pi}{\beta}}$
- if z is fixed, E_x^+ can be plotted against time

example 10.1:

b) ∵ $\beta^2 = \omega^2 \cdot \mu \cdot \epsilon \rightarrow \beta = \frac{1}{3}$ Rad/m

∵ $T = \frac{2\pi}{10^8} \rightarrow t$ for $T/2$ $= \pi \times 10^{-8}$ s ≈ 31.4 ns

practice exercise 10.1:

a) ∵ $k = \beta = \omega/u = \frac{2}{3}$ Rad/m

∵ $\lambda = \frac{2\pi}{\beta} \rightarrow \lambda = 3\pi$ m

∵ $T = \frac{2\pi}{2 \times 10^8} \approx 31.4$ ns

b) $t_1 = T/8 \approx 3.92$ ns

Subject Chapter 9 questions Date 5/3/2021 No.

Review questions:

9.1. ∵ $V_{emf} = \frac{-d\phi}{dt}\cdot N \rightarrow V_{emf} = N \cdot \frac{-\partial}{\partial t}[t^3-2t]m$

$\rightarrow V_{emf} = -100(3t^2-2)|_{t=1} m = -1V$ (b)

9.2: ~~(a)~~ (b) ∧ (d)

9.3: (a)

9.4: (c)

9.5: × (a)

9.7: ~~(a)~~ (a,c) $\frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) = \frac{1}{R^2}\cdot\cos(\theta)\cdot\cos(\omega t - R\omega\sqrt{\mu_0\epsilon_0}) \neq 0$

9.8: (b) × (c) ∧ (d)

9.9: ~~(a)~~ (b)

9.10: (d)

Problems: 11, 14, 15, 19, 20, 22, 27, 28, 31

11. ∵ Static magnetic field with moving conductor ∧ $B = 4.3\times10^{-5}$ ~~m~~Wb/m²

$V_{emf} = uBl\cdot\cancel{\sin\theta} = 120\frac{km}{hr}\cdot\frac{1000}{3600}\cdot 4.3\times10^{-5}\times 1.6m \cdot \cancel{\sin 65}$

$\rightarrow V_{emf} = \cancel{2.08}\, mV \quad uBl\cos\theta = 0.99\, mV$

14: $V_{emf} = \int_S \nabla\times(\bar{u}\times\bar{B})\, d\bar{S}$, $\bar{u} = \rho\cdot 60$ rad/s $\bar{a}_\phi$, $B = 15$ mWb/m $\bar{a}_z$

$\rightarrow \bar{u}\times\bar{B} = \rho\cdot 60\times 15m\ \bar{a}_\rho$

$\therefore V_{emf} = \int_{\rho_1}^{\rho_2}\rho\cdot 0.9\, d\rho = \frac{9}{20}\cdot[\rho^2]_{0.02}^{0.1} = 4.32 mV$

$\boxed{u = \rho\omega}$

15. ∵ $\bar{J}_d = \frac{\partial \bar{D}}{\partial t}$ ∧ $\bar{D} = \epsilon\bar{E}$ ∧ $E = \frac{V}{d}$ $\therefore \bar{D} = \epsilon\frac{V}{d}$

in phasors: $\bar{J}_d = j\omega D_s$ ∧ $D_s = \epsilon\frac{V_s}{d}$

$\rightarrow J_{d\,max} = \left|j\cdot(2\pi\cdot 10M)\cdot\frac{10^{-9}}{36\pi}\cdot\frac{50}{0.2m}\right| = 2.77.77\ A/m^2$

$I_{max} = J_d\cdot S = 0.0277778\ A$

19. $\frac{J_d}{J} = 1$ $J_d = \frac{\partial D}{\partial t}$ $J_d = j\omega D_s$ $D_s = 3\epsilon_0\bar{E}_s$

∧ $J = \sigma\cdot E_s \rightarrow \frac{3j\omega\epsilon_0}{\sigma} = 1 \rightarrow \omega = 3.77\, M$ Rad/s

$\rightarrow f = 0.6\, MHz = 600\, kHz$

Subject   Date   No.

Sixth edition problems:

11. $\because\ \bar{J} = \sigma \bar{E} \quad \wedge \quad \bar{J}_d = \frac{\partial \bar{D}}{\partial t}$

in phasors: $\bar{J} = \sigma \bar{E}_s \ \wedge \ \bar{J}_d = j\omega \bar{D}_s = j\omega\varepsilon \bar{E}_s$

a) $\frac{\bar{J}}{\bar{J}_d} = 1/\frac{\omega\varepsilon}{\sigma} = \frac{2\times10^{-3}}{81\varepsilon_0 \cdot 2\pi \cdot \times10^{9}} = 0.4444 \text{ m}$

b) $\frac{\bar{J}}{\bar{J}_d} = \frac{25}{4.5} = 5.5555$

c) $\frac{2\times10^{-4}}{5/18} = 0.72 \text{ m}$

14. $\frac{J}{J_d} = 10 \quad \bar{J} = \sigma \bar{E}_s \wedge \bar{J}_d = j\omega\varepsilon \bar{E}_s \rightarrow \frac{\sigma}{\omega\varepsilon} = 10$

$\frac{20}{81\varepsilon_0 \cdot 2\pi \cdot f} = 10 \rightarrow f = 444.44 \text{ MHz}$

$\omega = \frac{\sigma}{10\varepsilon} = 2\pi f \rightarrow f = \frac{\sigma}{20\pi\varepsilon} = \frac{1}{10^{-9}/36} = 36 \text{ GHz}$ if $\varepsilon = \varepsilon_0$

if $\varepsilon = 81\varepsilon_0 \rightarrow f = 444.44 \text{ MHz}$

15. $\because\ J = \sigma E \rightarrow E = \frac{0.2\sin(10^{9}t)}{2.5\times10^{6}}$

$\wedge\ J_d = \frac{\partial \bar{D}}{\partial t} = \varepsilon \frac{\partial E}{\partial t} \rightarrow J_d = 4.244\times10^{-9}\cos(10^{9}t)$

$\rightarrow J_{d(max)} = 4.2 \text{ nA/m}^2$

19. $\because\ \rho_v = \nabla \cdot \bar{D} \ \wedge\ \bar{J} = \sigma \bar{E} \ \wedge\ \bar{D} = \varepsilon \bar{E}$

$\rightarrow \bar{D} = \frac{\varepsilon}{\sigma}\bar{J} \rightarrow \rho_v = \frac{\varepsilon}{\sigma} \cdot \sin(10^{4}t) \cdot 3z^2\ \bar{a}_z$

~~$\because\ \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \ \wedge\ \nabla \cdot \bar{B} = 0 \ \wedge\ \nabla \times \bar{H} = J + \frac{\partial \bar{D}}{\partial t}$~~

~~$\therefore\ \nabla \times H = (2y\overset{\sin(10^4t)}{\uparrow} + 10^{4} \cdot 2y \cdot \frac{\varepsilon}{\sigma}\overset{\cos(10^4t)}{\uparrow})\bar{a}_x + (xz\overset{\sin(10^4t)}{\uparrow} + 10^{4} \cdot xz \cdot \frac{\varepsilon}{\sigma}\overset{\cos(10^4t)}{\uparrow})a_y + (z^3\overset{\sin}{\uparrow} + 10^{4}z^3 \cdot \frac{\varepsilon}{\sigma}\overset{\cos(10^4t)}{\uparrow})$~~

~~$\rightarrow (\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}) = 2y\sin(10^{4}t) + 2y \cdot 10^{4} \cdot \frac{\varepsilon}{\sigma} \cdot \cos(10^{4}t)$~~

~~$\rightarrow (\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}) = xz\sin(10^{4}t) + xz \cdot 10^{4} \cdot \frac{\varepsilon}{\sigma} \cdot \cos(10^{4}t)$~~

~~$\rightarrow \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = z^3\sin(10^{4}t) + 10^{4}z^3 \cdot \frac{\varepsilon}{\sigma}\cos(10^{4}t)$~~

$\because\ \nabla \times \bar{H} = J + \frac{\partial D}{\partial t}$

$\rightarrow \nabla \cdot (\nabla \times \bar{H}) = 0 = \nabla \cdot J + \frac{\partial \rho_v}{\partial t}$

$\rightarrow -\frac{\partial \rho_v}{\partial t} = \nabla \cdot \bar{J} \rightarrow -\frac{\partial \rho_v}{\partial t} = 3z^2\sin(10^{4}t)$

$\rightarrow \rho_v = 3\times10^{-4} \cdot z^2 \cdot \cos(10^{4}t) = 0.3 \cdot z^2 \cos(10^{4}t) \text{ mC/m}^3$

NOTEBOOK

9.20: $\because \bar{J}_d = \frac{\partial D}{\partial t} = j\omega D_s = j\omega E_s$

$\because \nabla \times \bar{H} = J + J_d \wedge \sigma = 0 \Rightarrow J = 0$

$\Rightarrow \nabla \times \bar{H} = \bar{J}_d = -\frac{\partial A_z}{\partial x} a_y = 2 \cdot 10 \cdot \cos(10^8 t - 2x)\bar{a}_y$

$\because \frac{\partial \bar{D}}{\partial t} = \bar{J}_d = 20\cos(10^8 t - 2x)\bar{a}_y$

$\Rightarrow \bar{D} = 20 \cdot 10^{-8} \sin(10^8 t - 2x)\bar{a}_y$

$\Rightarrow \bar{E} = \frac{20}{\varepsilon_r \varepsilon_0} \cdot 10^{-8} \sin(10^8 t - 2x)\bar{a}_y$

$\because \nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t} = -\mu_0 \cdot 10^9 \cos(10^8 t - 2x)\bar{a}_z$

$\Rightarrow \frac{\partial E_z}{\partial x} = -\frac{40}{\varepsilon_r \varepsilon_0} \cdot 10^{-8} \cos(10^8 t - 2x)\bar{a}_z = -\mu_0 10^{9+7} \cos(10^8 t - 2x)\bar{a}_z$

$\Rightarrow -4 = \varepsilon_r \varepsilon_0 - \mu_0 \cdot 10^{16} \Rightarrow 4 = \varepsilon_r \cdot \frac{10^{-9}}{36\cancel{\pi}} \cdot 4\cancel{\pi} \cdot 10^{-7} \cdot 10^{16}$

$\Rightarrow 4 = 4\varepsilon_r / 36 \Rightarrow \boxed{\varepsilon_r = 36}$

$\therefore \bar{E} = \frac{20}{36\varepsilon_0} \cdot 10^{-8} \sin(10^8 t - 2x)\bar{a}_y = 628.32 \sin(10^8 t - 2x)\bar{a}_y$

9.22: $\nabla \cdot \bar{D} = \rho_v \mid \nabla \cdot \bar{B} = 0 \mid \nabla \times \bar{E} = -\frac{\partial B}{\partial t} \mid \nabla \times \bar{H} = \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t}$

$\because \nabla \times \bar{E} = -\frac{\partial E_x}{\partial y}\bar{a}_z = -10\pi \cdot -\sin(\omega t + \pi y)\bar{a}_z$

$\Rightarrow -\frac{\partial B}{\partial t} = 10\pi \sin(\omega t + \pi y)\bar{a}_z \wedge B = \mu_0 H$

$\Rightarrow -\mu_0 \cdot \frac{\partial \bar{H}}{\partial t} = -\mu_0 \cdot -\frac{10}{\eta}\omega \sin(\omega t + \pi y)\bar{a}_z$

$\therefore \pi = \frac{\mu_0 \omega}{\eta}$ — ①

$\because \nabla \times \bar{H} = \frac{\partial H_z}{\partial y}\bar{a}_x = \frac{-10\pi}{\eta}\sin(\omega t + \pi y)\bar{a}_z = \sigma \bar{E} + \varepsilon \frac{\partial E}{\partial t}$

$\because \nabla \cdot D = \rho_v = 0 \Rightarrow \sigma = 0 \Rightarrow \nabla \times \bar{H} = \varepsilon \frac{\partial E}{\partial t}$

$\because \varepsilon \frac{\partial E}{\partial t} = -9\varepsilon_0 \cdot \omega \cdot 10 \cdot \sin(\omega t + \pi y)\bar{a}_x$

$\Rightarrow \boxed{\frac{\pi}{\eta} = 9\varepsilon_0 \omega}$ — ②

from ①: $\omega = \frac{\pi \eta}{\mu_0}$, and in ②: $\frac{\pi}{\eta} = 9\varepsilon_0 \frac{\pi \eta}{\mu_0}$

$\Rightarrow \mu_0 = 9\varepsilon_0 \eta^2 \Rightarrow \eta = 40\pi \approx 125.7\ \Omega$

$\wedge\ \omega = 100\pi$ Mrad/s $\approx 3.142 \times 10^8$

9.24: a) $\because E = \mathrm{Re}\{E_s e^{j\omega t}\} \wedge E = 4\cos(\omega t - 3x - 10^\circ) a_y - \cos(\omega t + 3x - 20^\circ)\bar{a}_z$

$\Rightarrow \bar{E}_s = 4e^{-j3x} e^{-j10^\circ}\bar{a}_y - e^{j(3x - 20^\circ)}\bar{a}_z$

b) $\bar{H}_s = \frac{\sin\theta}{r} e^{-j5r}\bar{a}_\theta$

c) $\bar{J}_s = 6e^{-3x} \cdot e^{-j(2x - 90^\circ)}\bar{a}_y + 10e^{-x} e^{-j5x}\bar{a}_z$

$\Rightarrow \bar{J}_s = 6j e^{-x(3+2j)}\bar{a}_y + 10e^{-x(1+5j)}\bar{a}_z$

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$3+4j = 5\angle 53.13^{\circ} = 5e^{j53.13^{\circ}}$

$-(3+4e^{j90})xe^{j\omega t}\,\bar{a}_y = -3x\cos(\omega t) - 4x\cos(\omega t+90^{\circ})$

$-x(3\cos(\omega t)+4\cos(\omega t+90^{\circ})$

9.31: a) $\bar{A} = -5x\cos(\omega t+53.13)\bar{a}_y + 5\cos(\omega t+90)\bar{a}_z$

b) $B = 10\cos(\omega t-\beta z)\bar{a}_z + 5\cos(\omega t+\beta z+3\pi/4)\bar{a}_y$

$= 10\cos(\omega t-\beta z)\bar{a}_z - 5\sin(\omega t+\beta z+\frac{\pi}{4})\bar{a}_y$

7th edition:

9.20: ∵ $\bar{J}_c = \sigma\bar{E}$, $\bar{J}_d = \frac{\partial D}{\partial t} = \varepsilon\frac{\partial\bar{E}}{\partial t}$

→ $E = 4000\cos(2\pi\times10^8 t)$ → $\bar{J}_d = -4.5\varepsilon_0\cdot2\pi\cdot10^8\cdot4000\sin(2\pi\times10^8 t)$ A/

→ $\bar{J}_d = -100\sin(2\pi\times10^8 t)$ A/m²

9.22: ∵ $\bar{J}_d = \frac{\partial D}{\partial t} = \varepsilon_0\frac{\partial E}{\partial t} = \varepsilon_0\cdot10^3\cdot25\cdot\cos(10^3 t)\bar{a}_z$ A/m²

→ $J_d = 0.221049\times10^{-6}\times\cos(10^3 t)\bar{a}_z$ A/m²

→ $I = \bar{J}_d\cdot\bar{S} = 2.21049\times10^{-6}\times\cos(10^3 t)$

9.29: ∵ $\nabla\cdot\bar{D} = \rho_v$ → $\varepsilon\nabla\cdot E = \rho_v$, $\nabla\times E =$ X

continuity equation: $\nabla\cdot\bar{J} = -\frac{\partial\rho_v}{\partial t}$

∵ $\nabla\times\bar{H} = \bar{J} + \frac{\partial D}{\partial t}$

→ $\nabla\cdot(\nabla\times\bar{H}) = \nabla\cdot\bar{J} + \nabla\frac{\partial D}{\partial t} = 0$

→ $\boxed{\nabla\cdot\bar{J} = -\frac{\partial\rho_v}{\partial t}}$

→ $3z^2\bar{a}_z\sin(10^4 t) = -\frac{\partial\rho_v}{\partial t}$

→ $3z^2\cdot10^{-4}\cdot\cos(10^4 t)\cdot\bar{a}_z = \bar{\rho}_v$

→ $\rho_v = 0.3z^2\cos(10^4 t)\bar{a}_z$ mC/m³

9.19: ∵ free space → $\sigma = 0$ → $\nabla\times\bar{H} = \frac{\partial D}{\partial t}$ $\beta^2 = \omega^2\cdot\mu\cdot\varepsilon$

→ $\frac{\partial H_y}{\partial x}\bar{a}_z = \varepsilon_0\frac{\partial\bar{E}}{\partial t}$

$10\beta\cos(10^8 t+\beta x)\bar{a}_z = \varepsilon_0\frac{\partial\bar{E}}{\partial t}$ → $\beta = 0.3333$

→ $\bar{E} = \frac{10\beta}{\varepsilon_0}\cdot10^{-8}\cdot\sin(10^8 t+\beta x)\bar{a}_z$

∵ $\nabla\times\bar{E} = -\frac{\partial B}{\partial t} = -\mu_0\cdot\frac{\partial\bar{H}}{\partial t} = -\mu_0\cdot10^8\cdot10\cdot\cos(10^8 t+\beta x)\bar{a}$

$-\frac{\partial E_z}{\partial x}\bar{a}_y = \frac{-10\beta}{\varepsilon_0}\cdot10^{-8}\cdot\beta\cdot\cos(10^8 t+\beta x)\bar{a}_y = -\mu_0\cdot10^8\cdot10\cos(10^8 t+\beta x)$

→ $\frac{\beta^2}{\varepsilon_0} = \mu_0\cdot10^{16}$ → $\beta = 1/3$ Rad/m

→ $\bar{E} = 1200\pi\sin(10^8 t+\beta x)\bar{a}_z$

$9.31:\ \because \nabla \times E = \frac{-\partial B}{\partial t} \rightarrow \bar{H} = \frac{-1}{\mu_0}\int (\nabla \times E)\, dt$

$\wedge\ \nabla \times \bar{E} = \frac{-1}{r\sin\theta}\left[\cancelto{0}{\frac{\partial A_\theta}{\partial \phi}}\right]\bar{a}_r + \frac{1}{r}\left[\frac{\partial (rA_\theta)}{\partial r}\right]\bar{a}_\phi$

$\rightarrow \nabla \times \bar{E} = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(10\sin\theta\cos(wt-\beta r)\right)\right]\bar{a}_\phi$

$\rightarrow \nabla \times \bar{E} = \frac{10}{r}\cdot\sin\theta\frac{\partial}{\partial r}\left[\cos(wt-\beta r)\right]\bar{a}_\phi$

$\rightarrow \nabla \times \bar{E} = \frac{10}{r}\sin\theta\cdot\beta\cdot\sin(wt-\beta r)\,\bar{a}_\phi$

$\rightarrow \bar{H} = \frac{-1}{\mu_0}\cdot\frac{10}{r}\cdot\sin\theta\cdot\beta\cdot\int\sin(wt-\beta r)\bar{a}_\phi\, dt$

$\rightarrow \bar{H} = \frac{-10\beta}{\mu_0\cdot r}\cdot\sin\theta\cdot(-\cos(wt-\beta r)\,a_\phi\cdot\frac{1}{w}$

$\rightarrow \bar{H} = \frac{10\beta}{\mu_0\cdot r\cdot w}\cdot\sin\theta\cdot\cos(wt-\beta r)\ a_\phi$ A/m

Sample homework

Q1: a) $\because \nabla\cdot D = \rho_v = 0 \wedge \nabla\cdot B = \mu\nabla\cdot\bar{H} = 0$

$\rightarrow \mu[k + 10 - 26] = 0 \rightarrow \boxed{k = 16}$

b) $\because 20y - kt = 0 \wedge y + 2\times10^6 t = 0$

$\wedge\ \nabla\times E = -\mu\frac{\partial H}{\partial t}$ X

$\rightarrow -\frac{\partial E_x}{\partial y}\bar{a}_y = -\mu\cdot2\times10^6\cdot\bar{a}_z$ X

$\because \nabla\times\bar{H} = \cancelto{0}{\sigma\bar{E}} + \varepsilon\frac{\partial\bar{E}}{\partial t} = -\varepsilon\cdot k\ \bar{a}_x$

$\rightarrow \frac{\partial H_z}{\partial y}\bar{a}_x = -\varepsilon k\ \bar{a}_x = 1 \therefore \boxed{k = -\frac{1}{\varepsilon}}$

Q2: $\because \nabla\times\bar{E} = -\frac{\partial B}{\partial t} = -\mu_0\frac{\partial H}{\partial t}$

$\wedge\ \nabla\times\bar{H} = \varepsilon_0\cdot\frac{\partial\bar{E}}{\partial t}$

$\rightarrow \nabla\times\bar{E} = \frac{\partial E_x}{\partial z}\bar{a}_y - \frac{\partial E_x}{\partial y}\bar{a}_z$

$= C\cdot a\cos(az)\cdot\sin(12y)\cdot\cos(2\times10^{10}t)\,\bar{a}_y$

$- C\cdot12\cdot\cos(12y)\cdot\sin(az)\cdot\cos(2\times10^{10})\,\bar{a}_z$

$\rightarrow \bar{H} = \frac{-C}{\mu_0}\cdot\sin(2\times10^{10}t)\left[a\cos(az)\sin(12y)\bar{a}_y - 12\cos(12y)\sin(az)\bar{a}\right.$

$\because \mu_0\nabla\cdot\bar{H} = 0 \Rightarrow a\cdot12\cos(az)\cos(12y) - a\cdot12 = 0$ ✓

$\because\ \bar{H} = 0.5\times10^{-10}\cdot\frac{-C}{\mu_0}\cdot\sin(2\times10^{10}t)\left[a\cos(az)\sin(12y)\,\bar{a}_y - 12\cos(12y)\sin(az)\,\bar{a}_z\right]$

$\wedge\ \nabla\times\bar{H} = \varepsilon_0\cdot\frac{\partial E}{\partial t} = -\varepsilon_0\cdot C\cdot\sin(12y)\cdot\sin(az)\cdot2\times10^{10}\cdot\sin(2\times10^{10}t)\,\bar{a}_x$

$\rightarrow \nabla\times\bar{H} = \frac{-C}{\mu_0}\cdot0.5\cdot10^{-10}\cdot\sin(2\times10^{10}t)\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]\bar{a}_x$

$\rightarrow \nabla\times\bar{H} = 0.5\cdot10^{-10}\cdot\frac{-C}{\mu_0}\cdot\sin(2\times10^{10}t)\cdot\left[12\sin(12y)\cdot12\sin(az) + a^2\sin(az)\sin(12y)\right]\bar{a}_x$

$\rightarrow 0.5\cdot10^{-10}\cdot\frac{-C}{\mu_0}\cdot\sin(2\times10^{10}t)\cdot\left[(144+a^2)\sin(12y)\cdot\sin(az)\right]\bar{a}_x$

$= -\varepsilon_0\cdot C\cdot\sin(12y)\cdot\sin(az)\cdot2\times10^{10}\cdot\sin(2\times10^{10}t)\,\bar{a}_x$

$\therefore\ 0.25\cdot10^{-20} = \varepsilon_0\cdot\mu_0/(144+a^2) \rightarrow a \approx 65.58$

Q3: a) $\because\ J_c = \sigma\bar{E} \rightarrow J_c = \frac{10}{\rho}\cos(10^5t)\,\bar{a}_\rho$

$I_c = \int_s J_c\,dS = \int_{z_1}^{z_2}\int_{\phi_1}^{\phi_2} J_c\,\rho\,d\phi\,dz\,\bar{a}_\rho$

$\rightarrow I_c = \int_0^{40cm}\int_0^{2\pi}\frac{10}{\rho}\cos(10^5t)\,\bar{a}_\rho\cdot\rho\,d\phi\,dz\,\bar{a}_\rho$

$\rightarrow I_c = 10\cos(10^5t)\cdot2\pi\cdot0.4 = 8\pi\cos(10^5t)\ A$

b) $\because\ J_d = \varepsilon\frac{\partial\bar{E}}{\partial t} = \frac{-10^{-5}}{\rho}\sin(10^5t)\cdot10^5 = \frac{-\sin(10^5t)}{\rho}\,\bar{a}_\rho$

$\rightarrow I_d = -\int_0^{0.4}\int_0^{2\pi}\frac{1}{\rho}\cdot\sin(10^5t)\cdot\rho\,d\phi\,dz = -0.8\pi\sin(10^5t)\ A$

c) $\frac{|I_d|}{|I_c|} = \frac{1}{10}$ $\frac{\omega\varepsilon}{\sigma}$

Q4: a) phase constant $= \beta \wedge \beta = \omega\sqrt{\varepsilon\mu}$ $\omega = 2\pi\cdot9.394\,G$

$\wedge\ \varepsilon = \varepsilon_r\cdot\varepsilon_0 \wedge \mu = \mu_0 \rightarrow \beta = \omega\cdot\frac{\sqrt{\varepsilon_r}}{c} = 294.18\ rad/m$

b) $\lambda = \frac{2\pi}{\beta} \wedge \lambda = \frac{u}{f} \wedge u = \frac{c}{\sqrt{\varepsilon_r\mu_r}}$

$\rightarrow \lambda = 0.02129\ m$

c) $u = c/\sqrt{\varepsilon_r\mu_r} = c/\sqrt{2.26} = 1.9956\times10^8\ m/s$

d) intrinsic impedance : $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_r\cdot\varepsilon_0}} = 250.79\,\Omega$

e) $H_0 = E_0/\eta = \frac{500}{250.79} = 1.994\ A/m$

Subject 10.3: Wave propagation in lossy dielectrics Date 6/3/2021 No.

* a lossy dielectric is one in which an EM wave loses power as it propagates (imperfect dielectric), $\sigma \neq 0$

* Vector wave equations (homogeneous vector helmholtz's equations):

- for a charge free imperfect dielectric:

$$\rightarrow \quad \boxed{\nabla^2 \bar{E}_s - \gamma^2 \cdot \bar{E}_s = 0}$$

& where $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

$$\rightarrow \quad \boxed{\nabla^2 \bar{H}_s - \gamma^2 \cdot \bar{H}_s = 0}$$

- γ : the propagation constant of the medium (in 1/m)

∵ γ is complex and can be written as $\alpha + j\beta$

& $-\mathrm{Re}\{\gamma^2\} = \beta^2 - \alpha^2 = \omega^2\mu\varepsilon$

& $|\gamma^2| = \alpha^2 + \beta^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2}$

$$\rightarrow \alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

&

$$\rightarrow \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

Quiz practice

Q1: 1. $\because \varepsilon \nabla \cdot E = \rho_v = 0 \wedge \mu \nabla \cdot \bar{H} = 0 \rightarrow k + 10 - 25 = 0 \rightarrow \boxed{k = 15}$

2. $\because \nabla \times \bar{E} = -\frac{\partial E_x}{\partial y} \bar{a}_z = \times$

$\because \nabla \times \bar{H} = \varepsilon \frac{\partial \bar{E}}{\partial t} = -\varepsilon k \bar{a}_x$

$\wedge \nabla \times \bar{H} = \frac{\partial H_z}{\partial y} \bar{a}_x = 1 \rightarrow \boxed{k = \frac{-1}{\varepsilon}}$

Q2: $\because$ lossless $\wedge$ source free $\rightarrow \sigma = 0 \wedge \rho_v = 0$

1. $\bar{E} = Re\{\bar{E}_s e^{j\omega t}\} = 1000 e^{-j\beta x} \bar{a}_z$

$\wedge \bar{H} = Re\{\bar{H}_s e^{j\omega t}\} = \frac{-1000}{\eta} e^{-j\beta x} \bar{a}_y$

2. $\because \nabla \times \bar{H}_s = \overset{\to 0}{\sigma \bar{E}_s} + \varepsilon \frac{\partial E_s}{\partial t} \rightarrow \nabla \times \bar{H}_s = j\omega\varepsilon \bar{E}_s$

$\wedge \nabla \times \bar{H}_s = \frac{\partial H_{sy}}{\partial x} \bar{a}_z = \frac{j\beta \cdot 1000}{\eta} e^{-j\beta x} \bar{a}_z$

$\rightarrow \frac{\cancel{j}\beta \cdot \cancel{1000}}{\eta} \cancel{e^{-j\beta x}} \cancel{\bar{a}_z} = \cancel{j}\omega\varepsilon \cdot \cancel{1000} \cancel{e^{-j\beta x}} \cancel{\bar{a}_z}$

$\rightarrow \boxed{\beta = \omega\varepsilon \cdot \eta}$

3. $\because \nabla \times \bar{E}_s = \frac{-\partial \bar{B}_s}{\partial t} = -\mu j\omega \bar{H}_s$

$\wedge \nabla \times \bar{E}_s = -\frac{\partial E_{sz}}{\partial x} \bar{a}_y = -1000 \cdot -j\beta \cdot e^{-j\beta x} \bar{a}_y$

$\rightarrow \cancel{1000} \cancel{j}\beta \cancel{e^{-j\beta x}} \cancel{\bar{a}_y} = \frac{-\cancel{1000}}{\eta} \cancel{e^{-j\beta x}} \cancel{\bar{a}_y} \cdot \cancel{j}\omega\mu$

$\rightarrow \boxed{\eta\beta = \omega\mu}$ $\boxed{\beta = \frac{\omega\mu}{\eta}}$

4. $\because \beta = \frac{\omega\mu}{\eta} = \omega\varepsilon \cdot \eta \rightarrow \eta^2 = \frac{\mu}{\varepsilon}$

$\rightarrow \eta = 60\pi \approx 188.5$

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Quiz 1: chapter 9 130806

أنا محمد سند هيثم الطاهر أشهد أني قرأت و فهمت تعليمات هذا الأمتحان القصير و تقيدت بها والله على ما أقول شهيد.

$\because \bar{E}(t) = 4.9\cos(1.8\times10^9\pi t - \alpha x - 2.5\alpha z)\,\bar{a}_y$

1. $\because \bar{J}_c = \sigma\bar{E} \wedge \sigma = 0 \because$ free space

$\therefore J_c = 0$ A/m²

$\because \bar{J}_d = \frac{\partial \bar{D}}{\partial t} = \varepsilon_0\frac{\partial \bar{E}}{\partial t} = -4.9\cdot1.8\cdot10^9\pi\cdot\sin\cdot -$

$\therefore \bar{J}_d = -0.245\cdot\sin(1.8\times10^9\pi t - \alpha x - 2.5\alpha z)\,\bar{a}_y$

2. $\because \bar{E} = \mathrm{Re}\{\bar{E}_s e^{j\omega t}\}$

$\therefore \boxed{\bar{E}_s = 4.9e^{-j\alpha(x+2.5z)}\,\bar{a}_y}$ V/m

3. $\because \beta = \omega/c = 6\pi \wedge \nabla^2\bar{E}_s + \beta^2\bar{E}_s = 0$

$\nabla^2\cdot E_{sy} + \beta^2\cdot E_{sy} = 0$

$\rightarrow \frac{\partial^2 E_{sy}}{\partial x^2} + \cancelto{0}{\frac{\partial^2 E_{sy}}{\partial y^2}} + \frac{\partial^2 E_{sz}}{\partial z^2} + 36\pi^2\cdot E_{sy} = 0$

$\rightarrow -\alpha^2\cdot4.9\cdot\cancel{\cos(\cdots)} + -(2.5)^2\cdot4.9\cdot\cancel{\cos(\cdots)} + 36\pi^2 E_{sy} = 0$

$\rightarrow -\alpha^2\cdot4.9 - 11.25 + 36\pi^2\cdot4.9 = 0$

$\rightarrow \alpha^2 = 352.9588$

$\rightarrow \boxed{\alpha = 18.7872 \text{ Rad/m}}$

- assuming $\bar{E}$ has only an x component and propagates in the positive z direction

$\rightarrow \bar{E}_s = \bar{a}_x E^+_{xs} \rightarrow \frac{\partial^2 E^+_{sx}}{\partial z^2} - \gamma^2 E^+_{xs} = 0$

$\therefore E^+_{xs}(z) = E^+_0 e^{-\gamma z}$ (after solving ODE)

$, \gamma = \alpha + j\beta \rightarrow E^+_{xs}(z) = E_0 e^{-\alpha z} e^{-j\beta z}$

$\therefore \bar{E}(z,t) = Re\{E^+_0 \cdot e^{-\alpha x} \cdot e^{-j\beta z} \cdot e^{j\omega t} \cdot \bar{a}_x\}$

$\rightarrow \boxed{\bar{E}(z,t) = |E_0| \cdot e^{-\alpha x} \cdot \cos(\omega t - \beta z) \cdot \bar{a}_x}$

where α is the attenuation coefficient & β is the phase constant

- α measures the spatial rate of decay of a wave in a medium
- β is a measure of phase shift per unit length (often called the wave number)

$\boxed{\lambda = \frac{2\pi}{\beta}} \rightarrow$ always

- for $\eta = \frac{E_0}{H_0}$, η is called the intrinsic impedance

$\boxed{\eta = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}}$

- similarly, $\bar{H}(z,t) = Re(H_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \bar{a}_y)$

- $|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$, $\tan(2\theta_\eta) = \frac{\sigma}{\omega\varepsilon}$, $0° \leq \theta_\eta \leq 45°$

$\underset{\text{lossless}}{\uparrow} \sigma = 0$ $\underset{\text{perfect conductor}}{\sigma = \infty} \uparrow$

$\therefore \bar{H} = \frac{E_0}{|\eta|} \cdot e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta) \bar{a}_y$

- hence, $\bar{H}$ always lags $\bar{E}$ by an angle θ_η where $0° \leq \theta_\eta \leq 45°$

- α can be measured in nepers per meter (Np/m) or decibels per meter (dB/m)
- generally, $\bar{H} = \frac{1}{\eta}(\bar{a}_n \times \bar{E})$ where $\bar{a}_n$ is the direction of propagation

$\rightarrow \bar{E} \times \bar{H}$ points in the direction of propagation ($\bar{a}_n$)

$\rightarrow \bar{E} \cdot \bar{H} = 0$, $\bar{E} \cdot \bar{a}_n = 0$, $\bar{H} \cdot \bar{a}_n = 0$ for TEM waves

- $\because \eta \cdot \gamma = j\omega\mu \rightarrow \theta_\eta + \theta_\gamma = 90°$ always

$\rightarrow \boxed{45° \leq \theta_\gamma \leq 90°}$

$\because \bar{H} = \frac{1}{\eta}(\bar{a}_n \times \bar{E}) \therefore \bar{E} = -\eta(\bar{a}_n \times \bar{H})$

- both conduction and displacement currents exist in conductors
  $\rightarrow \frac{|J_{cs}|}{|J_{ds}|} = \frac{|\sigma \bar{E}_s|}{|j\omega\varepsilon \bar{E}_s|} = \frac{\sigma}{\omega\varepsilon} = \tan\theta$, θ: loss angle $= 2\theta_\eta$
  - lossless dielectric (perfect dielectric) if $\tan\theta$ is very small $(\sigma << \omega\varepsilon)$
  - good conductor → $\tan\theta$ is very large. $(\sigma >> \omega\varepsilon)$
- $\nabla \times \bar{H}_s = (\sigma + j\omega\varepsilon)\bar{E}_s = j\omega\varepsilon\left[1 - \frac{j\sigma}{\omega\varepsilon}\right]\bar{E}_s$
  if $\varepsilon_c = \varepsilon\left[1 - \frac{j\sigma}{\omega\varepsilon}\right]$, where ε_c is the complex permittivity of the medium
  $\rightarrow \varepsilon_c = \varepsilon' - j\varepsilon''$, $\varepsilon' = \varepsilon$ and $\varepsilon'' = \varepsilon\tan\theta = \sigma/\omega$
  $\rightarrow \varepsilon''/\varepsilon' = \tan\theta$, loss tangent of the medium
- wave propagation in lossy media is the general case for wave propagation. equations for wave propagation in other media can be obtained from equations in lossy media.
- with $\varepsilon_c = \varepsilon\left[1 - \frac{j\sigma}{\omega\varepsilon}\right] \rightarrow \boxed{\gamma = j\omega\sqrt{\mu\varepsilon_c}} \wedge \boxed{\eta = \sqrt{\frac{\mu}{\varepsilon_c}}}$

example 10.2:

$\because \bar{H}_s = 10\, e^{-\alpha x} e^{-jx/2}\, \bar{a}_y \wedge H_0 = 10 \rightarrow E_0 = H_0 \cdot \eta = 2000\angle 30^\circ$

$\rightarrow \bar{E}_s = 2000\, e^{-\alpha x} e^{-jx/2} e^{j30^\circ} \cdot (-\bar{a}_k \times \bar{a}_y)$ $\quad \because E = -\eta(\bar{a}_k \times \bar{H})$

$\because x$ is the only variable given $\rightarrow \bar{a}_k = \bar{a}_x$

$\rightarrow \bar{E}_s = -2000\, e^{-\alpha x} e^{-jx/2} e^{j30^\circ} \cdot \bar{a}_z$

$\rightarrow \bar{E} = -2000\, e^{-\alpha x} \cos(\omega t - \frac{x}{2} + 30^\circ)$

$\because \beta = \frac{1}{2} \wedge \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$

$\wedge\ \alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$

$\rightarrow \frac{\alpha}{\beta} = 2\alpha = \left[\frac{\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}{\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1}\right]^{1/2}$ $\quad \because \tan(2\theta_\eta) = \frac{\sigma}{\omega\varepsilon}$

$\wedge\ \theta_\eta = 30^\circ \rightarrow \frac{\sigma}{\omega\varepsilon} = \sqrt{3}$

$\rightarrow 2\alpha = \left[\frac{\sqrt{4} - 1}{\sqrt{4} + 1} = \frac{1}{3}\right]^{1/2} \rightarrow \alpha = \frac{1}{2\sqrt{3}}$ Np/m ≈ 2.51 dB/m

practice exercise 10.2:

a) ∵ $\alpha = \frac{1}{3}$ $\wedge$ $\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$

$\rightarrow \frac{1}{3} = 10^8 \cdot \sqrt{\frac{16\varepsilon_0\mu_0}{2}\left[\sqrt{1+\left(\frac{\sigma}{10^8\cdot 8\varepsilon_0}\right)^2}-1\right]} \rightarrow \frac{81}{64} = 1+\left(\frac{\sigma}{10^8\cdot 8\varepsilon_0}\right)^2$

$\rightarrow \sigma \approx 3.64563 \times 10^{-3}$ S/m

$\rightarrow \beta = 1.3744$ rad/m

b) ∵ $\tan(\theta) = \frac{\sigma}{\omega\varepsilon} = 0.5154$

c) ∵ $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \Rightarrow |\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$ $\wedge$ $\theta_\eta = \frac{1}{2}\tan^{-1}(0.5154)$

$\rightarrow \eta = 177.97\angle 13.63° \ \Omega$

d) ∵ $u = \frac{\omega}{\beta} = \frac{10^8}{1.3744} = 7.276 \times 10^7$ m/s

e) ∵ $\bar{a}_k = \bar{a}_z$ $\wedge$ $\bar{a}_E = \bar{a}_x \rightarrow \bar{a}_H = \bar{a}_y$

$\wedge$ $H_0 = \frac{E_0}{|\eta|} \rightarrow \bar{H}_s = \frac{0.5}{177.97} e^{j13.63°} \cdot e^{-z/3} e^{-j\beta z} e^{-j90°}$

$\rightarrow \bar{H} = 2.813\, e^{-z/3} \sin(10^8 t - \beta z - 13.63°)\, \bar{a}_y$ mA/m

- in a lossy medium, $\frac{\omega}{\beta} \neq u$ (where $u = \frac{1}{\sqrt{\mu\varepsilon}}$)

lossy medium: $u_p = \omega/\beta$ $\wedge$ $\frac{u_p}{f} = \boxed{\frac{2\pi}{\beta} = \lambda}$ — generic

∵ $\theta_\gamma + \theta_\eta = 90°$ $\wedge$ $\eta \cdot \gamma = j\omega\mu$

$\rightarrow |\eta \cdot \gamma| = |j\omega\mu| = \omega \cdot \mu$

- when a dielectric is subjected to an applied static electric field, the centroids (center of mass of a geometric object of uniform density) of the positive and negative charges in atoms are displaced and form electric dipoles. The polarization vector ($\bar{P}$) is introduced to account for the formed dipoles.

If the applied field is made to alternate, the polarization vector is affected and becomes a function of frequency of the alternating field. hence, energy is lost in continuously flipping the dipoles, this is called polarization loss and it increases as the frequency increases.

This leads to complex permittivity : $\varepsilon_c = \varepsilon' - j\varepsilon''$

$\because \nabla \times \bar{H} = \sigma \bar{E} + \varepsilon_c \frac{\partial E}{\partial t} = \sigma \bar{E} + jw(\varepsilon' - j\varepsilon'')\bar{E}$

$\rightarrow \nabla \times \bar{H} = (\sigma + w\varepsilon'')\bar{E} + jw\varepsilon'\bar{E}$, & $\sigma_e = \sigma + w\varepsilon''$

$\rightarrow \nabla \times \bar{H} = jw(\varepsilon' - j\frac{\sigma_e}{w})\bar{E}$

where the effective conductivity : $\sigma_e = \sigma_s + \sigma_a$ (Static conductivity + alternating field conductivity)

The phenomenon that contributes to σ_a is called the dielectric hysteresis.

- the change in conductivity is responsible for heating of materials (using microwaves, for example).

* lossless dielectric : $\sigma << w\varepsilon$, $\sigma \simeq 0$, $\varepsilon = \varepsilon_r\varepsilon_o$, $\mu = \mu_o\mu_r$

- substitute into generalized equations.

$\because$

$$a = w\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \frac{\sigma}{w\varepsilon}} - 1\right]} \quad \Big| \quad b_r = w\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \frac{\sigma}{w\varepsilon}} + 1\right]}$$

$$\rightarrow \boxed{a = 0} \quad \& \quad \boxed{b_r = w\sqrt{\mu\varepsilon}}$$

$$\because \gamma = \sqrt{jw\mu(\sigma + jw\varepsilon)} \quad \therefore \quad \boxed{\gamma = jw\sqrt{\mu \cdot \varepsilon}}$$

$$\because \eta = \sqrt{\frac{jw\mu}{\sigma + jw\varepsilon}} \rightarrow \eta = \sqrt{\frac{\mu}{\varepsilon}} \rightarrow \boxed{\eta = \eta_o \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}}} \quad \& \quad \boxed{\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}}}$$

Subject Date $\beta = b$ No.

Where the free-space intrinsic impedance, $\boxed{\eta_0 = 120\pi}$ (in Ω)

& $u_p = u = 1/\sqrt{\mu\varepsilon} = c/\sqrt{\mu_r\varepsilon_r}$ | $\lambda = 2\pi/\beta = \frac{u}{f}$

& $\theta_\eta = 0 \rightarrow \bar{H}$ & $\bar{E}$ are in phase (in time domain).

* good dielectric : $\frac{\sigma}{\omega\varepsilon} << 1$

$\because \gamma = \sqrt{j\omega\mu(\sigma + \varepsilon\omega j)} \rightarrow \gamma = j\omega\sqrt{\mu\varepsilon} \cdot \left[1 - j\frac{\sigma}{\omega\varepsilon}\right]^{1/2}$

& $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$ binomial expansion

$\rightarrow \left[1 - j\frac{\sigma}{\omega\varepsilon}\right]^{1/2} \simeq \left[1 - \frac{1}{2}\cdot j \cdot \frac{\sigma}{\omega\varepsilon}\right]^1$

$\therefore \gamma = \alpha + j\beta \simeq j\omega\sqrt{\mu\varepsilon} \cdot \left[1 - j\frac{\sigma}{2\varepsilon\omega}\right]$

$\therefore \alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ & $\beta \simeq \omega\sqrt{\mu\varepsilon}$ & $\eta \simeq \sqrt{\mu/\varepsilon}$

* transverse electromagnetic wave (TEM) : EM waves with no electric or magnetic field components along the direction of propagation.

Subject 10.6: plane waves in good conductors: Date 14/3/2021 No.

good conductor: $\sigma >> \omega\varepsilon$, loss tangent $= \frac{\sigma}{\omega\varepsilon} >> 1$

$\rightarrow \sigma \simeq \infty$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0\mu_r$

$$\therefore \alpha = \beta = \sqrt{\frac{\omega \cdot \mu \cdot \sigma}{2}} = \boxed{\sqrt{\pi f \cdot \mu \cdot \sigma}}$$

$\because \gamma = j\omega\sqrt{\mu\varepsilon_c}$. $\wedge$ $\varepsilon_c = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right) \simeq -j\frac{\sigma}{\omega}$

$\rightarrow \gamma = \sqrt{j^2\omega^2 \cdot \mu \cdot \frac{\sigma}{j\omega}} = \sqrt{j\omega\mu\sigma}$, $\because j = e^{\pi/2} \rightarrow \sqrt{j} = e^{\pi/4}$

$$\therefore \boxed{\gamma = \sqrt{\omega\mu\sigma} \angle 45^\circ} = \sqrt{\omega\mu\sigma}\left(\frac{1+j}{\sqrt{2}}\right)$$

$\because \gamma = \alpha + j\beta \rightarrow \alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \wedge \beta(=\beta) = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \sigma \mu}$

$\wedge$ $\boxed{\lambda = \frac{2\pi}{\beta} = \frac{u_p}{f}}$ $\wedge$ $\boxed{u_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}}$

$\because \eta = \sqrt{\frac{j\omega\mu}{\sigma + \omega\varepsilon}} \simeq \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \cdot \angle 45^\circ$

$\rightarrow \theta_\eta = 45^\circ$, $\bar{E}$ & $\bar{H}$ are out of phase by 45° (max angle)

$\wedge$ $\theta_\eta + \theta_\gamma = 90^\circ = \theta$ (δ in prof. nihal's notes)

$\therefore \bar{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_x$ (for example)

$\rightarrow \bar{H} = (E_0/\eta) \cdot e^{-\alpha z} \cdot \cos(\omega t - \beta z - 45^\circ)\, \bar{a}_y$

* dispersive medium: a medium in which a signal is distorted due to wave velocity depending on frequency

$\because u_p = \sqrt{\frac{2\omega}{\mu\sigma}}$, a good conductor is dispersive since a wave's velocity depends on ω.

+ skin depth, δ (δ_c in prof. nihal's notes) is the distance through which the wave's amplitude attenuates by a factor of e^{-1} (decreases to 37% of its original value) exact

$\because e^{-\alpha \cdot z} = e^{-1} \rightarrow \alpha \cdot z = 1 \ , \ z = \boxed{\frac{1}{\alpha} = \delta}$ (in meters)

- the skin depth is a measure of the depth to which an EM wave can penetrate.
- the amplitude of E_o reduces to less than 1% its original in fivefold the skin depth $(5 \cdot \delta)$

+ for a good conductor:

$\because \alpha = \sqrt{\pi f \cdot \sigma \mu} \rightarrow \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

$\therefore$ the skin depth decreases as the frequency increases

& for a perfect conductor $(\sigma = \infty)$, the skin depth is zero and no penetration occurs.

$\because \eta = \sqrt{\frac{j \omega \mu}{\sigma}}$ & $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \rightarrow \boxed{\eta = \frac{\sqrt{2}}{\sigma \delta} \angle 45^\circ}$

$\rightarrow \boxed{\eta = \frac{1+j}{\sigma \delta}}$

& skin effect: the phenomenon in which charges move from the bulk of the material to its surface.
therefore, the frequency increase causes an increase in resistance of a material

-skin depth is useful in calculating the ac resistance

$\because R_{dc} = l / \sigma S$, l: length , σ: conductivity , S: surface area

& $R_{ac} = l / \sigma A$, A: effective area

$\because$ skin effect causes charges to only travel at around the surface of a material $\rightarrow$ for a cylindrical conductor, $A = \pi a^2 - \pi (a - \delta)^2$

$\rightarrow A \simeq 2 \pi a \delta \quad \therefore R_{ac} = l / (\sigma \cdot 2 \pi a \delta) \rightarrow \frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta}$ a = radius

$\therefore R_{ac} >> R_{dc}$ for high frequencies

a = radius

- the skin resistance is defined as : $R_s = R_e\{\eta\}$, $\eta = \sqrt{\frac{w\mu}{\sigma}} \cdot \left(\frac{1+j}{\sqrt{2}}\right)$ for good conductors
  → for good conductors: $R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = \boxed{1/(\sigma\delta)}$
- the skin resistance is the resistance for a plane conductor with unity length and width and thickness equal to the skin depth
- for a given length and width, the ac resistance is given by:

$$R_{ac} = R_s \cdot \frac{l}{w} = \frac{l}{\sigma\delta w} \quad , \quad w: \text{width}$$

hence for a cylindrical conductor with radius a → $w = 2\pi a$

$\therefore R_{ac} = l/(\sigma \cdot 2\pi \cdot a \cdot \delta)$ as shown before

- note for solving problems: always check the type of medium (by checking the loss tangent) as specific media equations greatly simplify the problems.

example 10.3:

- no attenuation factor → lossless medium

$\because \bar{H} = -0.1\cos(wt - z)\bar{a}_x + 0.5\sin(wt - z)\bar{a}_y$ A/m

→ direction of travel: +z

$\because \beta = 1$ & $\eta = 60\pi = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \cdot \varepsilon_r}} = \frac{120\pi}{\sqrt{\varepsilon_r}}$

→ $\varepsilon_r = 4$

$\because \beta = 1 \text{ rad/m} = w\sqrt{\mu\varepsilon}$ → $w = \frac{1 \cdot c}{2} = 1.5 \times 10^8$ rad/s

$\because \bar{E} = -\eta \cdot (\bar{a}_z \times \bar{H})$, $\bar{a}_z \times \bar{H} = -0.5\sin(wt - z)\bar{a}_x - 0.1\cos(wt - z)\bar{a}_y$

→ $\bar{E} = 30\pi \cdot \sin(wt - z)\bar{a}_x + 6\pi\cos(wt - z)\bar{a}_y$

practice exercise 10.3:

$\because \bar{E} = 50 \sin(10^8 t + 2z)\, \bar{a}_y$ V/m

a) wave propagates in $-\bar{a}_z$ direction

b) $\because$ lossless $\wedge$ $\lambda = \frac{2\pi}{\beta} \rightarrow \lambda = \pi = 3.142$ m

$\because \beta = w\sqrt{\mu\varepsilon} \rightarrow w = \frac{2}{\sqrt{\mu\varepsilon}} = 10^8 t \rightarrow f = 15.915$ MHz

& $\mu = \mu_0 \rightarrow \sqrt{\varepsilon_r} = \frac{2c}{w} \rightarrow \varepsilon_r = 36$

c) $\because \bar{H} = \frac{1}{\eta} \cdot (a_k \times \bar{E}) \rightarrow a_k \times \bar{E} = -\bar{a}_z \times \bar{E} = 50 \sin(10^8 t + 2z)\, \bar{a}_x$

$\wedge$ $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{6} = 20\pi \rightarrow \bar{H} = 0.796 \sin(10^8 t + 2z)\, \bar{a}_x$ A/m

example 10.4:

$\because \bar{E} = 2e^{-\alpha z} \cdot \sin(10^8 t - \beta z)\, \bar{a}_y$ V/m , $\varepsilon_r = 1$, $\mu_r = 20$, $\sigma = 3$ S/m

$\rightarrow$ loss tangent : $\frac{\sigma}{w\varepsilon} = 1080\pi \gg 1 \rightarrow$ good conductor

$\rightarrow \alpha = \beta = \sqrt{\pi f \mu \sigma}$ $\wedge$ $f = \frac{10^8}{2\pi}$

$\rightarrow \alpha = 61.399 = \beta$ (Np/m)

$\because \bar{H} = \frac{1}{\eta}(\bar{a}_k \times \bar{E})$ $\wedge$ $\bar{a}_k = +\bar{a}_z \rightarrow \bar{a}_z \times \bar{E} = -2e^{-\alpha z} \sin(10^8 t - \beta z)\, \bar{a}_x$

$\wedge$ $\eta = (1+j) \cdot \frac{61.4}{3} = \frac{\sqrt{2}}{3} \cdot 61.4 \angle 45^0$

$\rightarrow \bar{H} = -2e^{-\alpha z} \cdot \sin(10^8 t - \beta z - 45^0) \Big/ \frac{\sqrt{2}}{3} 61.4$

$\rightarrow \bar{H} = -69.1\, e^{-\alpha z} \sin(10^8 t - \beta z - 45^0)\, \overline{a_x}$ mA/m

practice exercise 10.4: $\sigma = 10^{-2}$ S/m , $\varepsilon_r = 4$, $\mu_r = 1$, $w = 10^9\pi$

$\because$ lossy medium $\rightarrow \alpha = 0.9416$ -Ⓐ Np m^{-1} $\beta = 20.965$ -Ⓑ rad m^{-1}

$\because \bar{E} = E_0 \cdot e^{-\alpha y} \cdot \cos(wt - \beta y + \theta)\, \bar{a}_z$

$\rightarrow \bar{E}(1\text{m}, 2\text{ns}) = 2.7889\, \bar{a}_z$ V/m ≈ 2.844

b) $\because \beta$ in rad/m $\wedge$ $\beta \cdot y =$ rad

$\rightarrow y = \frac{10^\circ \cdot \frac{\pi}{180^\circ}}{\beta} \approx 8.325$ mm

c) $\because$ Reduced by 40% $\rightarrow 0.6\, E_0 \rightarrow \ln(0.6) = -0.51082$

$\rightarrow e^{-0.5108} = e^{-\alpha y} \rightarrow y = \frac{-0.5108}{-\alpha} \approx 0.542$m

d) ∵ $\bar{H} = \frac{1}{\eta}[\bar{a}_y \times \bar{E}]$, $\bar{a}_y \times \bar{E} = 30 e^{-\alpha z} \cos(\ )\,\bar{a}_x$

& $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \approx \sqrt{35389.54 \angle 0.08975819}$

$\approx 188.116 \angle 0.04488$ rad

$\to \bar{H} = \frac{30}{188.116} e^{-\alpha y} \cdot \cos(10^9 \pi t - \beta y + \frac{\pi}{4} - 0.04488)$

at $\bar{H}(y=2m, t=2ns)$, $\bar{H} = -0.0228\,\bar{a}_x$ A/m

∴ $\bar{H} = -22.8$ mA/m

example 10.6 : 15/3/2021

∵ $\bar{J}_s = \sigma \bar{E}_s$ & $\nabla^2 \cdot \bar{E}_s - \gamma^2 \bar{E}_s = 0$

$\to \nabla^2 \bar{J}_s - \gamma^2 \bar{J}_s = 0$; ∵ direction of propagation $= +\bar{a}_z$

& $E_y = E_z = 0$ ∴ $\nabla^2 \bar{J}_s - \gamma^2 J_s = 0 \to \frac{\partial^2 J_{xs}}{\partial z^2} - \gamma^2 \bar{J}_{sx} = 0$

∵ $\frac{\partial^2 J_{sx}}{\partial z^2} - \gamma^2 J_{sx} = 0$ is an ODE

$m^2 - \gamma^2 = 0 \to m = \pm\gamma \to J_{sx} = c_1 e^{-\gamma z} + c_2 e^{+\gamma z}$

$c_2 = 0$ ∵ propagation in $+z$ direction

∵ good conductor $\to \alpha = \beta = \frac{1}{\delta} \to \gamma = \alpha + j\beta = \frac{1+j}{\delta}$

$\to J_{sx} = c_1 e^{-(\frac{1+j}{\delta} \cdot z)}$ & $c_1 = J_{sx}$ for $z = 0$

$\to J_{sx}(0)$: current density on the conductor surface.

practice exercise 10.5:

∵ $\bar{J} = J_{sx}(0)\, e^{-z(\frac{1+j}{\delta})}$ & $I = \int_s \bar{J} \cdot ds$

$\to I = \int_0^\infty \int_w J_{sx}(0) \cdot e^{-z(\frac{1+j}{\delta})} \cdot dy\, dz$

$\to I = J_{sx}(0) \cdot \frac{-\delta}{1+j} \cdot w \cdot \left[e^{-z\frac{1+j}{\delta}}\right]_0^\infty = \frac{J_{sx}(0) \cdot \delta \cdot w}{1+j}$

$\to I = \frac{J_{sx}(0) \cdot \delta \cdot w}{\sqrt{2}} \angle -45°$, $|I| = \frac{J_{sx}(0) \cdot \delta \cdot w}{\sqrt{2}}$

example 10.6:

$R_{dc} = l/\sigma S$ ∵ $S_i = \pi a^2$ λ $S_o = \pi(b+t)^2 - \pi b^2$

$\rightarrow S_i = 12.57\mu$ λ $S_o = 40.84\mu$ $\sigma_{copper} = 5.8\times10^7$ S/m

$R_{dc} = \frac{l}{\sigma S_i} + \frac{l}{\sigma S_o} = 2.943m + 0.844m = 3.587\, m\Omega$

∵ $R_{ac} = \frac{l}{\sigma \delta W}$ λ $\delta = \frac{1}{\sqrt{\pi f \sigma \mu}}$ at 100MHz, $\delta = 6.6085\mu$

$\rightarrow R_{ac\,i} = \frac{2}{\sigma\cdot\delta\cdot 2\pi a} = 0.415\,\Omega$

$\rightarrow R_{ac\,o} = \frac{2}{\sigma\delta\cdot 2\pi b} = 0.138$ } $R_{ac\,total} = 0.553\,\Omega$

∴ $\frac{R_{ac}}{R_{dc}} \approx 150$ → R_{ac} is about 150 times greater than R_{dc}

practice exercise 10.6: ∵ $\sigma_{aluminum} = 3.5\times10^7$

$W = 2\pi(2.6m) = \frac{26}{5}\pi$ mm

λ $R_{ac} = \frac{l}{\sigma \delta W}$ λ $R_{dc} = \frac{l}{\sigma S}$ $S = \pi\cdot 2.6^2\times10^{-6}$

$\rightarrow \frac{R_{ac}}{R_{dc}} = \frac{\sigma S}{\sigma \delta W} = \frac{\pi\cdot d^2}{4\cdot\pi d\cdot\delta} = \frac{d}{4\delta}$

∵ $\delta = \frac{1}{\alpha}$, for good conductor, $\alpha = \sqrt{\pi f \mu \sigma}$

$\rightarrow \delta = 26.902\mu$ at 10MHz λ 1.902μ at 2GHz

∴ $\frac{R_{ac}}{R_{dc}}$ at 10MHz = 24.16

λ $\frac{R_{ac}}{R_{dc}}$ at 2GHz = 341.7

examples from notes:

example: $\bar{E} = 2\cos(10^8 t - \frac{z}{\sqrt{3}})\bar{a}_x - \sin(10^8 t - \frac{z}{\sqrt{3}})\bar{a}_y$

∵ $\alpha = 0 \rightarrow$ lossless λ $\beta = 1/\sqrt{3}$, direction of propagation $= \bar{a}_z$

λ $\lambda = \frac{2\pi}{\beta} = 2\sqrt{3}\,\pi$, ∵ $\beta = W\sqrt{\mu\varepsilon}$, $\mu = \mu_o$ λ $\varepsilon = \varepsilon_r\varepsilon_o$

$\rightarrow \beta = [W/C]\cdot\sqrt{\varepsilon_r} \rightarrow \varepsilon_r = 3$

$\wedge\ \bar{H} = \frac{1}{\eta}(\bar{a}_z \times \bar{E})$, $\bar{a}_z \times \bar{E} = \sin(10^8t$ [illegible] $\bar{a}_x + 2\cos$ [illegible] $\bar{a}_y$

$\wedge\ \eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{3}} \rightarrow \bar{H} = \frac{120\pi}{\sqrt{3}}\sin(10^8t - \frac{z}{\sqrt{3}})\bar{a}_z + \frac{240\pi}{\sqrt{3}}\cos(10^8t - \frac{z}{\sqrt{3}})\bar{a}_y$

example : $\because$ $\bar{E} = 100e^{-\gamma z}$, $\varepsilon_r = 4$, $\sigma = 0.1$, $\mu = \mu_0$, $f = 2.45\,GHz$

check loss tangent : $\frac{\sigma}{\varepsilon_r\varepsilon_0 \cdot w} = \frac{0.1}{4\varepsilon_0 \cdot 2\pi \cdot f} = 0.1837$,

$\because$ loss tangent < 0.2 is good dielectric (low-loss)

$\rightarrow \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{0.1}{4}\cdot 120\pi = 3\pi \approx 9.43$ Np/m

$\wedge\ \beta = w\sqrt{\mu\varepsilon} = 2\pi f \cdot \frac{2}{c} = \frac{98}{3}\pi \approx 102.6$ rad/m

$\rightarrow \gamma = 9.43 + 102.6j$

$\because \delta = 1/\alpha = \frac{1}{3\pi} \approx 10.7$ cm

$\rightarrow$ |E| decreases by 63% after 10.7 centimeters

($|E|_{10.7} = 0.37 \cdot |E|_0$

example: $\because$ $\bar{E} = 100e^{-\alpha z}\cos(10^7\pi t - \beta z)\bar{a}_x$, $\varepsilon_r = 72$, $\mu_r = 1$, $\sigma = 4$

$\wedge$ $\tan(\theta)$: loss tangent $= \frac{\sigma}{\varepsilon \cdot w} = \frac{4}{10^7\pi \cdot 72 \cdot \frac{10^{-9}}{36\pi}} = \frac{2}{10^{-2}} = 200$

$\rightarrow$ 200 >> 1 $\rightarrow$ medium is good conductor

$\therefore \alpha = \beta = \sqrt{\pi f \mu\sigma} = \sqrt{\pi \cdot \frac{10^7\pi}{2\pi} \cdot 4\pi \times 10^{-7} \cdot 4} = 2\cdot\sqrt{2}\pi$

$\rightarrow \alpha = \beta \approx 8.86$

$\wedge\ \eta = (1+j)\frac{\alpha}{\sigma} = (\sqrt{2})\frac{\sqrt{2}}{2}\pi\angle 45° = \pi\angle 45°$

$\because u_p = \frac{w}{\beta} \rightarrow u_p = \frac{10^7\pi}{2\sqrt{2}\pi} = 3.53$ Mm/s

$\because \delta = 1/\alpha = 0.113$ m $\rightarrow$ 11.3 cm

example: $\because$ $\varepsilon' = 80\varepsilon_0$, $\mu_r = 1$ $\wedge$ $\sigma = 4$ S/m , $\varepsilon'' = 44\,\varepsilon_0$

a) at $f = 30\,Hz$, $\tan(\theta) \gg 1 \rightarrow$ conductor

$\therefore \alpha = \sqrt{\pi f \mu\sigma} = 0.02176$ Np/m

$\rightarrow 0.1|E_0| = |E_0| \cdot e^{-\alpha \cdot d}$ $\rightarrow d = \frac{-2.3025}{-0.02176} = 105.813$ m

b) at $f = 10\,GHz$, loss tangent $= \frac{\varepsilon''}{\varepsilon'} = \frac{44}{80} \not\gg 1$

$\therefore$ we must use exact expressions :

$\alpha = w\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{w\varepsilon}\right)^2} - 1\right]} =$

$a = 2\pi \cdot 10^{10} \cdot \left[\frac{\boxed{\frac{80}{9}\cdot 10^{-16}}}{2} \cdot \sqrt{1+\left(\frac{45}{80}\right)^2} - 1 \right]^{1/2} = 508.9$ Np/m

$\because \gamma = j\omega\sqrt{\mu\varepsilon_c}$ $\varepsilon_c = \varepsilon' - j\varepsilon''$ $\wedge$ $a = R\{\gamma\}$

$\rightarrow a = Re\{j\omega\sqrt{\mu(80\varepsilon_0 - j45\varepsilon_0)}\} = 1.0198644\times10^{-15} \angle -29.35994°$

$\rightarrow a = Re\{j\omega\cdot(3.1934316\times10^{-8}\angle -14.678895°)\}$

$\Rightarrow a = Re\{508.463 + 1941.063j\} = 508.5$

or $\because a = Re\{j\omega\sqrt{\mu_0(\varepsilon' - j\varepsilon'')}\}$

$\rightarrow a = Re\{j\omega\sqrt{\mu_0\varepsilon_0(80-j45)}\} = \frac{\omega}{c}\cdot Re\{j\sqrt{80-j45}\}$

$\therefore a = 508.9$ Np/m

$\wedge$ $0.1|E_0| = e^{-a\cdot d}$ $\rightarrow d = 4.526\times10^{-3}$ m ≈ 4.5 mm

example: $\because \bar{H} = 10\cdot e^{-ax}\cos(\omega t - \frac{x}{2})\,\bar{a_y} \rightarrow \beta = 0.5$

$\wedge$ $\eta = 200\angle 30°$

$\because \eta = 200\angle30° \rightarrow \theta_\eta = 30° \rightarrow \theta$: loss tangent $= 60°$

$\therefore \tan\theta = \tan(60°) = \sqrt{3} \cancel{\gg} 1$ $\tan(\theta) = \frac{\sigma}{\varepsilon\omega}$

→ must use exact equations : $\because \theta = \tan^{-1}(\frac{\beta}{a}) =$

$\rightarrow \tan\theta = \sqrt{3} = \frac{\beta}{a} = \frac{0.5}{a} \rightarrow a = \frac{\sqrt{3}}{6}$

$\because \bar E = -\eta(\bar{a_k}\times\bar H)$

$\rightarrow \bar E = -200\angle30°(\bar{a_x}\times\bar H)$ $\wedge$ $\bar{a_x}\times\bar H = 10\cdot e^{-ax}\cos(\omega t - \frac{x}{2})\,\bar{a_z}$

$\rightarrow \bar E = -2000\cdot e^{-\frac{\sqrt{3}}{6}x}\cdot\cos(\omega t - \frac{x}{2} + 30°)\,\bar{a_z}$

$\because \eta = \frac{j\omega\mu}{\gamma} \rightarrow \omega = \frac{-j\cdot\eta(a+j\beta)}{\omega\mu}$

example: $\because f_0 = 450$ MHz, $\delta_c = 15$ mm $\rightarrow a = \frac{200}{3}$ Np/m

$\wedge$ $\beta =$ rad m^{-1} $\rightarrow \beta = \frac{93°\times\frac{\pi}{180}}{15\text{mm}} = \frac{310}{9}\pi$ rad/m

$\because$ phase $= \beta z$

$\therefore a \approx 66.67$ Np/m $\wedge$ $\beta \approx 108.21$ rad/m

$\because \tan(\theta) = \frac{\beta}{a} = \frac{\sigma}{\varepsilon\omega}$

$\wedge$ $\gamma = a + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$\rightarrow a^2 - \beta^2 = -\omega^2\mu\varepsilon$ $\wedge$ $2a\beta = \omega\mu\sigma$

$\rightarrow \sigma = \frac{2a\beta}{\omega\mu_0} = 4.0615$/m $\wedge$ $\varepsilon_r = 81.98$

example: $\because |\bar{E}|_{z=10m} = 19.025$ V/m $\wedge$ $|\bar{E}|_{z=100m} = 12.13$ V/m

$\rightarrow |\bar{E}|_{z=10m} = |\bar{E}_0| \cdot e^{-\alpha \cdot 10}$

$\therefore \frac{|\bar{E}|_{z=10m}}{|\bar{E}|_{z=100m}} = \frac{\cancel{|\bar{E}_0|} \cdot e^{-\alpha \cdot 10}}{\cancel{|\bar{E}_0|} \cdot e^{-\alpha \cdot 100}}$

$\rightarrow \frac{19.025}{12.13} = e^{90\alpha} \rightarrow \alpha = \frac{\ln\left(\frac{19.025}{12.13}\right)}{90}$

$\therefore \alpha = 5.0008$ mNP/m

samples 2::

first exam: 3/11/2018

2. a) $\because \omega = 2\pi \cdot 2.5GHz$ $\wedge$ $\alpha = 50$ $\frac{\beta}{\alpha} = \tan\theta = \frac{\sigma}{\omega\varepsilon}$

$\wedge\ (\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

$\rightarrow \alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$ $\wedge$ $2\alpha\beta = \omega\mu\sigma$

$\rightarrow \beta^2 - 2500 = 4\pi^2 (2.5)^2 \cdot 10^{18} \cdot \mu_r \mu_0 \cdot 9\varepsilon_0$ —①

$\rightarrow \frac{5\pi \cdot 10^9}{2 \cdot 50} \cdot \mu_r \mu_0 \cdot \sigma = \beta$ $\wedge$ $\beta = \alpha\frac{\sigma}{\omega\varepsilon_r\varepsilon_0}$

$\rightarrow 5\pi \cdot 10^7 \cdot \mu_r \cdot \mu_0 \cdot \cancel{\sigma} = \frac{50\cancel{\sigma}}{5\pi 10^9\ \varepsilon_r\varepsilon_0}$

$\rightarrow \mu_r \cdot \mu_0 \Rightarrow \mu_0 = 0.20264$

$\rightarrow \beta^2 = 5000 \rightarrow \boxed{\beta = 50\sqrt{2}}$

$\rightarrow \sigma = 1.967969$ S/m

$\therefore \eta = \frac{j\omega\mu}{\alpha + j\beta} = 37.9123619 + 26.666667j\ \Omega$

$\wedge\ \bar{E} = -\eta(\bar{a}_k \times \bar{H})$, $\bar{a}_k \times \bar{H} = \bar{a}_x \times \bar{z}$

$\rightarrow \bar{E} = -\eta(10e^{-50x}\cos(\omega t - \beta x) - \bar{a}_y)$

$\therefore \bar{E} = \boxed{461.88e^{-50x}\cos(5\pi \times 10^9 - 50\sqrt{2} \cdot x + 35.26)\bar{a}_y}$

problem 2: assuming $\mu_r = 1$

$\beta^2 - \alpha^2 = \omega^2 \cdot \mu\varepsilon \rightarrow \beta = 164.845$

$\because \eta = \frac{j\omega\mu}{\gamma} = 114.6\angle 16.873^\circ\ \Omega$

$\rightarrow \bar{E} = -\eta(\bar{a}_k \times \bar{H}) = 1146\, e^{-40x} \cdot \cos(5\pi\times10^{9} - 164.845x + 16.873)$

1/11/2016 homework:

1. $\because \varepsilon''/\varepsilon' = 0.1 \ll 1 \rightarrow$ good dielectric

$\rightarrow \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \wedge \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon'} \rightarrow \varepsilon' = \frac{\varepsilon''}{0.1} = 10\varepsilon'' = 10\frac{\sigma}{\omega}$

$\rightarrow \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu\omega}{10\sigma}} = \sqrt{\frac{\sigma\cdot\mu\omega}{40}}$ X

$\because \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon'}} \wedge \varepsilon'' = \frac{\sigma}{\omega}$

$\rightarrow \alpha = \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} \quad \because \frac{\varepsilon''}{\varepsilon'} = 0.1 \rightarrow \varepsilon' = 10\varepsilon''$

$\rightarrow \alpha = \frac{\omega}{2}\sqrt{\frac{\mu\cdot\varepsilon_r\varepsilon_0}{100}} = \frac{1.5G}{2}\cdot\sqrt{\frac{4\pi\times10^{-9}\cdot3\cdot\frac{10^{-9}}{36\pi}}{100}} = \frac{1.5G}{6\times10^{8}}\cdot\sqrt{\frac{3}{100}}$

$\rightarrow \alpha = 0.433$ Np/m $\rightarrow 3\,\text{Np} = \alpha\cdot d \rightarrow d = 6.93$ m

- dielectric constant: ε_r

b) $|E_0|\cdot\frac{1}{2} = |E_0|\cdot e^{-\alpha d} \rightarrow \frac{\ln(0.5)}{-\alpha} = d = 1.601$ m

c) $\beta\cdot d = 240^\circ \rightarrow d = \frac{240\cdot\frac{180}{\pi}}{\beta} \wedge \beta = \omega\sqrt{\mu\varepsilon}$

$\rightarrow \beta = 5\sqrt{3} \rightarrow d = 0.504$ m

2) $\because \delta = 1.44$ m $\rightarrow \alpha = 0.6944$ Np/m $\wedge\ \eta = 60\pi\angle 30^\circ\ \Omega$

$\wedge\ 2\theta_\eta = \theta_{loss} \rightarrow \tan\theta = \sqrt{3} \rightarrow \beta = 1.203$

$\wedge\ \frac{\sigma}{\omega\varepsilon} = \sqrt{3} \quad \because \eta = \frac{j\omega\mu}{\gamma} \quad \gamma = \alpha + j\beta$

$\rightarrow 60\pi\angle 30^\circ = \frac{j\,\omega\,4\pi\times10^{-7}}{0.6944 + j1.203} \rightarrow |\eta|\cdot|\gamma| = \omega\mu$

$\rightarrow \omega = 2.0835\times10^{8}$

$\because \beta^2 - \alpha^2 = \omega^2\cdot\mu\cdot\varepsilon \rightarrow \boxed{\varepsilon_r \approx 2}$

$\wedge\ 2\alpha\beta = \omega\cdot\mu\cdot\sigma$

$\rightarrow \sigma = 6.3811\times10^{-3}, \ \lambda = \frac{2\pi}{\beta} = 5.22\text{ m} \wedge u_p = \frac{\omega}{\beta} = 173.2$ Mm/s

3. $|\bar{E}|_{0.01} = |E_0| \cdot e^{-\alpha \cdot 0.01} \quad \wedge \quad \alpha = Re\{\gamma\}$

$\wedge \quad \gamma = j w \sqrt{\mu \varepsilon_c} \Rightarrow \gamma = \sqrt{j^2 w^2 \mu_0 (2-j) \varepsilon_0}$

$\Rightarrow \gamma^2 = 6130.31 \angle 153.435^\circ$

$\Rightarrow \gamma = 17.9887 + 76.202j \Rightarrow \alpha \approx 18 \wedge \beta \approx 76.2$

$\Rightarrow |\bar{E}|_{0.01} = 100 \cdot e^{-18 \cdot 0.01} = 83.536 \text{ V/m}$

$\wedge \quad \beta \cdot 0.01 = \Theta = 0.76202 \text{ rad} = 43.66^\circ$

Subject 10.9: wave polarization   Date 19/3/2021   No.

* polarization: the locus of the tip of the electric field at a given point as a function of time

  - if the plane is perpendicular to the direction of propagation

- AM radio broadcasting has polarization vertical to the earth's surface
- FM broadcasting is generally, circularly polarized.

+ a uniform plane wave is said to be polarized if it has only one vector component or when its transverse components are in phase.

- a wave is linearly polarized if the phase difference between its components is some integer multiple of π (i. in phase)

  if $\bar{E} = E_{0x}\cos(\omega t - \beta z + \phi_x)\bar{a}_x + E_{0y}\cos(\omega t - \beta z + \phi_y)\bar{a}_y$

  $\wedge \quad \Delta\phi = \phi_x - \phi_y = n\pi \quad , n = 0, 1, 2, \ldots$

  hence, the two components will maintain the same ratios at all times

- a wave is circularly polarized if its components are equal in magnitude and the phase difference is an odd multiple of $\pi/2$: (90° out of phase)

  $E_{0y} = E_{0x} = E_0 \quad \wedge \quad \Delta\phi = \phi_x - \phi_y = (2n+1)\cdot\frac{\pi}{2} \quad , n = 0, 1, 2 \ldots$

- if $E_{0y} \neq E_{0x}$ then the wave is elliptically polarized

$\tan\theta_\gamma = \frac{\beta}{\alpha} = \tan(90 - \theta_\eta)$ $\theta \tan(\theta_\gamma) = \frac{\beta}{\alpha}$

$\rightarrow$ (ot

$\theta_\eta + \theta_\gamma = 90$

$\tan(2\theta_\eta) = \frac{\sigma}{\omega\varepsilon}$

$2\theta_\eta = 180 - 2\theta_\gamma$

$\tan \quad \rightarrow \tan(-2\theta_\gamma) = -\tan(\theta_\gamma)$

$\frac{2}{\cos^2\theta} - 2$

$-\tan(2\theta_\gamma)$

$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta} \quad \frac{2-2\cos^2\theta}{\cos^2\theta} \quad \frac{2\tan\theta}{1-\tan^2\theta}$

$1 - 2\cos^2\theta - 1$

.NOTEBOOK

Subject practice for quiz 2 Date 21/3/2021 No.

first exam:

② $\epsilon_r = 9$, $\mu_r = 1$, $\alpha = 50$, $\omega = 5\pi\times10^9$

$\eta = \frac{j\omega\mu}{\gamma}$ ∧ $\gamma = \alpha + j\beta$

$\Rightarrow \eta = \frac{j5\pi\times10^9\times\mu_0}{50 + j\beta}$

∵ $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon) = \alpha^2 - \beta^2 + 2j\alpha\beta$

$\Rightarrow \alpha^2 - \beta^2 = -\omega^2\mu\epsilon$

$\Rightarrow \beta^2 = \alpha^2 + \omega^2\mu\epsilon = 2500 + 25\pi^2\times10^{18}\times4\pi\times10^{-7}\times9\times\frac{10^{-9}}{36\pi}$

$\Rightarrow \beta = 164.8454$ rad/m

∴ $\eta = \frac{j120\pi^2\times10^2}{50 + 164.8454j} = 114.5886\angle16.893^\circ\ \Omega$

∵ $\bar{E} = -\eta(a_n\times\bar{H})$, $\bar{a}_n = +\bar{a}_x$

$\Rightarrow \bar{a}_x \times H = -\bar{a}_y$

$\Rightarrow \bar{E} = -114.5886\cdot10\cdot e^{-50x}\cdot\cos(5\pi\times10^9 - 164.845x + 16.89^\circ)\cdot -\bar{a}_y$

∴ $\bar{E} = 1140.59\cdot e^{-50x}\cdot\cos(5\pi\times10^9 - 164.845x + 16.89^\circ)\,\bar{a}_y$

HW2 - and - HW3

① ∵ $\frac{\epsilon''}{\epsilon'} = \frac{\sigma/\omega}{\epsilon_r\epsilon_0}$ = loss tangent , $\omega = 1.5G$

a) $\alpha\cdot d = 3$ Np ∵ $\frac{\epsilon''}{\epsilon'} << 1$, good dielectric

$\Rightarrow \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$, $\epsilon = \epsilon_r\epsilon_0$, $\epsilon_r = 3$

$\Rightarrow \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu_0}{3\epsilon_0}}$, ∵ $\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} \rightarrow \sigma = 0.1\cdot\omega\epsilon$

∴ $\alpha = \frac{0.1\cdot\omega\cdot\epsilon_0\cdot3}{2}\cdot\frac{120\pi}{\sqrt{3}} = \frac{\sqrt{3}}{4}$

$\Rightarrow d = \frac{4\cdot3}{\sqrt{3}} = 6.928$ m

① $\beta \cdot d = 250^\circ \cdot \frac{\pi}{180}$ $\wedge$ $\beta = \omega \cdot \sqrt{\mu\varepsilon} = (\omega/c)\cdot\sqrt{3}$

$\to \beta = 5\sqrt{3} \to d = 0.504\ m$

② $\eta = 60\pi \angle 30^\circ\ \Omega \to \theta_\eta = 30^\circ$ $\tan(\theta_\eta) = \frac{\alpha}{\beta}$

$\wedge$ $\alpha = \frac{1}{\delta} \to \beta = 1.203\ Rad\ m^{-1}$

$\because \gamma = \alpha + j\beta \to \gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu(\sigma + j\omega\varepsilon)$

$\to \beta^2 - \alpha^2 = \omega^2\mu\varepsilon$

$\wedge$ $2\alpha\beta = \omega\mu\sigma$

$\because \eta = \frac{j\omega\mu}{\gamma} \to |\gamma\eta| = \omega\mu \to \omega = 208.36\ M\,Rad/s$

$\therefore \varepsilon_r = \frac{\beta^2 - \alpha^2}{\omega^2 \cdot \mu \cdot \varepsilon_0} \approx 2$

$\wedge$ $\sigma = \frac{2\alpha\beta}{\omega\mu} = 6.38\ mS/m$

$\because \lambda = \frac{2\pi}{\beta} = 5.22\ m$ $\wedge$ $u_p = \frac{\omega}{\beta} = 173.2\ Mm/s$

③ $|E|_{0.01} = |E_0|\, e^{-\alpha \cdot 0.01}$ $\wedge$ $\alpha = \omega\left[\frac{\mu\varepsilon}{2}\cdot\left[\sqrt{1+(\tan\theta)^2} - 1\right]\right]^{0.5}$

$\wedge$ $\tan(\theta) = \frac{\varepsilon''}{\varepsilon'} = \frac{1}{2} \to \alpha = 17.989$ $\wedge$ $\beta = 76.2\ Rad/m$

$\to |E|_{0.01} = 100\, e^{-17.989 \cdot 0.01} = 83.54\ V/m$

$\wedge$ $\theta = \beta \cdot d = 76.2 \cdot 0.01 \approx 43.66^\circ$

④ $\lambda = 54.6\ m \to \beta = \frac{2\pi}{54.6} = 0.1151\ Rad\ m^{-1}$

$\because |E_0| = 110\ V/m$ $\wedge$ $|E|_{40} = 41\ V/m$

$\to \frac{|E_0|}{|E|_{40}} = e^{\alpha \cdot 40} \to \alpha = 0.02467\ Np/m$

$\omega = 2\pi/0.5\mu = 4\pi \times 10^6$, non-magnetic $\to \mu_r = 1$

γ^2: $2\alpha\beta = \omega\mu\sigma$

$\wedge$ $\beta^2 - \alpha^2 = \omega^2\mu\varepsilon \to \varepsilon_r = 7.2035$

$\wedge$ $\sigma = 0.3496\ mS/m$

Quiz 2 :

① $\because\ \lambda = 24 \times 10^{-3} = \frac{2\pi}{\beta} \rightarrow \beta = 261.8 \text{ rad m}^{-1}$

$\because\ \tan(\theta_\eta) = \frac{\alpha}{\beta} \rightarrow \alpha = 108.44 \text{ Np/m} \rightarrow \boxed{\delta = 9.22 \text{ mm}}$ X

$\wedge\ \gamma^2 = \alpha^2 - \beta^2 + 2\alpha\beta = j\omega\mu(\sigma + j\omega\varepsilon)$

$\eta = \frac{j\omega\mu}{\gamma} \rightarrow |\eta|\cdot|\gamma| = \omega\cdot\mu \rightarrow \boxed{\mu_r = 1.599}$

$\because\ \beta^2 - \alpha^2 = \omega^2 \mu\cdot\varepsilon \rightarrow \varepsilon_r = 24.98$

$\wedge\ 2\alpha\beta = \omega\mu\sigma \rightarrow \boxed{\sigma = 2.4985 \text{ S/m}}$

② $\because\ \theta_\eta = \frac{\pi}{10} \rightarrow \frac{\alpha}{\beta} = \tan\frac{\pi}{10} \rightarrow \beta = 61.5537$

$\wedge\ \tan\theta = \tan(2\theta_\eta) = 0.7265 \not\gg 1$, neither good dielectric nor good conductor

$\because\ |\eta| = \frac{10}{0.05} = \frac{|E_0|}{|H_0|} = 200\ \Omega \ \wedge\ \eta = \frac{j\omega\mu}{\gamma}$

$\rightarrow |\eta|\cdot|\gamma| = \omega\cdot\mu_r\cdot\mu_0 \rightarrow |\gamma| = 74.3023$

$\therefore\ \mu_r = 1.639 \rightarrow$ non-magnetic

$\because\ \beta^2 - \alpha^2 = \omega^2\cdot\mu\cdot\varepsilon \rightarrow \boxed{\varepsilon_r = 4.71364}$

$\because\ u_p = \frac{\omega}{\beta} \rightarrow u_p = 102.0765 \text{ Mm/s}$

$\rightarrow$ in 1 μsec, wave travels 102.0765 m

Quiz 2: Chapter 10 (part 1) :

أنا محمد سند الطاهر أشهد أني قرأت وفهمت تعليمات هذا الامتحان القصير وتقيدت بها والله على ما أقول شهيد.

$\because \quad \bar{E} = 2840\, e^{-\alpha z} \cos(2\pi \times 10^3 t - \beta z)\ \bar{a}_y$

$\wedge \quad \sigma = 4\ S/m\ ,\ \varepsilon_r = 81\ ,\ \mu_r = 1$

1) $\because \lambda = \frac{2\pi}{\beta} \quad \wedge \quad \beta = \omega\left[\frac{\mu\varepsilon}{2}\left[\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{2}}+1\right]\right]^{\frac{1}{2}}$

$\rightarrow \beta = \omega \cdot 2\times10^{-5} = \boxed{0.12566\ \mathrm{rad\ m^{-1}}}$

$\therefore \lambda = \frac{2\pi}{\beta} \approx \boxed{50\ m}$

$\boxed{\therefore \text{antenna size} = \frac{1}{2}\lambda = 25\ m}$

2) $|E|_d = 1\ \mu V/m = 3.5211\times10^{-13}\,|E_0|$

$\therefore \quad 3.5211\times10^{-13} \cdot |E_0| = |E_0| \cdot e^{-\alpha \cdot d}$

$\wedge \quad \alpha = \omega\left[\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]\right]^{0.5} \approx \beta = 0.12566\ \mathrm{Np\,m^{-1}}$

$\therefore \quad \boxed{d \approx 228.19\ m}$

3) $\because \tan\theta = \frac{\sigma}{\omega\varepsilon} = \frac{4}{2\pi\times10^3\times81\varepsilon_0} = 8.888...\times10^5 >> 1$, conductor

$\rightarrow \eta = (1+j)\frac{\alpha}{\sigma} \rightarrow |\eta| = 0.0444279$

$\wedge \quad \theta_\eta = 45^\circ$

$\because \quad \bar{H} = \frac{1}{\eta}(\bar{a}_k \times \bar{E})$

$\rightarrow \bar{H} = \frac{-1}{0.044...}\left(2840\, e^{-\alpha z} \cdot \cos(2\pi\times10^3 t - \beta z - 45^\circ)\right)\bar{a}_x$

$\therefore \boxed{\bar{H} = -63924\, e^{-0.126 z} \cdot \cos(2\pi\times10^3 t - 0.126 z - 45^\circ)\ \bar{a}_x}$

Subject 10.8: power and poynting vector Date 21/3/2021 No.

$$\because \quad \nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \quad \wedge \quad \nabla \times \bar{H} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t}$$

$$\Rightarrow \bar{H} \cdot (\nabla \times \bar{E}) = \bar{H} \cdot \left(-\mu \frac{\partial \bar{H}}{\partial t}\right) \quad \because \frac{\partial |\bar{H}|^2}{\partial t} = 2\bar{H} \ldots$$

$$\therefore \bar{H} \cdot (\nabla \times \bar{E}) = -\mu \cdot \frac{1}{2} \frac{\partial |\bar{H}|^2}{\partial t}$$

$$\Rightarrow \bar{E} (\nabla \times \bar{H}) = \sigma |\bar{E}|^2 + \frac{\varepsilon}{2} \frac{\partial |\bar{E}|^2}{\partial t} \qquad |\bar{E}| = E$$

$$\because \nabla \cdot (\bar{A} \times \bar{B}) = \bar{B}(\nabla \times \bar{A}) - \bar{A} \cdot (\nabla \times \bar{B})$$

$$\Rightarrow \bar{E} (\nabla \times \bar{H}) = \nabla \cdot (\bar{H} \times \bar{E}) + \bar{H} \cdot (\nabla \times \bar{E}) \quad \wedge \nabla (\bar{H} \times \bar{E}) = -\nabla (\bar{E} \times \bar{H})$$

$$\therefore -\nabla (\bar{E} \times \bar{H}) = \frac{\mu}{2} \frac{\partial |\bar{H}|^2}{\partial t} + \sigma |\bar{E}|^2 + \frac{\varepsilon}{2} \frac{\partial |\bar{E}|^2}{\partial t}$$

$$\therefore \boxed{\nabla \cdot (\bar{E} \times \bar{H}) = \frac{-\mu}{2} \cdot \frac{\partial H^2}{\partial t} - \left[\sigma E^2 + \frac{\varepsilon}{2} \cdot \frac{\partial E^2}{\partial t}\right]}$$

$$\Rightarrow \int_v \nabla \cdot (\bar{E} \times \bar{H}) \, dv = -\frac{\partial}{\partial t} \int_v \left[\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2\right] dv - \int_v \sigma E^2 \, dv$$

divergence theorem gives:

$$\underbrace{\oint_s (\bar{E} \times \bar{H}) \, d\bar{s}}_{\text{total power leaving the volume}} = \underbrace{-\frac{\partial}{\partial t} \int_v \left[\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2\right] dv}_{\text{rate of decrease in energy stored in electric and magnetic fields}} - \underbrace{\int_v \sigma E^2 \, dv}_{\text{ohmic power dissipated}}$$

→ poynting's theorem: shows conservation of power

poynting vector: $\bar{P} = \bar{E} \times \bar{H}$, in W/m^2

- integrating poynting vector over a closed surface gives the net power flowing out of the surface.

- the poynting vector is normal to both $\bar{E}$ & $\bar{H}$, and is parallel to the direction of propagation of the wave

assume $\bar{E}(z,t) = E_0\, e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_x$ ∧ $\bar{H}(z,t) = \frac{E_0}{|\eta|} \cdot e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \bar{a}_y$

$\rightarrow \bar{P}(z,t) = \bar{E}(z,t) \times \bar{H}(z,t) = \frac{E_0^2}{|\eta|} e^{-2\alpha z} \cdot \cos(\omega t - \beta z) \cdot \cos(\omega t - \beta z - \theta_\eta)\, \bar{a}_z$

$$\therefore \boxed{\bar{P}(z,t) = \frac{E_0^2}{2|\eta|} \cdot e^{-2\alpha z} \cdot \left[\cos(\theta_\eta) + \cos(2\omega t - 2\beta z - \theta_\eta)\right] \bar{a}_z}$$

(for a uniform plane wave)

- the time average poynting vector : $\bar{P}_{avg}(z) = \frac{1}{T}\int_0^T P(z,t)\, dt$ (in W/m^2)

- the total time average power crossing a surface(S) : $P_{avg} = \int_S \bar{P}_{avg} \cdot d\bar{S}$ (in W)

example 10.8 :

a) $\because \bar{E} = 4 \sin(2\pi \times 10^7 t - 0.8x)\, \bar{a}_z = 4 \cos(2\pi \times 10^7 t - 0.8x - \frac{\pi}{2})\, \bar{a}_z$

$\rightarrow$ lossless medium $\rightarrow \beta = \omega\sqrt{\mu\varepsilon} \rightarrow (\omega/c) \cdot \sqrt{\varepsilon_r}$

$\therefore \varepsilon_r = 14.59$ ∧ $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{\varepsilon_r}} = 98.69\ \Omega$

b) $\because \bar{P}_{avg} = \frac{1}{T}\int_0^T \bar{P}(x,t)\, dt$ ∧ $\bar{P}(x,t) = \bar{E} \times \bar{H}$

∧ $\bar{H} = \frac{1}{\eta}(\bar{a}_x \times \bar{E}) = \frac{4}{98.69} \cdot \cos(2\pi \times 10^7 t - 0.8x - \frac{\pi}{2})\, \bar{a}_y$

$\therefore \bar{P}(x,t) = \frac{16}{2 \cdot 98.69} \cdot \left[1 + \cos(4\pi \times 10^7 t - 1.6x - \pi)\right] \bar{a}_x$

$\rightarrow \frac{1}{T}\int^T \bar{P}(x,6)\, dt = \frac{16}{2 \cdot 98.69} = 81 a_x$ mW/m^2

c) $P_{avg} = \int_S 81m \cdot d\bar{S} = 10^{-3} \times 81 \cdot S\, \bar{a}_n$

normal $\overset{\text{unit}}{\uparrow}$ vector to a plane is $\frac{\text{gradient}}{\text{magnitude}} = \frac{2\bar{a}_x + \bar{a}_y}{\sqrt{5}}$

$\rightarrow P_{avg} = (81m\, \bar{a}_x) \cdot (\frac{2}{\sqrt{5}} \bar{a}_x + \frac{1}{\sqrt{5}} \bar{a}_y) \cdot 100 \times 10^{-4}$

$\rightarrow \boxed{0.7245 \text{ mW}}$

practice exercise 10.8:

$\because \eta = 120\pi \rightarrow E_0 = 24\pi V/m \wedge \bar{P}_{avg} = \frac{E_0^2}{2|\eta|} = 7.539 \bar{a}_x W/m^2$

a) $P_{avg} = \int_S \bar{P}_{avg} \, dS = 7.539 \bar{a}_x \cdot S a_n$

$a_n = \frac{a_x + a_y}{\sqrt{2}} \rightarrow P_{avg} = 53.309 mW$

b.) $P_{avg} = 7.539 \bar{a}_x \cdot \pi \cdot 5^2 \times 10^{-4} \bar{a}_x = 59.21 mW$

$$\nabla \cdot (\bar{E} \times \bar{H}) = -\sigma E^2 - \frac{\partial}{\partial t}(W_e + W_m)$$

where W_e: electric energy density, $\frac{1}{2}\varepsilon E^2$ (J/m^3)

joules per meter cube

and W_m: magnetic energy density, $\frac{1}{2}\mu H^2$ (J/m^3)

* average power density:

$$P(z,t) = Re\{\bar{E}_s e^{j\omega t}\} \times Re\{\bar{H}_s e^{j\omega t}\}$$

$\because Re(\bar{A}) \times Re(\bar{B}) = \frac{1}{2}(\bar{A} + \bar{A}^*) \times \frac{1}{2}(\bar{B} + \bar{B}^*) = \frac{1}{2} Re\{\bar{A} \times \bar{B}^* + \bar{A} \times \bar{B}\}$

$$\rightarrow \boxed{P(z,t) = \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^* + \bar{E}_s \times \bar{H}_s e^{j2\omega t}\}}$$

$$\rightarrow \bar{P}_{av}(z) = \frac{1}{2} Re\{E_s \times H_s^*\} \quad (W/m^2)$$

example:

$\because \tan\theta << 1 \rightarrow$ good dielectric, $\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \wedge \sigma = \tan\theta \cdot \omega\varepsilon$

$\rightarrow \alpha = 0.01481 Np/m \wedge \eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{2}} = 266.573 \Omega$

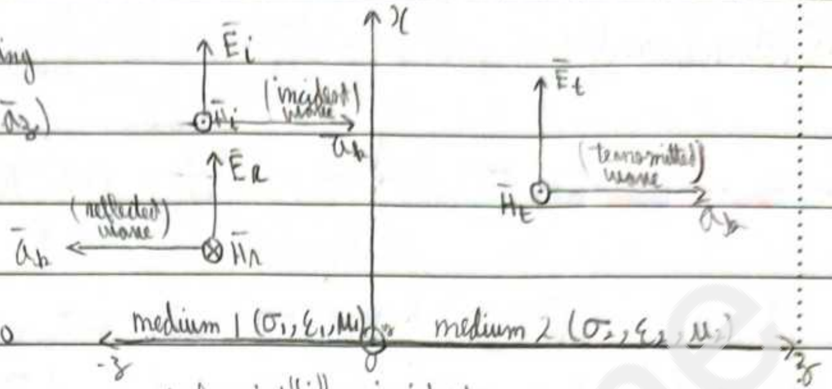
$\therefore \bar{P}_{av} = \frac{1}{2} \cdot \frac{|E_0|^2}{|\eta|} \cdot e^{-2\alpha x}$ ~~$\cos(\theta_\eta)$~~ $\bar{a}_x$

$\rightarrow \bar{P}_{avg} = 1.876 e^{-0.02962 \cdot x} (mW/m^2) \bar{a}_x$

$P_{diss} = [P_{avg}(x=0) - P_{avg}(x=1)] \cdot S \approx 54.75 mW$

dissipated

Subject 10.9 : Reflection of a plane wave at normal incidence Date 30/3/2021 No.



+ assuming incident wave is traveling in the positive z direction $(\bar{a}_k = \bar{a}_z)$

* $\bar{E}_{is}(z) = E_{io} e^{-\gamma_1 z} \bar{a}_x$

$\because \bar{H} = \frac{1}{\eta}(\bar{a}_k \times \bar{E})$, $\frac{E_{io}}{\eta_1} = H_{io}$

$\rightarrow \bar{H}_{is} = H_{io} e^{-\gamma_1 z} \bar{a}_y$

subscript "i" : incident

* $\bar{E}_{rs}(z) = E_{ro} \cdot e^{-\gamma_1 z} \bar{a}_x$ λ $\bar{a}_k = -\bar{a}_z \rightarrow \bar{H}_{rs} = H_{ro} \cdot e^{-\gamma_1 z} \cdot (-\bar{a}_y)$

subscript "r" : reflected

* $\bar{E}_{ts}(z) = E_{to} \cdot e^{-\gamma_2 z} \bar{a}_x$ λ $\bar{H}_{ts}(z) = H_{to} e^{-\gamma_2 z} \bar{a}_y$ subscript "t" : transmitted

- $\bar{E}$ and $\bar{H}$ are zero at the interface (the area where medium 1 & 2 connect)
  - total electric field in medium 1, $\bar{E}_1 = \bar{E}_i + \bar{E}_r$
  - total magnetic field in medium 1, $\bar{H}_1 = \bar{H}_i + \bar{H}_r$
  - total electric field in medium 2, $\bar{E}_2 = \bar{E}_t$
  - total magnetic field in medium 2, $\bar{H}_2 = \bar{H}_t$
- boundary conditions require that the tangential components of $\bar{E}$ & $\bar{H}$ fields be continuous, since $\bar{E}$ & $\bar{H}$ are TEM waves, all their components are tangential to the interface.

$$\bar{E}_{1tan} = \bar{E}_{2tan} \rightarrow \bar{E}_i(0) + \bar{E}_r(0) = \bar{E}_t(0)$$

$$E_{io} + E_{ro} = E_{to}$$

$$λ \ \bar{H}_{1tan} = H_{2tan} \rightarrow \bar{H}_i(0) + \bar{H}_r(0) = \bar{H}_t(0)$$

$$H_{io} - H_{ro} = H_{to}$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2}$$

$$\therefore \boxed{E_{ro} = \left[\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right] \cdot E_{io}} \ λ \ \boxed{E_{to} = \left[\frac{2\eta_2}{\eta_2 + \eta_1}\right] \cdot E_{io}}$$

* Reflection coefficient, $\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, $E_{r0} = \Gamma \cdot E_{i0}$

* transmission coefficient, $\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_1 + \eta_2}$, $E_{t0} = \tau \cdot E_{i0}$

$\therefore \quad 1 + \Gamma = \tau$, both $\Gamma \wedge \tau$ are dimensionless, may be complex

$0 \leq |\Gamma| \leq 1$

* case 1: medium 1 is a perfect dielectric (lossless, $\sigma_1 = 0$)

medium 2 is a perfect conductor ($\sigma_2 \simeq \infty$)

$\rightarrow \eta_2 = 0 \rightarrow \Gamma = -1 \wedge \tau = 0$

∴ the wave is totally reflected as nothing is transmitted.

hence, the reflected wave has the same amplitude as the incident wave

combining the waves forms a standing wave, which doesn't travel:

$\rightarrow \alpha_1 = 0 \wedge \gamma_1 = j\beta_1$

$\rightarrow \bar{E}_{1s} = \bar{E}_{is} + \bar{E}_{rs} = (E_{i0} e^{-\gamma_1 z} + \Gamma E_{i0} \cdot e^{\gamma_1 z})\, \bar{a}_x$

$\rightarrow \bar{E}_{1s} = E_{i0} (e^{-j\beta_1 z} - e^{j\beta_1 z})\, \bar{a}_x = 2E_{i0} \cdot (-j\sin(\beta_1 z))\, \bar{a}_x$

$\rightarrow \bar{E}_1 = \mathrm{Re}\{\bar{E}_{1s} \cdot e^{j\omega t}\} = 2E_{i0} \cdot \sin(\beta_1 z) \cdot \sin(\omega t)\, \bar{a}_x$

$\wedge \; \bar{H}_1 = \frac{2E_{i0}}{\eta_1} \cdot \cos(\beta_1 z) \cos(\omega t)\, \bar{a}_y$

$\sigma_1 = \sigma_2 = 0$

* case 2: medium 2 has intrinsic impedance larger than medium 1: $\eta_2 > \eta_1$

$\wedge \; \Gamma > 0$

a standing wave is produced in medium 1 but $|E_{i0}| \neq |E_{r0}|$

and a transmitted wave is observed in medium 2

a relative maximum is observed in medium 1 when:

$-\beta_1 z_{max} = n \cdot \pi \rightarrow |E_1|_{max}$

for $n = 0, 1, 2, \ldots$

$\rightarrow z_{max} = \frac{-n \cdot \pi}{\beta_1} = \boxed{\frac{-n \cdot \lambda_1}{2}}$

$\wedge \; \frac{2\pi}{\beta} = \lambda$

and a minimum of $|E_1|$ occurs at : $-\beta_1 z_{min} = (2n+1)\cdot\frac{\pi}{2}$

$$\rightarrow \boxed{z_{min} = \frac{-(2n+1)\cdot\lambda_1}{4}} \quad \forall\ n = 0,1,2,\ldots$$

$\sigma_1 = \sigma_2 = 0$ | lossless

* case 3: if $\eta_2 < \eta_1 \wedge \Gamma < 0$:

$$|E_1|_{max} \text{ at } z_{max} = \frac{-(2n+1)\lambda_1}{4} \quad \forall\ n = 0,1,2,\ldots$$

$$\wedge\ |E_1|_{min} \text{ at } z_{min} = \frac{-n\cdot\lambda_1}{2} \quad \forall\ n = 0,1,2,\ldots$$

- $|H_1|_{min}$ occurs wherever $|E_1|_{max}$ occurs and vice versa
- the transmitted wave in the above cases is a purely traveling wave and hence there are no max or min values in medium 2.

* standing wave ratio: Ratio of max $|E_1|$ (or H_1) to min $|E_1|$ (or $|H_1|$)

$$\rightarrow SWR = \frac{|E_1|_{max}}{|E_1|_{min}} = \frac{|H_1|_{max}}{|H_1|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\rightarrow |\Gamma| = \frac{SWR-1}{SWR+1}$$

$$\because |\Gamma| \leq 1 \rightarrow 1 \leq SWR \leq \infty$$

- SWR is dimensionless but often expressed in dB

example 10.9 :

freespace, $\eta_1 = 120\pi\,\Omega$, medium 2 : $\eta_2 = 120\pi\cdot\sqrt{\frac{8}{2}} = 240\pi\,\Omega$

$\because \overline{H_i} = 10\cos(10^8 t - \beta_1 z)\,a_x \rightarrow \overline{E_i} = -1200\pi\cos(10^8 t - \beta_1 z)\,a_y$

$\because \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \wedge \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$

$\frac{H_{r0}}{H_{i0}} = -\Gamma$

$\frac{H_{t0}}{H_{i0}} = \tau\,\frac{\eta_1}{\eta_2}$

$\rightarrow \Gamma = \frac{1}{3} \wedge \tau = \frac{4}{3}$

$\rightarrow \overline{H_{r0}} = -\Gamma\cdot\overline{H_{i0}} = \frac{-10}{3} \rightarrow \overline{H_r} = \frac{-10}{3}\cos(10^8 t + \beta z)\,\overline{a_x}$

$\wedge\ H_{t0} : \tau\cdot\frac{1}{2}\cdot 10 = \frac{20}{3} \rightarrow H_t = \frac{20}{3}\cos(10^8 t - \beta z)\,\overline{a_x}$

$\wedge\ \overline{E_r} = -400\pi\cdot\cos(10^8 t + \beta z)\,\overline{a_y} \wedge \overline{E_t} = -1600\pi\cos(10^8 t - \beta z)\,\overline{a_y}$

check that $\overline{E_i}(0) + \overline{E_r}(0) = \overline{E_t}(0)$: $-400\pi + -1200\pi = -1600\pi$

check that $\overline{H_i}(0) + \overline{H_r}(0) = \overline{H_t}(0)$: $10 - \frac{10}{3} = \frac{20}{3}$

practice exercise 10.9:

$\because \bar{E}_{is} = 10 e^{-j\beta z} \bar{a}_x \wedge \eta_1 = 120\pi , \eta_2 = 60\pi \rightarrow \Gamma = -\frac{1}{3}$

$\rightarrow \bar{E}_{rs} = -10 \cdot \frac{1}{3} \cdot e^{j\beta_1 z} \bar{a}_x$

$\because T = \frac{2}{3} \rightarrow \bar{E}_{ts} = \frac{20}{3} \cdot e^{-j\beta_2 z} \bar{a}_x$

example 10.10: $\bar{E}_i = 40 \cos(wt - \beta_1 z) \bar{a}_x + 30 \sin(wt - \beta_1 z) \bar{a}_y$

a) assume $\eta_1 = 120\pi \rightarrow \bar{H}_i = \frac{40}{120\pi} \cos(wt - \beta_1 z) \bar{a}_y - \frac{30}{120\pi} \sin(wt - \beta_1 z) \bar{a}_x$

b) perfectly conducting $\rightarrow \bar{E}_t = 0 = \bar{H}_t \rightarrow \Gamma \simeq -1 \wedge T = 0$

$\therefore \bar{E}_r = -40 \cos(wt + \beta z) \bar{a}_x - 30 \sin(wt + \beta z) \bar{a}_y$

$\wedge \bar{H}_r = \frac{-1}{4\pi} \sin(wt + \beta z) \bar{a}_x + \frac{1}{3\pi} \cos(wt + \beta z) \bar{a}_y$

c) $\bar{E}_1 = \bar{E}_i + \bar{E}_r \wedge \bar{H}_1 = \bar{H}_i + \bar{H}_r$ standing

d) $\because P_{avg}(z) = \frac{|E_0|^2}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta) \bar{a}_z , \alpha = 0 \wedge \theta_\eta = 0$

$\rightarrow P_{avg}(z) = \frac{|E_1|^2}{240\pi} \wedge |E_{1s}|^2 = (E_{i0})^2 \bar{a}_z - (E_{r0})^2 \bar{a}_z$

$\rightarrow |E_{1s}|^2 = (40^2 + 30^2) \bar{a}_z - (40^2 + 30^2) \bar{a}_z$

$\rightarrow P_{avg}(z) = 0$

practice exercise 10.10: $\bar{E} = 50 \sin(wt - 5x) \bar{a}_y \rightarrow \beta_1 = 5$

a) $\because \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \wedge \eta_2 = \sqrt{\frac{jw\mu}{\sigma + jw\varepsilon}} \wedge \eta_1 = \sqrt{\frac{\mu}{\varepsilon}}$

$\because \beta_1 = 5 = w\sqrt{\mu\varepsilon} \rightarrow w = 0.75 \text{ G rad } s^{-1}$

- w is the same in both media

$\rightarrow \eta_1 = 240\pi \ \Omega \wedge \eta_2 = \sqrt{9109.936} \angle 0.65596 \text{ rad}$

$= 95.445 \angle 0.65596 \text{ rad } \Omega$

$\therefore \Gamma = 0.818626 \angle 2.986 \text{ Rad}$

$\wedge \tau = 0.22953 \angle 0.5857 \text{ Rad}$

$\wedge SWR = 10.027$

b) ∘∘ $\bar{E}_r = \Gamma \cdot \bar{E}_i \rightarrow \bar{E}_r = 50 \cdot 0.818626 \cdot \sin(wt + 5x + 2.986)\, \bar{a}_y$

$\wedge\ \bar{H}_r = -\Gamma \cdot \bar{H}_i \Rightarrow \bar{H}_r = -\frac{50}{240\pi} \cdot 0.818626 \cdot \sin(wt + 5x + 2.986)\, \bar{a}_z$

c) ∘∘ $\alpha_2 = 6.02104$, $\beta_2 = 7.826$ Using exact expressions

$\rightarrow \bar{E}_t = \tau \cdot e^{-\alpha x} \cdot \bar{E}_i = 11.4765\, e^{-6.021x} \sin(wt - 7.8x + 0.5857)$

$\rightarrow \bar{H}_t = \tau \cdot \frac{\eta_1}{\eta_2} e^{-\alpha x} \cdot \bar{H}_i = 120.24\, e^{-6.021x} \sin(wt - 7.8x - 0.0701)$ mA/m

d) ∘∘ $P_{avg}(x) = \frac{|E|^2}{2|\eta|} \cdot e^{2\alpha x} \cdot \cos(\theta_\eta)$

$\rightarrow P_{avg}(x)\ 1 = \frac{50^2 - 40.9313^2}{2 \cdot 240\pi} = \frac{1.0936}{2} = 0.5468$

$\rightarrow P_{avg}(x)\ 2 = \frac{11.4765^2}{2 \cdot 95.\text{[illegible]}} \cdot e^{-6.021 \cdot x \cdot 2} \cdot \cos(0.65576)$

$= \frac{1.0937}{2} \cdot e^{-12.042x} = 0.5468 \cdot e^{-12.042x}$

- for real Γ, if $\Gamma > 0$: $\bar{E}_r$ will have the same direction as $\bar{E}_i$ and $\bar{H}_r$ will have an opposite direction to $\bar{H}_i$

  if $\Gamma < 0$: $\bar{E}_r$ and $\bar{E}_i$ will have opposite directions while $\bar{H}_r$ and $\bar{H}_i$ will have the same direction

- if a wave is completely reflected (travels from lossless to perfect conductor) the time (t) and distance (z) are decoupled:

  $\sin(\beta z) \cdot \sin(wt)$ or $\cos(\beta z) \cdot \cos(wt)$ or ...

- if $\Gamma = 0$, then there is no reflection (no standing wave) and SWR=1

  $\eta_2 = \eta_1$

- index of refraction, n, for a non-magnetic material:

  $n = \sqrt{\varepsilon_r} \rightarrow \Gamma = \frac{n_1 - n_2}{n_1 + n_2}$ | n_1: refraction index of medium 1 | n_2: refraction index of medium 2

  $\rightarrow SWR = \begin{cases} n_2/n_1 & \text{if } n_2 > n_1 \\ n_1/n_2 & \text{if } n_1 > n_2 \end{cases}$ larger over smaller example 1 from notes:

$\bar{E}_1 = 50\cos(377\times10^9 t - 3770x)\bar{a}_y + 25\cos(377\times10^9 t + 3770x)\bar{a}_y$

∵ first part has a $-\beta x$ phase, and the second has a $+\beta x$

→ first part is traveling in $+x$ direction while the second in $-x$

1) ∴ $50 = E_{i0} \wedge 25 = E_{r0}$, lossless

∵ $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} \wedge |\Gamma| = \frac{E_{r0}}{E_{i0}} = \frac{25}{50} = \frac{1}{2}$

→ $SWR = \frac{1.5}{0.5} = 3$

2) ∵ $\beta_1 = 3770$ ∧ lossless → $\beta = \omega\sqrt{\mu\varepsilon}$

assume non-magnetic: $\mu = \mu_0$

→ $3770 = 377\times10^9 \cdot \sqrt{\mu\cdot\varepsilon_r\varepsilon_0} = \frac{377\times10^9}{3\times10^8}\cdot\sqrt{\varepsilon_r}$ -

∴ $\boxed{\varepsilon_{r1} = 9}$

∵ $\eta_1 = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{3} = 40\pi\ \Omega$

∧ $\Gamma = \frac{1}{2} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} \to \frac{1}{2}\eta_2 = 1.5\eta_1 \to \boxed{\eta_1 = 120\pi}$

assume lossless: $\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi/\sqrt{\varepsilon_r} \to \boxed{\varepsilon_{r2} = 1}$

3) $P^{med1}_{avg} = \frac{|E_1|^2}{2|\eta_1|}\bar{a}_x = \frac{50^2-25^2}{2\cdot40\pi} = 7.46\ W/m^2$

$P_{avg,med1} = P^{incident}_{avg} - P^{reflected}_{avg} = \frac{|E_{i0}|^2}{2\eta_1}\left[1-|\Gamma|^2\right]\bar{a}_x$

$P^{transmitted}_{avg} = \frac{|E_{t0}|^2}{2\eta_2} \wedge E_{t0} = \tau E_{i0}$

→ $P^{transmitted}_{avg} = \frac{\tau^2}{2\eta_2}\cdot|E_{i0}|\bar{a}_x$

∧ $\tau = 1+\Gamma = 1.5 \to P^{transmitted}_{avg} = 7.46$

note: $|E_{max}| = |E_{i0}|(1+|\Gamma|) = |E_{i0}| + |E_{R0}|$

$|E_{min}| = |E_{i0}|(1-|\Gamma|) = |E_{i0}| - |E_{r0}|$

example 2 from notes: $\bar{E}_i = 50 \sin(wt - 5x)\,\bar{a}_y$ lossless to lossy

① ∵ medium 1 is lossless → $\eta_1 = \sqrt{\frac{\mu}{\varepsilon}} = 240\pi\ \Omega$

∧ ∵ $\beta_1 = 5 = w\sqrt{\mu\varepsilon} = w\cdot\frac{1}{c}\cdot 2 \rightarrow w = 750\ M\ rad\ s^{-1}$

→ $\eta_2 = \sqrt{\frac{jw\mu}{\sigma + jw\varepsilon}}$, w doesn't change → $\eta_2^2 = 9109.7362\angle 1.31151$ rad

→ $\eta_2 = 95.445\angle 0.65575$ rad

→ $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.818625\angle 2.9859$ rad

→ $\tau = 0.229526\angle 0.584715$ rad

→ SWR $= \frac{1 + |\Gamma|}{1 - |\Gamma|} = 10.028878$

② $\bar{E}_r = \Gamma\cdot\bar{E}_i = |\Gamma|\cdot 50\sin(wt + 5x + \theta_r)$

→ $\bar{E}_r = 41\sin(750\times10^6 t + 5x + 171.1^\circ)\,\bar{a}_y$ $\bar{a}_{kr} = -\bar{a}_x$

∧ $\bar{H}_r = \frac{1}{\eta_1}(a_{kr}\times\bar{E}_r) = -54.378\cdot\sin(750\times10^6 t + 5x + 171.1)\,\bar{a}_z$

③ $\bar{E}_t = \tau\cdot\bar{E}_i = e^{-\alpha x}\cdot 11.4763\sin(750\times10^6 t - \beta_2 x + 33.559^\circ)\,\bar{a}_y$

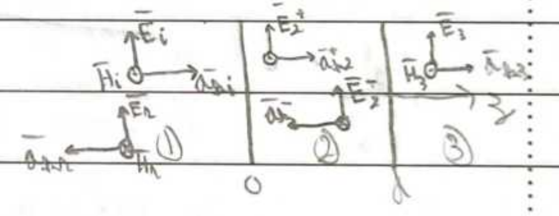
$\bar{H}_t = \frac{1}{\eta_2}(\bar{a}_{kt}\times\bar{E}_t) = e^{-\alpha x}\cdot 120.24\cdot\sin(750\times10^6 t - \beta_2 x + 33.56^\circ - 37.57^\circ)\,\bar{a}_z$

$= e^{-\alpha x}\cdot 120.24\sin(750\times10^6 t - \beta_2 x - 4.01^\circ)\,\bar{a}_z$

example 3 from notes:

$\bar{E}_1 = (E_{i0}e^{-j\beta_1 z} + E_{r0}e^{j\beta_1 z})\,\bar{a}_x$

$H_1 = \frac{1}{\eta_1}(E_{i0}\cdot e^{-j\beta_1 z} - E_{r0}\cdot e^{j\beta_1 z})\,\bar{a}_y$



$\bar{E}_2 = (E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z})\,\bar{a}_x$

$\bar{H}_2 = \frac{1}{\eta_2}(E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z})\,\bar{a}_y$

$\bar{E}_3 = E_3^+ e^{-j\beta_3 z}\,\bar{a}_x$ ∧ $\bar{H}_3 = \frac{E_3^+}{\eta_3}\cdot e^{-j\beta_3 z}\,\bar{a}_y$

boundary conditions:

$\bar{E}_1(z=0) = \bar{E}_2(z=0)$, $\bar{H}_1(z=0) = \bar{H}_2(z=0)$

$\bar{E}_2(z=d) = \bar{E}_3(z=d)$, $H_2(z=d) = H_3(z=d)$

at $z=0$:

$E_{i0} + E_{r0} = E_2^+ + E_2^-$ & $\frac{\eta_2}{\eta_1}(E_{i0} - E_{r0}) = E_2^+ - E_2^-$

at $z=d$:

$E_2^+ \cdot e^{-j\beta_2 d} + E_2^- \cdot e^{j\beta_2 d} = E_3^+ e^{-j\beta_3 d}$

& $\frac{\eta_3}{\eta_2}(E_2^+ \cdot e^{-j\beta_2 d} - E_2^- e^{j\beta_2 d}) = E_3^+ e^{-j\beta_3 d}$

$\rightarrow E_2^+ e^{-j\beta_2 d} + E_2^- e^{j\beta_2 d} = \frac{\eta_3}{\eta_2}(E_2^+ \cdot e^{-j\beta_2 d} - E_2^- e^{j\beta_2 d})$

$\rightarrow E_2^+ e^{-j\beta_2 d}\left[\frac{\eta_2 - \eta_3}{\eta_2}\right] + E_2^- e^{j\beta_2 d}\left[\frac{\eta_2 + \eta_3}{\eta_2}\right] = 0$

$\rightarrow E_2^+\left[\frac{\eta_2 - \eta_3}{\eta_2}\right] = -E_2^- e^{2j\beta_2 d}\left[\frac{\eta_2 + \eta_3}{\eta_2}\right]$

$\rightarrow E_2^+ \cdot \left[\frac{\eta_2}{\eta_2 + \eta_3} \cdot \frac{\eta_2 - \eta_3}{\eta_2}\right] = E_2^+\left[\frac{\eta_2 - \eta_3}{\eta_2 + \eta_3}\right] = -E_2^- e^{2j\beta_2 d}$

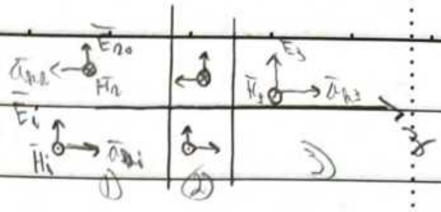
$\therefore E_2^- = E_{i0} + E_{r0} - E_2^+ \rightarrow \frac{\eta_2}{\eta_1}(E_{i0} - E_{r0}) = 2E_2^+ - E_{i0} - E_{r0}$

$\rightarrow 2E_2^+ = \left(\frac{\eta_1 + \eta_2}{\eta_1}\right)E_{i0} + \left(\frac{\eta_1 - \eta_2}{\eta_1}\right)E_{r0}$

$\frac{E_2^+}{E_2^-} = \frac{\eta_2 + \eta_3}{\eta_3 - \eta_2} \cdot e^{2j\beta_2 d} = \frac{\eta_2 + \eta_3}{\eta_3 - \eta_2} \cdot [\cos(2\beta_2 d) + j\sin(2\beta_2 d)]$

$\eta_2 \cos(2\beta_2 d) + \eta_2 j\sin(2\beta_2 d) + \eta_3 \cos(2\beta_2 d) + j\,\cancel{\beta_3}\sin(2\beta_2 d)$

Subject first exam practice Date 13/4/2021 No.



Three media:

example:

$$\bar{E}_1 = \bar{a}_x E_{io}\, e^{-j\beta_1 z} + \bar{a}_x E_{ro}\, e^{j\beta_1 z}$$

$$\bar{H}_1 = \bar{a}_y \frac{E_{io}}{\eta_1} e^{-j\beta_1 z} - \bar{a}_y \frac{E_{ro}}{\eta_1} e^{j\beta_1 z}$$

$$\bar{E}_2 = \bar{a}_x E_2^+ e^{-j\beta_2 z} + \bar{a}_x E_2^- e^{j\beta_2 z}$$

$$\bar{H}_2 = \bar{a}_y \frac{E_2^+}{\eta_2} e^{-j\beta_2 z} - \bar{a}_y \frac{E_2^-}{\eta_2} e^{j\beta_2 z}$$

$$\bar{E}_3 = \bar{a}_x E_3^+ e^{-j\beta_3 z} \quad , \quad \bar{H}_3 = \bar{a}_y \frac{E_3^+}{\eta_3} \cdot e^{-j\beta_3 z}$$

boundary conditions that must be satisfied:

$$\bar{E}_1(z=0) = \bar{E}_2(z=0) \quad \Big| \quad H_1(z=0) = H_2(z=0)$$

$$\bar{E}_2(z=d) = \bar{E}_3(z=d) \quad \Big| \quad \bar{H}_2(z=d) = \bar{H}_3(z=d)$$

four unknowns: E_{ro}, E_2^+, E_2^-, E_3^+

① $E_{io} + E_{ro} = E_2^+ + E_2^-$

② $\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2}(E_2^+ - E_2^-)$

③ $E_2^+ e^{-j\beta_2 d} + E_2^- e^{j\beta_2 d} = E_3^+ e^{-j\beta_3 d}$

④ $\frac{1}{\eta_2}\left(E_2^+ e^{-j\beta_2 d} - E_2^- e^{+j\beta_2 d}\right) = \frac{E_3^+}{\eta_3} \cdot e^{-j\beta_3 d}$

$$\rightarrow \Gamma = \frac{Z - \eta_1}{Z + \eta_1} \quad , \quad Z = \eta_2 \cdot \frac{\eta_3 \cos(\beta_2 d) + j\eta_2 \sin(\beta_2 d)}{\eta_2 \cos(\beta_2 d) + j\eta_3 \sin(\beta_2 d)}$$

for no reflection to occur, $\Gamma = 0 \rightarrow \eta_1 = Z$

$\rightarrow \eta_1 = \eta_2 \cdot \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}$ $\tan(\beta_2 d)$ must equal zero

$\rightarrow \beta_2 d = n\pi \rightarrow d = \frac{\pi n}{\beta_2} = \frac{n\lambda_2}{2}$

$n = 0, 1, 2$

$\therefore \eta_1 = \eta_2 \cdot \frac{\eta_3}{\eta_2} \rightarrow \eta_1 = \eta_3$

or $\eta_1 = \eta_2 \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}$

rearrange: $\frac{\eta_1}{\eta_2}\left[\eta_2 + j\eta_3 \tan(\beta_2 d)\right] = \eta_3 + j\eta_2 \tan(\beta_2 d)$

$\eta_1 + j\frac{\eta_1\eta_3}{\eta_2}\tan(\beta_2 d) = \eta_3 + j\eta_2 \tan(\beta_2 d)$

$\rightarrow \eta_1 - \eta_3 = j\tan(\beta_2 d)\left[\eta_2 - \frac{\eta_1\eta_3}{\eta_2}\right]$

$\rightarrow \boxed{j\tan(\beta_2 d)} = \frac{\eta_1 - \eta_3}{\eta_2 - \frac{\eta_1\eta_3}{\eta_2}} = \boxed{\frac{\eta_1\eta_2 - \eta_3\eta_2}{\eta_2^2 - \eta_1\eta_3}}$

for $j\tan(\beta_2 d) = \frac{\eta_1\eta_2 - \eta_3\eta_2}{\eta_2^2 - \eta_1\eta_3} = \infty$

$\rightarrow \beta_2 d = (2n+1)\frac{\pi}{2}$ $\wedge$ $\eta_2^2 - \eta_1\eta_3 = 0$

$d = (2n+1)\frac{\lambda_2}{4}$ $\eta_2 = \sqrt{\eta_1\eta_3}$

- coating a camera lense: all visible light should pass $f = 10^{14}$ Hz

$\rightarrow \lambda = \frac{c}{f_0}$ in freespace $= 3$ um

$\lambda_2 = \frac{2\pi}{\beta_2}$ $\wedge$ $\beta_0 = w\sqrt{\mu\varepsilon}$ assume lossless and non-magnetic

$\rightarrow \beta_1 = w\sqrt{\mu_0\varepsilon_0}$ free space

$\beta_2 = w\sqrt{\mu_0\mu_r^1\varepsilon_0\varepsilon_r}$ non-mag.

$\therefore \lambda_1 = \frac{2\pi}{w\sqrt{\mu_0\varepsilon_0}}$ $\wedge$ $\lambda_2 = \frac{2\pi}{w\sqrt{\mu_0\varepsilon_0\varepsilon_r}}$ $\frac{\lambda_2}{\lambda_1} = \frac{1}{\sqrt{\varepsilon_r}}$

$\rightarrow \boxed{\lambda_2 = \lambda_1/\sqrt{\varepsilon_r}}$

$\therefore d = \frac{\lambda_2}{4} = \frac{\lambda_1}{4\sqrt{\varepsilon_r}} = \frac{0.75}{\sqrt{\varepsilon_r}}$ um

$\frac{\frac{2\pi}{w\sqrt{\mu_0\varepsilon_0\varepsilon_r}}}{\frac{2\pi}{w\sqrt{\mu_0\varepsilon_0}}} = \frac{w\sqrt{\mu_0\varepsilon_0}}{w\sqrt{\mu_0\varepsilon_0\varepsilon_r}} = \sqrt{\frac{1}{\varepsilon_r}}$

- now choose ε_r for $\eta_2 = \sqrt{\eta_0 \eta_{lens}}$

* Radome: a dome-like structure designed to protect a radar antenna from the elements while allowing waves to pass.

example: transparent non-magnetic coating to eliminate UV reflection

$\lambda_0 = 0.4\ \mu m$ ∧ glass' $\mu_r = 1, \varepsilon_r = 6$ (given)

a) ε_r of coating: $\because \eta_2 = (\eta_1 \eta_3)^{1/2}$

assuming glass is lossless: $\eta_1 = \eta_0 \wedge \eta_3 = \eta_0 / \sqrt{\varepsilon_r}$

$\rightarrow \eta_2 = 240.876\ \Omega \rightarrow \sqrt{\varepsilon_{r2}} = 1.56 \rightarrow \varepsilon_r \approx 2.45$

$\wedge\ d = \frac{\lambda_2}{4} \rightarrow \frac{0.4\mu}{4 \cdot \sqrt{\varepsilon_r}} = d = 63.9\ nm$

b) power reflectivity: $\frac{P_{ref}}{P_{inc}} \times 100\% = \frac{E_{r0}^2}{E_{i0}^2} \times 100 = \boxed{|\Gamma|^2 \times 100\%}$

$\wedge\ \beta_2 = \omega\sqrt{\mu\varepsilon} = 2\pi/\lambda$

$\because \Gamma = \frac{Z - \eta_1}{Z + \eta_1} \wedge Z = \eta_2 \cdot \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}$

$\beta_2 = \frac{2\pi}{\lambda} \wedge \lambda_2^{red} = \frac{0.7\mu}{\sqrt{\varepsilon_r}}$

$\rightarrow Z = 211.49 + 108.93 j\ \Omega$

$\rightarrow |\Gamma| = 0.279 \rightarrow$ power ref. ≈ 7.7% for red light

Dr Majid's quizzes:

2) $\delta = \frac{1}{\alpha}$ $\quad \frac{E_0}{E_2} = 5$

$E \cdot e^{-\alpha \cdot z} \quad E \cdot e^{-2\alpha} = \frac{1}{5} E_0 \rightarrow -2\alpha = -1.609$

$\rightarrow \alpha = 0.805 \rightarrow \delta = 1.243$

$E \cdot e^{-\alpha \cdot 0} = 5 \cdot E \cdot e^{-\alpha \cdot 2} \quad e^{-\alpha \cdot 2} = \frac{1}{5} \rightarrow \frac{1}{\alpha} = 1.2427$

3) wave impedance = intrinsic impedance for TEM,

$\because \beta = 1.6 \wedge f = 2 \times 10^7\ Hz \wedge \alpha = 0 \rightarrow$ lossless

$\therefore \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = \frac{\eta_0}{\sqrt{\varepsilon_r}}$

$\because \beta = \omega\sqrt{\mu\varepsilon} \rightarrow 1.6 = 4\pi \times 10^7 \cdot \sqrt{\mu_0 \cdot \varepsilon_0 \varepsilon_r}$

$\rightarrow \varepsilon_r = \cancel{1.9444}\ 14.6 \rightarrow \eta = 98.7\ \Omega$

$\cdot \beta = 1 \rightarrow \sqrt{\varepsilon_r} = \frac{\beta \cdot c}{\omega} = 3 \rightarrow \varepsilon_r = 9$

* Sample problems: chapter 9 $\bar{D} = \epsilon \bar{E}$

[1] if $\mu = 10^{-5}$ $\wedge$ $\epsilon = 4\times10^{-9}$ & $\sigma = 0$, $\rho_v = 0$ $B = \mu \bar{H}$

1) $\because \nabla \cdot \bar{B} = 0 \rightarrow \nabla \cdot H = 0 \rightarrow k + 10 - 25 = 0 \rightarrow k = 15 \text{ A/m}^2$

2) $\because \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = \frac{-\partial E_x}{\partial y} = \frac{-\partial B}{\partial t}$ ✗

$\wedge \ \nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$ $J = \sigma \bar{E} = 0$

$\rightarrow \nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t}$ ✗

$\rightarrow \nabla \times \bar{H} = 1 = -\epsilon k \rightarrow k = -0.25 \times 10^{9} \frac{V}{ms}$

[2] 1) $E_s = \bar{a}_y 1000 e^{-j\beta x}$ $\wedge$ $H = -\bar{a}_y \frac{1000}{\eta} e^{-j\beta x}$

2) ampere's law: $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \bar{J}_s + j\omega \bar{D}_s$

$\rightarrow \nabla \times \bar{H}_s = \bar{J}_s + j\omega\epsilon \bar{E}$

$\wedge \ \nabla \times \bar{H}_s = \frac{\partial H_y}{\partial x} \bar{a}_z = + \frac{+j\beta\, 1000}{\eta} e^{-j\beta x} \bar{a}_z$

$\rightarrow \frac{j\beta \cdot 1000}{\eta} \cancel{e^{-j\beta x}} = j\omega\epsilon(\cancel{1000 \cdot e^{-j\beta x}})$

$\rightarrow \boxed{\frac{\beta}{\eta} = \omega\epsilon}$

3) faraday's law: $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -j\omega \bar{B}_s$

$\nabla \times \bar{E} = -\frac{\partial E_x}{\partial x} \bar{a}_y = -(-1000) j\beta e^{-j\beta x} \cdot \bar{a}_y$

$= -j\omega\mu_0 \bar{H}_s$

$\rightarrow \cancel{+1000} j\beta \cancel{e^{-j\beta x}} = -j\omega\mu_0 \cdot \frac{\cancel{-1000}}{\eta} \cancel{e^{-j\beta x}}$

$\rightarrow \beta = \frac{+\mu_0\omega}{\eta} \rightarrow \beta\eta = \mu_0\omega$

4) $\because$ ~~β~~ $\eta = \frac{\mu_0\omega}{\beta} = \cancel{\frac{\omega\epsilon}{\beta}}$ $\frac{\beta}{\omega\epsilon}$ $\frac{\beta^2}{\omega\epsilon} = \mu_0\omega$

$\rightarrow \beta = \omega\sqrt{\mu_0\epsilon}$

$\rightarrow \eta = \frac{\mu_0 \cancel{\omega}}{\cancel{\omega}\sqrt{\mu_0\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon}}$

$\because \epsilon_r = 4 \rightarrow \eta = \sqrt{\frac{\mu_0}{4\epsilon_0}} = 60\pi$

Subject sample homework chapter 4 Date 14/4/2021 No.

1) $\mu = 10^{-5}$ H/m , $\varepsilon = 4\times10^{-9}$ F/m , $\rho_v = 0$

a) $\because \rho_v = 0 \rightarrow \nabla\cdot\bar{D} = 0$ ✓

& $\nabla\cdot\bar{B} = 0 \rightarrow \nabla\cdot\bar{H} = 0 \rightarrow k + 10 - 25 = 0 \rightarrow k = 15$

b) $\nabla\times\bar{H} = \cancel{\sigma\bar{E}} + \varepsilon\frac{\partial \bar{E}}{\partial t} = 1 \rightarrow -k\varepsilon = 1$

$\rightarrow k = \frac{-1}{4\times10^{-9}} = \boxed{-0.25\times10^{9}}$

2) $E = C\sin(12y)\sin(az)\cos(2\times10^{10}t)\hat{a}_x$

$\rightarrow \nabla\times\bar{E} = -\frac{\partial B}{\partial t} = \frac{\partial A_x}{\partial z}a_y - \frac{\partial A_x}{\partial y}a_z$

$= C\left[a\cos(az)\cdot\sin(12y)\cdot\cos(2\times10^{10}t)\bar{a}_y - 12\cos(12y)\sin(az)\cos(2\times10^{10}t)\bar{a}_z\right]$

$\rightarrow \bar{B} = \frac{-C}{2\times10^{10}}\sin(2\times10^{10}t)\left[a\cos(az)\sin(12y)\bar{a}_y - 12\cos(12y)\sin(az)\bar{a}_z\right]$

$\rightarrow \bar{H} = \frac{C}{2\times10^{10}\cdot\mu_0}\cdot$ ''

$\because \nabla\times\bar{H} = \cancelto{0}{\sigma\bar{E}} + \varepsilon_0\frac{\partial\bar{E}}{\partial t}$

$\rightarrow \nabla\times\bar{H} = -2\times10^{10}\sin(2\times10^{10}t)\cdot C\sin(12y)\sin(az)\cdot\varepsilon_0$

& $\nabla\times\bar{H} = \frac{-C}{2\times10^{10}\mu_0}\sin(2\times10^{10}t)\cdot\left[a^2\sin(az)\cdot\sin(12y)\bar{a}_x + 12^2\sin(12y)\cdot\sin(az)\bar{a}_x\right]$

$\therefore \frac{\cancel{-C}}{\mu_0 2\times10^{10}}\cancel{\sin(2\times10^{10}t)}\cdot\cancel{\sin(az)}\cdot\cancel{\sin(12y)}\cdot\left[a^2 + 144\right]$

$= \cancel{-2\times10^{10}}\cdot\cancel{C}\cdot\cancel{\sin(12y)}\cdot\cancel{\sin(az)}\cdot\cancel{\sin(2\times10^{10}t)}\cdot\varepsilon_0$

$\rightarrow 4\times10^{20}\cdot\varepsilon_0\cdot\mu_0 = \left[a^2 + 144\right] \rightarrow a = 65.577$

3) $\because E = \frac{10^6}{\rho}\cos(10^9 t)\bar{a}_\rho$ V/m

$\rightarrow J_c = \sigma\bar{E} = 10^{-5}\cdot\bar{E} = \frac{10}{\rho}\cos(10^9 t)(\bar{a}_\rho)$ A/m²

→ double integral with respect to ρ:

$$\int_0^{2\pi}\int_0^{0.4}\frac{10}{\cancel{\rho}}\cdot\cos(10^9 t)\cancel{\rho}\,d\phi\,dz = 8\pi\cos(10^9 t)\ \text{A}$$

$\because J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial \bar{E}}{\partial t} = \frac{-1}{\rho} \sin(10^3 t)\bar{a}_\rho$ A/m^2

$\rightarrow I_d = -\int_0^{2\pi}\int_0^{0.4} \frac{\rho}{\rho} \sin(10^5 t) \cdot d\phi\, dz = -0.8\pi \sin(10^5 t)$ A

$\frac{I_d}{I_c} = 0.1 = \frac{1}{\text{loss tangent}} = \frac{\omega\varepsilon}{\sigma} = \frac{10^{-11} \cdot 10^5}{10^{-6}} = 0.1$

a) lossless:

a) $\beta = \omega\sqrt{\mu\varepsilon} = 294.18$ rad/m

b) $\lambda = \frac{2\pi}{\beta} = 21.286$ mm

c) $u = \frac{\omega}{\beta} = 1.9956 \times 10^8$ m/s

d) $\eta = \frac{120\pi}{\sqrt{2.26}} = 250.79\ \Omega$

e) $|H| = \frac{|E|}{\eta} \approx 2$ A/m

Chapter 9 quiz:

1) $\because$ free-space $\rightarrow \sigma = 0 \rightarrow J_c = 0 \quad \because J_c = \sigma\bar{E}$

$\wedge \quad J_d = \varepsilon \frac{\partial \bar{E}}{\partial t} = -\varepsilon \cdot 4.9 \cdot 1.8\times10^9 \times\pi \cdot \sin(1.8\times10^9\pi t - \alpha x - 2.5\alpha z)\bar{a}_y$

$\rightarrow J_d = -0.245 \sin(1.8\times10^9\pi t - \alpha x - 2.5\alpha z)\bar{a}_y$

2)

$\bar{E}_s = 4.9\, e^{-j\alpha x} e^{-j2.5\alpha z} \bar{a}_y$

3) $\because \nabla^2\bar{E}_s - \gamma^2\bar{E}_s = 0 \quad \wedge\ \alpha + j\beta = \gamma,\ \alpha = 0$ in free-space

$\rightarrow \nabla^2\bar{E}_s + \beta^2\bar{E}_s = 0$

$\nabla^2 \bar{E}_{sy} = \frac{4.9}{-j\alpha} e^{-j\alpha x} e^{-j\alpha z} + \frac{j4.9}{\alpha} e^{-j\alpha x} e^{-j\alpha z}$

$\rightarrow \nabla^2\bar{E}_{sy} + \beta^2\bar{E}_s = 0$

$\rightarrow 2\cdot\frac{\cancel{4.9}j}{\alpha} \cancel{e^{-j\alpha x}} \cancel{e^{-j\alpha z}} = -\beta^2 \cdot \cancel{4.9} \cdot \cancel{e^{-j\alpha x}} \cancel{e^{-j\alpha z}}$

$\wedge\ \beta = 6\pi$

$\rightarrow 2 = -\alpha \cdot \beta^2 \rightarrow \alpha = \frac{2}{1}$

3) $\bar{\nabla}^2 \cdot \bar{E}_s = \nabla^2 \cdot E_y = \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 F_z}{\partial z^2}$

& $\nabla^2 \cdot E_y = 4.9\left[-a^2 e^{-jax} e^{-j2.5az} - (2.5)^2 a^2 e^{-jax} e^{-j2.5az}\right]$

$\rightarrow -\cancel{4.9 \cdot e^{-jax} e^{-j2.5az}}\left[a^2 + (2.5)^2 a^2\right] = -\beta^2\left[\cancel{4.9 e^{-jax} e^{-j2.5az}}\right]$

$\rightarrow a^2 + (2.5)^2 a^2 = \beta^2 \rightarrow a^2\left[1+(2.5)^2\right] = 36\pi^2$

$\rightarrow a \approx 7$ rad/m

first exam 2018:

1) a) $\overline{E \times H}_{avg} = P_{avg}(z) = \frac{1}{T}\int_0^T P(z,t)\,dt = \frac{E_0^2}{2\eta} e^{-az} \cos(\theta_\eta)\bar{a}_z$

& $a = 0 \rightarrow 1.1 = \frac{|E_{i0}|^2}{2|\eta|} \cdot \bar{a}_z$

$\because \eta_{lossless} = \frac{120\pi}{\sqrt{\epsilon_r}}$ non-mag

$\because \beta = 2\pi = \omega\sqrt{\mu_0 \epsilon} \rightarrow \sqrt{\epsilon_r} = \frac{2\pi \cdot c}{\omega}$

$\rightarrow \epsilon_r = 1 \rightarrow \eta = 120\pi$

$\therefore E_{i0} = 28.8$ V/m

b) $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ $\quad P_{reflected} = 0.1\ W/m^2$

$\rightarrow 0.1 = \frac{|E_{r0}|^2}{2|\eta_1|} \rightarrow E_{r0} = 8.68$ V/m

$\therefore |\Gamma| = \frac{|E_{r0}|}{|E_{i0}|} = \frac{8.68}{28.8} \approx 0.3$

c) $\because \lambda = \frac{2\pi}{\beta} \rightarrow \lambda_1 = \frac{2\pi}{2\pi} = 1\,m$

β_2: $\because \frac{|E_{t0}|}{|E_{i0}|} = \tau$ & $\tau = 1 + \Gamma = 1.3$

$\because \tau = \frac{2\eta_2}{\eta_1 + \eta_2} \rightarrow \eta_2(2-\tau) = \tau\eta_1$ & $\eta_1 = 120\pi$

$\rightarrow \eta_2 = 66.528\ \Omega$

$\because \eta_2 = \frac{120\pi}{\sqrt{\epsilon_{r2}}} \rightarrow \epsilon_{r2} = 32.111$

$\rightarrow \beta_2 = 35.605 \rightarrow \lambda_2 = \frac{2\pi}{\beta_2} = 17.65\,cm$

$\because \frac{P_{ref}}{P_{inc}} = |\Gamma|^2$ & $P_{ref} = P_{inc} - P_{trans} = 0.1\ W/m^2$

$\rightarrow |\Gamma|^2 = \frac{0.1}{1.1} \rightarrow |\Gamma| = 0.3015$

$\because \eta_1 = \eta_0$ & $\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r2}}}$, unless $\varepsilon_{r2} = 1$, $\eta_2 < \eta_1$

$\therefore \boxed{\Gamma = -0.3015}$

$\sqrt{\varepsilon_{r2}} > \sqrt{1}$

$\because n_2 > n_1$

d) $\lambda_1 = \frac{2\pi}{\beta_1} = 1\ m$

$\lambda_2 = \frac{2\pi}{\beta_2}$, $\beta_2 = \frac{\omega \cdot \sqrt{\varepsilon_{r2}}}{c}$ $\because SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{n_2}{n_1}$

$\rightarrow \frac{1.3015}{0.6985} = \frac{\sqrt{\varepsilon_{r2}}}{1} \rightarrow \varepsilon_{r2} = 3.47181$

$\therefore \lambda_2 = \frac{2\pi c}{\omega(\sqrt{\varepsilon_{r2}})} = \frac{\lambda_1}{\sqrt{\varepsilon_{r2}}} = 53.69\ cm$

$\boxed{2}$ a) $\because H \cancel{= 10}\, e^{-50x} \cos(\cancel{\omega t} - \beta x) \cancel{/\eta_0}$

$\cancel{\eta_0 = \frac{120\pi}{\sqrt{4}} = 40\pi}$ $\cancel{, \omega = 5\pi \cdot 10^9}$

$\cancel{\&\ \beta = 5\pi \cdot 10^9 \sqrt{\mu_0 \cdot 4\varepsilon_0} = 50\pi}$

$\because \alpha = 50 \rightarrow$ lossy, use exact

$\omega = 5\pi \times 10^9$ rad/s

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$\rightarrow$ ~~loss tangent = 61.78446~~ / $\rightarrow$ ~~ϕ = 25.9436~~

$$\left(\frac{50}{\omega}\right)^2 = \frac{\mu\varepsilon}{2}\left[\sqrt{1+(\tan\theta)^2} - 1\right]$$

$\rightarrow \boxed{\tan\theta = 0.6681} = \frac{\sigma}{\omega\varepsilon} \rightarrow \sigma =$ ~~[illegible]~~

$\because \tan\theta = \tan(2\theta_\eta) \rightarrow \theta_\eta = 0.29497$

$\because \tan(\theta_\eta) = \frac{\alpha}{\beta} \Rightarrow \beta = 164.844$

$\therefore \eta_p = \frac{j\omega\mu_0}{\alpha + j\beta} = 114.589 \angle 0.2945$ rad Ω

$\therefore \bar{E} = -\eta(\bar{a}_{n} \times \bar{H}) \ \& \ \bar{a}_{n} \times \bar{H} = -\bar{a}_y$

$\rightarrow \boxed{E = 1145.89\, e^{-50x} \cos(5\pi\times10^{4} - 164.8442x + 0.2945)\ V/m}$

b) ~~$\because \frac{|E_0|^2}{2|\eta|} \cdot e^{-50x} \cdot \cos(0.2945) = 2589.883\ W/m^2$~~

~~$\rightarrow P_{avg} = P_{avg} \cdot S \ \& \ S = \frac{\phi}{\cancel{2\pi}} \cdot \pi r^2 = \frac{\phi}{2} r^2$~~

~~$\rightarrow 15.74 = 2589.883 \cdot \frac{\phi}{2} \cdot (0.11)^2 \rightarrow \phi = 1.0045\ rad$~~

~~$\rightarrow \phi = 57.55°$~~

$\because P_{diss} = (P_{avg}^{in} - P_{avg}^{out}) \cdot S \quad \because P_{avg} = \int_S P_{avg} \cdot dS$

$\& \ P_{avg}^{in} = \frac{|E_0|^2}{2|\eta|} \cdot \cos\theta_\eta \quad \because$ still at $x = 0$

$P_{avg}^{out} = \frac{|E_0|^2}{2|\eta|} \cdot e$

$\rightarrow P_{diss} = 15.74 = (5482.782 - \quad) \cdot \left[\frac{\phi}{2\pi} \cdot \pi r^2\right]$

$\rightarrow 15.74 = 4259.4 \cdot \frac{\phi}{2} \cdot (0.11)^2$

$\rightarrow \phi = 0.6108\ rad \approx 34.99°$

$\boxed{3}$ a) $\because SWR = \frac{|E_1|_{max}}{|E_1|_{min}} = \frac{13}{7} = \frac{1+|\Gamma|}{1-|\Gamma|} \rightarrow \left(\frac{13}{7}+1\right)\cdot|\Gamma| = \frac{13}{7} - 1$

$\rightarrow |\Gamma| = 0.3$

$\because \eta_2 > \eta_1 \rightarrow \Gamma > 0 \rightarrow \boxed{\Gamma = 0.3}$

b) $\because \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \rightarrow \eta_2 = \frac{\eta_1(1+\Gamma)}{(1-\Gamma)} \rightarrow \eta_2 \approx 700.264\ \Omega$

$\because \Gamma$ complex $\eta_2 + \Gamma\eta_1 = \eta_2 - \eta_1$

$\rightarrow (\Gamma - 1)\eta_2 = -\eta_1(\Gamma + 1)$

$\rightarrow \eta_2 = \frac{\eta_1(\Gamma+1)}{1-\Gamma}$

$\because$ E_{max} occurs when $\cos = 1$

$\because$ $\bar{E}_1 = E_{i0}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right)$

$\rightarrow \bar{E}_1 = E_{i0}\, e^{-j\beta_1 z}\left(1 + \Gamma \boxed{e^{j2\beta_1 z}}\right)$ but $\Gamma = |\Gamma| \cdot e^{j\theta_\Gamma}$

$\cos(\theta_\Gamma + 2\beta_1 z) = 1$ when $\theta_\Gamma + 2\beta_1 z = 2n\pi$, $n = 0, 1, 2 \ldots$

$\rightarrow \theta_\Gamma = -2\beta_1 z$ & max at $-60.6\ cm = z$

& $\beta_1 = \frac{2\pi}{\lambda_1}$ & $\lambda_1 = 4.75\ cm$

$\rightarrow \theta_\Gamma = -2 \cdot \frac{2\pi}{3} \cdot (-0.606) = 2.53841\ rad$

$\therefore \boxed{\Gamma = 0.3\angle 2.53841\ rad}$

b) $\because \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma \rightarrow \eta_2 = \eta_1 \frac{(1+\Gamma)}{(1-\Gamma)} = 231.215\angle 0.358\ rad$

$\mu = \mu_0$ non-mag

$\because \eta = \frac{j w \mu}{\alpha + j\beta} \rightarrow \eta = \frac{j \cdot w \cdot \mu_0}{\alpha + j\beta}$

$w = \frac{\beta_1}{\sqrt{\mu\varepsilon}} = 2\pi \times 10^8\ rad\ s^{-1}$

$\rightarrow 231.215\angle 0.358 = \frac{j \cdot 2\pi \times 10^8 \cdot 4\pi \times 10^{-7}}{\alpha + j\beta}$

$\rightarrow \alpha_2 + j\beta_2 = 1.1966 + 3.1984j$

$\therefore \lambda_2 = \frac{2\pi}{3.1984} = 1.9645\ m$

c) $\because \bar{E}_1 = E_{i0}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right)$ ~~at $z = 0$, $E_1 = E_{i0}$~~

at $z = 0$ $\bar{E}_1 = E_{i0} \cdot (1 + \Gamma)$

~~$\because E_1 = 13$ at $z = -60.6\ cm \rightarrow E_{i0}\left(e^{-j\beta_1(-0.606)} + \Gamma e^{j\beta_1(-0.606)}\right) = 13$~~

~~$\therefore E_{i0} \cdot e^{-j\beta_1(-0.606)}\left[1 + \Gamma\right] = 13$~~

~~$E_{i0}\left[\cos(-\beta_1 0.606) - j\sin(-\beta_1 0.606)\right] = \frac{13}{1+\Gamma}$~~

$\because |E_1(z=0)| = |E_{t0}|$ & $|E_{i0}| = \frac{|E_1|_{max}}{1+|\Gamma|} = \frac{|E_1|_{min}}{1-|\Gamma|}$

$\rightarrow |E_{i0}| = 10 \rightarrow \tau \cdot |E_{i0}| = |E_{t0}|$

$\rightarrow |E_{t0}| = |1+\Gamma| \cdot |E_{i0}| = 7.72\ V/m = |E_1\ z=0|$

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① a) $\because \frac{\varepsilon''}{\varepsilon'} = 0.1 = \tan\theta \rightarrow$ material is a good dielectric

$\rightarrow \alpha = \frac{\sigma}{2} \cdot \sqrt{\frac{\mu}{\varepsilon}}$, $\mu = \mu_0 \wedge \varepsilon = 3\varepsilon_0$

$\because \varepsilon'' = \frac{\sigma}{\omega} \rightarrow \sigma = \frac{\varepsilon''}{\varepsilon'} \cdot 3 \cdot \varepsilon_0 \cdot \omega$

$\rightarrow \sigma = 3.98\ mS/m^2 \rightarrow \alpha = 0.433013$

$3\ N_p = \alpha \cdot d \quad \because \alpha = N_p/m \wedge d = m$

$\rightarrow d = 6.93m$

b) $P_0 = \frac{1}{2} P_d \quad P_0 = |E_0|^2 \wedge P_d = |E_d|^2 \quad \because E_d = E_0 \cdot e^{-\alpha d}$

$\rightarrow \frac{1}{2}|E_0|^2 = |E_0|^2 \cdot e^{-2\alpha d} \rightarrow \ln(0.5) = -2\alpha d$

$\rightarrow d = 0.8\ m$

c) $\beta \cdot d = \theta \rightarrow \beta \cdot d = 250^\circ = \frac{25}{18}\pi$

$\because \beta = \omega\sqrt{\mu\varepsilon} = 8.66 \rightarrow d = 0.504\ m$

② $\eta = 60\pi\angle 30^\circ \wedge 1/\alpha = 1.44$

$\because \theta_\eta = 30 \wedge$ loss tangent $= \tan(2\theta_\eta) = \sqrt{3}$ lossy

$\because \eta = \frac{j\omega\mu}{\gamma}$ ~~$\wedge \gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu\sigma - \omega^2\mu\varepsilon$~~

~~$\rightarrow \alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$~~

~~$\wedge j2\alpha\beta = j\omega\mu\sigma$~~

$\because \boxed{\alpha = \frac{1}{1.44}} \wedge \tan(\theta_\eta) = \frac{\alpha}{\beta} \rightarrow \boxed{\beta = 1.2028}$ rad/m

$\rightarrow \eta \cdot \gamma = j\omega\mu \rightarrow \mu\omega = 261.799 \rightarrow \omega = 2.0833 \times 10^8$ rad s^{-1}

$\boxed{\frac{\sigma}{\omega\varepsilon} = \sqrt{3}} \wedge \beta^2 - \alpha^2 = \omega^2\mu\varepsilon$ — ①

$2\alpha\beta = \omega\mu\sigma$ — ②

from ① : $\boxed{\varepsilon_r \approx 2}$ | $\lambda = \frac{2\pi}{\beta} = 5.224m$

from ② : $\boxed{\sigma = 6.38 \times 10^{-3}}$ S/m | $u_p = \frac{\omega}{\beta} = 1.7321 \times 10^8$ m/s

$\rightarrow$ check : $\frac{6.38 \times 10^{-3}}{2.0833 \times 10^8 \times \frac{10^{-9}}{36\pi} \cdot 2}$

NOTEBOOK

B) $\because \varepsilon_c = 2\varepsilon_0 - j\varepsilon_0 \rightarrow \tan\theta = \frac{1}{2}$, lossy, non-magnetic

$$\alpha = w\sqrt{\frac{\mu\varepsilon}{2}\cdot\left[\sqrt{1+\left(\frac{\sigma}{w\varepsilon}\right)^2} - 1\right]} \quad \because \frac{1}{2} = \frac{\sigma}{w\varepsilon} \wedge \varepsilon = 2\varepsilon_0$$

$\rightarrow \sigma =$

$$\rightarrow \alpha = w\sqrt{\frac{2}{(3\times10^8)^2\cdot 2}\cdot\left[\sqrt{1+0.25} - 1\right]} = 17.9888 \text{ Np/m}$$

$$\wedge \beta = w\sqrt{\frac{1}{9\times10^{16}}\cdot\left[\frac{\sqrt{5}}{2}+1\right]} = 76.202 \text{ Rad/m}$$

$\therefore E_1 = 100\cdot e^{-\alpha\cdot 0.01} = 83.5364$ V/m

$\wedge \theta = \beta\cdot 0.01 = 0.76202$ rad

$$P_{diss} = \left[P^{in}_{avg} - P^{out}_{avg}\right]\cdot S = \frac{|E_0|^2}{2|\eta|}\cos(\theta_\eta)\cdot\left[1 - e^{-2\cdot\alpha\cdot 0.01}\right]$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{63551.02263}\angle 0.46365 \text{ rad}$$

$$\rightarrow P_{diss} = \frac{|100|^2}{2\cdot 252.11}\cdot\cos(0.231825)\cdot\left[1 - e^{-17.9888\cdot 2\cdot 0.01}\right]\cdot S$$

$\rightarrow P_{diss} = 5.83245\cdot S \wedge S = 10$ cm & 8 cm

$\rightarrow P_{diss} = 0.04666$ W

$$P_{diss} = \frac{|E_0|^2}{2|\eta|}\cdot\cos(\theta_\eta)\cdot\left[1 - e^{-2\alpha z}\right]\cdot S \text{ (Area)}$$

① ∵ $SWR = 5 = \frac{1+|\Gamma|}{1-|\Gamma|} \rightarrow |\Gamma| = \frac{5-1}{1+5} = \frac{4}{6} = \frac{2}{3}$

∵ $\eta_2 < \eta_1 \rightarrow \Gamma = \frac{-2}{3}$

$\rightarrow \tau = 1/3$, ∵ $SWR = \frac{\eta_2}{\eta_1} = n_2 = \sqrt{\varepsilon_{r_2}}$

$\rightarrow \varepsilon_{r2} = 25$ λ $\lambda_2 = \frac{2\pi}{\beta_2}$, $\beta_2 = \omega\sqrt{\mu_0 \cdot \varepsilon_r \varepsilon_0}$

∵ $\lambda_1 = 3m \rightarrow \beta_1 = \frac{2}{3}\pi = \omega\sqrt{\mu_0 \varepsilon_0} \rightarrow \omega = 2\pi \times 10^8 \, rad\, s^{-1}$

$\rightarrow \beta_2 = \beta_1 \cdot \sqrt{\varepsilon_r} = \frac{10}{3}\pi \rightarrow \boxed{\lambda_2 = 0.6 m}$

② ∵ loss tangent = 0.05 → good dielectric

Γ ~~SWR~~ $= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, $\eta_1 = 120\pi$, $\eta_2 = \frac{120\pi}{\sqrt{5}} = 168.6\,\Omega$

$\rightarrow \Gamma = -0.381966$

$\rightarrow SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \boxed{2.23607}$

~~$|P_{avg}| = 2|P_{avg\,d}|$~~ →

∵ $P^{d}_{avg} = 5 W/m^2$, $P^{o}_{avg} =$

~~$|P^{o}_{avg}| = |\tau \cdot E_{io}|^2$, $P^{d}_{avg} = |\tau \cdot E_{io} \cdot e^{-\alpha d}|^2 = \frac{1}{2}|\tau E_{io}|^2$~~

~~$\rightarrow e^{-2\alpha d} = 8/2 \rightarrow -2\alpha d =$~~

~~$\rightarrow d = \frac{[illegible]}{2\alpha}$~~ , $\alpha = \frac{\sigma}{2}\sqrt{\mu/\varepsilon}$

$\rightarrow \boxed{\alpha = 0.02341605 \, Np/m}$

~~$\rightarrow d = 14.8 m$~~

reflected power

$|P^{o}_{avg}| = |\boxed{1 - |\Gamma|^2} \cdot P^{i}_{avg}| = 8.541 \, W/m^2$

∴ $e^{-2\alpha d} = \frac{8.541}{5} \rightarrow +2\alpha d = 0.53544$

$\rightarrow d = \frac{0.53544}{2 \cdot 0.02341605} \approx 11.4332$

$\boxed{3}$ $\because$ $\bar{E}_1 = 100 e^{-j10z}\bar{a}_x + 50 e^{j10z} e^{j\pi/4} \bar{a}_x$

$100 = E_{i0}$, $50 = E_{r0}$ $e^{j\pi/4} \rightarrow \boxed{\frac{\pi}{4} = \theta_r}$

$\rightarrow \Gamma = \frac{1}{2}\angle\frac{\pi}{4} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

$\rightarrow \eta_2 = \eta_1 \frac{\Gamma+1}{1-\Gamma} = 1001.444\angle\overset{0.478}{\cancel{2.7134}}$ $\quad \eta_2\Gamma + \eta_1\Gamma = \eta_2 - \eta_1$

$\eta_2\gamma_2 = j\omega\mu$ $\quad \eta_2(\Gamma-1) = -\eta_1(\Gamma+1)$

$\rightarrow \gamma_2 = \frac{j\omega\mu}{\eta_2}$ & $\mu = \mu_0$ (non-mag) $\quad \eta_2 = \eta_1\frac{\Gamma+1}{1-\Gamma}$

& $\omega = \frac{\beta}{\sqrt{\mu_0\varepsilon_0}} = 30\times10^8$ rad s^{-1}

$\rightarrow \gamma_2 = \alpha_2 = +1.56195$ & $\beta_2 = +3.425$ rad m^{-1}

$\therefore \lambda_2 = \frac{2\pi}{3.425} = 1.83449$ m

~~Emin at $\theta_r + 2\beta_1 z = (2n+1)\frac{\pi}{2}$, $n = 0, 1, \lambda$~~

~~$\rightarrow \theta_r = \frac{\pi}{2} - 2\beta_1 z \rightarrow z = \frac{\frac{\pi}{2} - \theta_r}{2\beta_1}$~~

~~$\therefore z = 0.057 = 5.714$ cm~~

Emin at $\cos(\theta_r + 2\beta_1 z) = -1$

$\rightarrow \theta_r + 2\beta_1 z =$ odd multiples of $-\pi$

$\rightarrow z = \frac{-\pi - \theta_r}{2\beta_1} = -17.85$ cm

$\boxed{1}$ $\because$ $\beta_1 = 1 \rightarrow \omega\sqrt{\mu_0\varepsilon_0} = 1 \rightarrow \boxed{\omega = 3\times10^8 \text{ rad s}^{-1}}$

$\because \eta_1 = 120\pi$ & $E_{r4} = -\eta_1(a_{n} \times \bar{H}_{r4})$

$\bar{a}_y \cdot \frac{-120\pi}{12\pi}(\cos(\omega t + z))$

$\rightarrow SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$ & $|\Gamma| = \frac{10}{30} = \frac{1}{3}$

$\rightarrow SWR = 2$ $\quad$ or $SWR = \frac{|E|_{max}}{|E|_{min}} = \frac{|E_{i0}| + |E_{r0}|}{|E_{i0}| - |E_{r0}|}$

~~$\because \eta_2 = \frac{\Gamma+1}{1-\Gamma}\cdot\eta_1 = SWR\cdot\eta_1 = 240\pi$~~

~~$\wedge\ \eta_2 = \frac{j\omega\mu}{\gamma} \rightarrow \gamma = \frac{j\omega\mu}{\eta_2} = 2i$~~

~~$\because \eta_2 < \eta_1$~~ $\because \Gamma = \frac{E_{ro}}{E_{io}}$, $E_{io} = 30$, $E_{ro} = -10$

$\rightarrow \Gamma = \frac{-1}{3} \quad \therefore \eta_2 = \eta_1 \frac{1+\Gamma}{1-\Gamma} = 60\pi$

$\therefore \gamma = \frac{j\omega\mu}{60\pi} = 8j$

$\boxed{2}$ $\because P_{avg} = \frac{|E_0|^2}{2|\eta|}\cdot\cos(\theta_\eta)\cdot\left(1-e^{-2\alpha z}\right)\cdot S$

$\because \varepsilon_c = 16\varepsilon_0 - j\varepsilon_0 \rightarrow \frac{\sigma}{\omega\varepsilon} = \frac{1}{16} << 1$ → good dielectric

$\therefore \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$, $\sigma = \frac{\omega\varepsilon}{16} =$

$\rightarrow \alpha = 6.545$ Np/m

$\wedge\ \eta = \sqrt{\frac{\mu}{\varepsilon}} = 30\pi \quad \therefore \theta_\eta = 0$

~~$\therefore 0.1 = \frac{10^4}{60\pi}\cdot\cos(0)\cdot\left(1-e^{-\alpha z}\right)\cdot 0.1\cdot 0.1$~~

~~$\rightarrow 60\pi = 1\cdot\left(1-e^{-\alpha z}\right) \rightarrow \alpha z = 0.208865$~~

$\rightarrow z = 0.0319 = 3.2$ cm

$P^{z}_{avg} = \left(\frac{|E_0|^2}{2|\eta|}\cdot\cos(\theta_\eta)\cdot e^{-2\alpha z}\right)\cdot S$

$P_{diss} = P^{0}_{avg} - P^{z}_{avg} = \frac{|E_0|^2}{2|\eta|}\cdot\cos(\theta_\eta)\cdot\left[1-e^{-2\alpha z}\right]\cdot S$

$\rightarrow \frac{|100|^2}{60\pi}\cdot\left[1-e^{-2\alpha z}\right]\cdot S = 0.1$

$\rightarrow \left[1-e^{-2\alpha z}\right] = \frac{0.1}{S}\cdot\frac{60\pi}{10^4}$

$\rightarrow -2\alpha z = -0.208865$

$\rightarrow z = 1.6$ cm

$\boxed{3}$ non-mag

a) $\alpha_2 = 1.2$ Np/m, $\beta_2 = 3.2$ rad/m, $\omega = 2\pi 10^8$ rad s^{-1}

∵ $\eta_2 = \frac{j\omega\mu}{\gamma}$ → $\eta_2 = 231.03\angle 0.35877$ rad Ω

$\eta_2 < \eta_1$

∵ $E = -\eta(\bar{a}_n \times \bar{H}) = (\eta) 3.34 \cdot \bar{a}_x \cdot e^{-1.2z} \cos()\pi$

→ $\bar{E}_t = 771.6402 \cdot e^{-1.2z} \cdot \cos(2\pi\times10^8 t - 3.2z + 0.35877)\bar{a}_x$ mV/m

∵ $T = \frac{E_{t0}}{E_{i0}}$ → $E_{i0} = 771.6402 / T$ & $T = \frac{2\eta_2}{\eta_1+\eta_2}$

→ $E_{i0} = \cancel{596.4}\ 1000\angle -0.22289$ rad

→ $\bar{E}_i = 1000 \cdot \cos(2\pi\times10^8 t - \beta_1 z - 0.22289 + 0.35877)\bar{a}_x$

& $\beta_1 = \omega / c =$

→ $\bar{E}_i = 1000 \cdot \cos(2\pi\times10^8 t - \frac{2}{3}\pi \cdot z + 0.13588)\bar{a}_x$ mV/m

b) ∵ $E_{max} = \theta_r + 2\beta_1 z = -2n\pi$, $n = 0, 1, 2 \ldots$

$\theta_r = 2.5382$ & $\beta_1 = \frac{2}{3}\pi$

∴ $z = \frac{-2.5382}{4/3\pi} = -0.606$ m $= -60.6$ cm

$\boxed{4}$ $\bar{E} = E_0 \cdot e^{-\alpha z} \cdot \cos(\omega t - \beta z)$

∵ $\cancel{E_0}$ $|E|_{z=0} = 110$ & $|E|_{z=40m} = 41V = E_0 \cdot e^{-\alpha 40}$

→ $\cancel{\frac{110}{41}}$ $\frac{41}{110} = e^{-\alpha \cdot 40}$ → $\alpha = 0.024673$ Np/m

$\lambda = 27.3m \cdot 2 = \cancel{\beta}54.6 m = \frac{2\pi}{\beta}$ → $\beta = 0.115077$ rad/m

∵ $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

→ $\alpha^2 - \beta^2 + j2\alpha\beta = -\omega^2\mu\varepsilon + j\omega\mu\sigma$

→ $\beta^2 - \alpha^2 = \omega^2\mu\varepsilon$ — ① $\omega \approx 2\pi \cdot \frac{1}{0.5\mu}$

& $j2\alpha\beta = j\omega\mu\sigma$ — ②

from ①: $\varepsilon_r = \frac{\beta^2 - \alpha^2}{\omega^2\mu_0 \cdot \varepsilon_0} = \boxed{7.2005}$

from ②: $\sigma = 1/\frac{\omega\mu_0}{2\alpha\beta} = 1/2780.86 = \boxed{3.6\times10^{-4}\ S/m}$

quiz 2 redone:

1) $\sigma = 4\ S/m,\ \varepsilon_r = 81,\ \mu_r = 1$ $\because \frac{\sigma}{\omega\varepsilon} = \frac{4}{2\pi\times10^3 \cdot 81\varepsilon_0} >> 1$ → conductor

→ $\alpha = \beta = \sqrt{\pi f \mu \sigma} = \frac{\pi}{25}$ Np/m & Rad/m

$\because \lambda = \frac{2\pi}{\beta}$ → $\lambda_0 = 50$. → antenna $= 25$ m

2) $\because$ sensitivity $= 1\mu V$ → $|E_s| = E_0 \cdot e^{-\alpha z} = 1\mu V$

→ $e^{-\alpha z} = \frac{1\mu}{2840}$ ⟹ $-\alpha z = -21.7671$

→ $\boxed{z = 173.217\ m}$

3) $\because E(t) = 2840\, e^{-\alpha z} \cos(2\pi\times10^3 t - \beta z)\, \overline{a_y}$

& $\overline{H} = \frac{1}{\eta}(a_{k} \times \overline{E})$ & $\cancel{\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{40}{3}\pi}$

$\cancel{= \frac{-3}{40\pi} \cdot 2840 \cdot e^{-\alpha z} \cos(2\pi\times10^3 - \beta z)\, \overline{a_x}}$

$\cancel{= -67.8\, e^{-\frac{\pi}{25}z} \cdot \cos(2\pi\times10^3 - \frac{\pi}{25}z)\, \overline{a_x}}$

$\eta = (1+j)\frac{\alpha}{\sigma} = 0.044.. \angle \boxed{\frac{\pi}{4}}$

→ $\overline{H} = -63922.6 \cdot e^{-\frac{\pi}{25}z} \cdot \cos(2\pi\times10^3 t - \frac{\pi}{25} z - \frac{\pi}{4})\, \overline{a_x}$

$\approx -64 \cdot e^{-0.1257z} \cdot \cos(2\pi\times10^3 t - 0.1257z - \frac{\pi}{4})\, \overline{a_x}$

$\boxed{1}$ $\delta,\ \mu_r,\ \varepsilon_r,\ \sigma$

$\eta = 80.2 \angle 22.5°$ & $\lambda = 24$ mm

→ $\beta = \frac{2\pi}{\lambda} = 261.799$ rad/s

& $\because \tan(\theta_\eta) = \frac{\alpha}{\beta}$ → $\alpha = 108.441$ Np/m

→ $\boxed{\delta = 1/\alpha = 9.2216\ mm}$

& $|\gamma\eta| = |j\omega\mu|$

$\because \beta^2 - \alpha^2 = \omega^2 \mu \varepsilon$ $22726.1881 = 14212.23\,\mu_r$

& $2\alpha\beta = \omega\mu\sigma$ → $\boxed{\mu_r = 1.6}$

$\boxed{\varepsilon_r = 24.97 \approx 25}$

$\boxed{\sigma = 2.5\ S/m}$

$\boxed{2}$ $\because \theta_\eta = \frac{\pi}{10} \rightarrow \tan(\theta_\eta) = \frac{\alpha}{\beta}$ & $\alpha = 20 \rightarrow \beta = 61.554$ rad/m

$\because \tan(2\theta_\eta) = \tan(\theta)$ loss tangent

$= 0.72654 \not\gg 1 \;\; \not\ll 1$ hence lossy

not good conductor nor good dielectric

$\because |\eta| = \frac{|E_0|}{|H_0|} = \frac{10}{0.05} = 200\ \Omega \rightarrow \eta = 200 \angle \frac{\pi}{10}$ rad

$\because \eta = \frac{j\omega\mu}{\gamma} \rightarrow \mu_r = \frac{|\eta\gamma|}{|\omega\mu_0|} = 1.64 \rightarrow$ mag material

$\because \beta^2 - \alpha^2 = \omega^2\mu\varepsilon \rightarrow \varepsilon_r = \frac{\beta^2 - \alpha^2}{\omega^2 \cdot \mu \cdot \varepsilon_0} = 4.7108$

& $V = \frac{\omega}{\beta} = 1.0208 \times 10^8$ m/s $\rightarrow$ 102.08 m in 1 μs

u_p for a lossless dielectric $= \frac{1}{\sqrt{\mu\varepsilon}}$

$\theta_\eta = 0$ for lossless $\rightarrow$ E & H in phase

$\theta_\gamma = \theta_\eta$ for a good conductor $(45°)$

$$\gamma = \sqrt{\omega\mu\sigma}\ \boxed{\angle 45°} \rightarrow \boxed{\frac{1+j}{\sqrt{2}}}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\ \boxed{\angle 45°}$$

Samples 3:

$\boxed{5}$ $\because$ $\Gamma = \frac{Z-\eta_1}{Z+\eta_1}$ & $\boxed{\eta_1 = 120\pi}$

$\because$ $Z = \eta_2 \cdot \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}$

$\rightarrow \Gamma = 0$

$$\eta_1 = Z = \eta_2 \frac{\eta_3 + j\eta_2 \tan \beta_2 d}{\eta_2 + j\eta_3 \tan(\beta_2 d)}$$

$\because$ quarter-wave transformer $\rightarrow$

$$\eta_2 = \sqrt{\eta_1 \eta_3}$$

$\because$ $\Gamma = 0.6$ at $f = 0 \rightarrow \beta_2 = 0$ $\because$ $\beta = \omega\sqrt{\mu\varepsilon}$

$\therefore$ $Z = \cancel{\eta_2} \cdot \frac{\eta_3 + 0}{\cancel{\eta_2} + 0} = \eta_3$

$\therefore$ $|0.6| = \frac{|\eta_3 - \eta_1|}{\eta_3 + \eta_1} \rightarrow \eta_3 = \frac{1+\Gamma}{1-\Gamma} \cdot \eta_1$

$\because$ $\sqrt{\varepsilon_{R3}} > 1 \rightarrow \eta_3 < \eta_1 \rightarrow \Gamma = -0.6$

$\therefore$ $\eta_3 = 30\pi = \sqrt{\frac{\mu_0}{\varepsilon_R \varepsilon_0}} = \frac{120\pi}{\sqrt{\varepsilon_R}} \rightarrow \boxed{\varepsilon_{R3} = 16}$

$\because$ η_3 at 900 MHz $= \eta_3$ at 0

$\rightarrow \eta_2 = \sqrt{120\pi \cdot 30\pi} = \pi \cdot 60$

$\rightarrow \varepsilon_{R2} = 4$

$\because$ $(2n+1)\frac{\lambda_2}{4} = d$ $\lambda_2 = \frac{2\pi}{\beta_2}$ at 900 MHz

$\beta_2 = 12\pi \rightarrow \lambda_2 = \frac{1}{6}$ m

$\rightarrow d = \frac{1/6}{4} = \frac{1}{24} \approx 4.17$ cm

$\boxed{1}$ $\varepsilon_r = 4$, $\Gamma = 0 \rightarrow \eta_1 = Z$

$$Z = \eta_2 \cdot \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}$$

$d = n \cdot \frac{\lambda_2}{2}$ half-wave section

$\lambda_2 = \frac{2\pi}{\beta_2}$, $\beta_2 = (w/c) \cdot \sqrt{\varepsilon_r} =$

$d_a = n \cdot \frac{\lambda_{2a}}{2}$ $\lambda_{2a} = \frac{1}{10} \rightarrow d_a = 5n_1$ (cm)

$d_b = n \cdot \frac{\lambda_{2b}}{2}$ $\lambda_{2b} = \frac{3}{50} \rightarrow d_b = 3n_2$ cm

if $d_a = d_b \Rightarrow 15$ (common multiple)

$\boxed{2}$ $\because d_a = \frac{0.7u}{\cancel{2}} \cdot n$ $\wedge$ $d_b = \frac{0.5u}{2} \cdot n \rightarrow \boxed{d = \cancel{1}.75 um}$

$$\because \eta_1 = \eta_2 \cdot \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}$$

quarter wave transformer:

$\rightarrow \eta_2 = \sqrt{\eta_1 \eta_3} = \sqrt{120\pi \cdot \eta_3} = 120\pi / \sqrt{1.52}$

$\because n_3 = 1.52 = \sqrt{\varepsilon_r} \rightarrow \eta_3 = 248.02$

$\rightarrow \boxed{\eta_2 = 305.78\ \Omega}$ $\therefore \boxed{\varepsilon_r = 1.52}$

$\because d = (2n+1)\frac{\lambda_2}{4}$ $\wedge$ $\lambda_2 = \frac{2\pi}{\beta_2} = \frac{\lambda_1}{\sqrt{\varepsilon_{r2}}}$

$\rightarrow d_a = (2n+1) \cdot 0.142$ $\wedge$ $d_b = (2n+1) \cdot 0.1014$

$\rightarrow d = 0.7097$ um

$\boxed{2}$ a) $\because SWR = \frac{\cancel{\eta_2} \cdot 1 + |\Gamma|}{1 - |\Gamma|}$ $\wedge$ $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

$\eta_1 = 120\pi$ $\eta_2 = 80\pi \rightarrow \Gamma = -0.2$

$\rightarrow SWR = 1.5$

$|E|_{max}$ at $\beta_1 \cdot d = -2n\pi$

$\rightarrow d = \frac{-2n\pi}{\beta_2} = 0$

$\boxed{2}$:

a) $\because \eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \frac{2}{3}\eta_0 \wedge \eta_1 = \eta_3$

$\rightarrow Z = \eta_2 \cdot \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}$

where $\beta_2 d = \beta_2 \cdot 0.1$, $\beta_2 = \omega\sqrt{\mu\varepsilon} = 24\pi$ rad/m

$\rightarrow \beta_2 d = 2.4\pi$

$\therefore Z = \frac{2}{3}\eta_0 \cdot \frac{\eta_0 + j\frac{2}{3}\eta_0 \tan(2.4\pi)}{\frac{2}{3}\eta_0 + j\eta_0 \tan(2.4\pi)} = 182.1675\angle -0.240183$ rad

$\therefore SWR = \frac{1+|\Gamma|}{1-|\Gamma|} \wedge \Gamma = \frac{Z-\eta_1}{Z+\eta_1} = 0.3684\angle -2.8402$

$\rightarrow SWR = 2.16656$ $\beta_1 = \frac{\omega}{c}$

$\wedge \quad \theta_r + 2\beta_1 z = -2n\pi$, $\theta_r = 2\pi - 2.8402$

$\rightarrow z = \frac{-\theta_r}{2\beta_1} = -13.66$ cm

b) ~~$\beta_2 d = (2n+1)\frac{\pi}{2} \rightarrow d = \frac{\pi}{2\beta_2} = 20.833$ cm~~

$\tan(\beta_2 d) = 0 \rightarrow \beta_2 d = n\pi$

$\rightarrow d = \frac{\pi}{\beta_2} = \frac{1}{24} = 4.17$ cm

$\boxed{3}$ $\frac{P_{ref}}{P_{inc}} = |\Gamma|^2 = \frac{1}{4} \rightarrow |\Gamma| = \frac{1}{2}$, $\varepsilon_{r2} = ?$

$\because \Gamma = \frac{Z-\eta_1}{Z+\eta_1}$, $Z = \eta_2 \cdot \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}$

$\rightarrow Z = \frac{1+\Gamma}{1-\Gamma} \cdot \eta_1$ but $\boxed{\beta_2 d = \beta_2 \cdot \frac{\lambda_2}{4}}$

$\wedge \lambda_2 = \frac{2\pi}{\beta_2} \rightarrow \beta_2 d = \frac{\pi}{2} \rightarrow \tan(\beta_2 d) = \infty$

$\rightarrow Z = \eta_2 \cdot \frac{j\eta_2}{j\eta_3} = \frac{\eta_2^2}{\eta_3} = \frac{\eta_0^2/\varepsilon_r}{\eta_0}$

$\rightarrow Z = \frac{\eta_0}{\varepsilon_r} \rightarrow \Gamma = \frac{\frac{\eta_0}{\varepsilon_r} - \eta_0}{\frac{\eta_0}{\varepsilon_r} + \eta_0} = \frac{1/\varepsilon_r - 1}{\frac{1}{\varepsilon_r} + 1}$

$\varepsilon_r > 1 \rightarrow \Gamma < 0$

$\rightarrow \Gamma = \frac{1-\varepsilon_r}{1+\varepsilon_r} \rightarrow \Gamma = -\frac{1}{2} = \frac{1-\varepsilon_r}{1+\varepsilon_r} \rightarrow 1 = \varepsilon_r \cdot \frac{1-0.5}{1.5} = 3$

$\boxed{5}$ quarter-wave transformer : $\eta_2 = \sqrt{\eta_1 \eta_3}$

$\wedge\ \theta_\Gamma + 2\beta_2 d = -(2n+1)\pi$

$\because \eta_1 = 120\pi$, when $f = 0 \rightarrow \beta_2 = \omega\sqrt{\mu\epsilon} = 0$

$\therefore Z = \eta_2 \cdot \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)} = \eta_3$

$\wedge\ |\Gamma| = 0.6$

$\because \epsilon_{r3} > 1 \rightarrow \Gamma = -0.6 = \frac{\eta_3 - \eta_1}{\eta_3 + \eta_1}$

$\rightarrow \eta_3 = \frac{1+\Gamma}{1-\Gamma} \cdot \eta_1 = 30\pi\ \Omega$ freq. independent

at $f = 900\text{MHz}$, $\eta_3 = 30\pi$, $\eta_2 = \sqrt{\eta_1 \cdot \eta_3} = 60\pi$

$\rightarrow \frac{\eta_0}{\sqrt{\epsilon_{r2}}} = 60\pi \rightarrow \boxed{\epsilon_{r2} = 4}$, $\epsilon_{r3} = 16$

$\because 2\beta_2 d = \pi \rightarrow d = \frac{\pm\pi}{2\beta_2} \wedge \beta_2 = \frac{\omega \cdot \sqrt{\epsilon_r}}{c}$

$\rightarrow d = \frac{\pm\pi}{24\pi} = \frac{-1}{24} = 4.169$ cm

$\because d = \frac{\lambda_2}{4} \wedge \lambda_2 = \frac{2\pi}{\beta_2} \wedge \beta_2 = 12\pi$

$\rightarrow d = \frac{1}{24} = 0.041669$ m

$\boxed{1}$ $\because \eta_1 = \eta_3 \rightarrow d = n \cdot \frac{\lambda_2}{2} \wedge \lambda_2 = \frac{2\pi}{\beta_2}$

$d_a = n \cdot \frac{\lambda_{2a}}{2}$, $\beta_{2a} = (1.5G \cdot 2\pi \cdot \sqrt{\epsilon_r})/c = 20\pi$ rad/m

$\rightarrow d_a = n \cdot \frac{1}{20}$

$\wedge\ d_b = n \cdot \frac{\lambda_{2b}}{2}$, $\beta_{2b} = (2.5G \cdot 2\pi \cdot \sqrt{\epsilon_r})/c = \frac{100}{3}\pi$ rad/m

$\rightarrow d_b = n \cdot \frac{3}{100} \rightarrow d_b = 3n_1$ cm

$\wedge\ d_a = 5n_2$ cm $\rightarrow d = 15$ cm

$\boxed{2}$ $\because \eta_1 \neq \eta_3 \rightarrow \eta_2 = \sqrt{\eta_1 \eta_3}$, $\eta_1 = \eta_0$

$\eta_3 = \frac{\eta_0}{\sqrt{\epsilon_{r3}}}$, $\epsilon_{r3} = n^2 = 2.3104 \rightarrow \eta_3 = 248.0205$

$\rightarrow \eta_2 = 305.78017\Omega = \frac{\eta_0}{\sqrt{\epsilon_{r2}}} \rightarrow \boxed{\epsilon_{r2} = 1.52}$

$\because\ d = (2n+1)\frac{\lambda_2}{4}$, $\lambda_2 = \frac{\lambda_0}{\sqrt{\varepsilon_r}} = 0.567775\ \lambda\ 0.4055535$

$\rightarrow d = (2\cdot2+1)\cdot\frac{0.567775}{4} = 0.709\ \mu m$

$\lambda_2 = \frac{2\pi}{\beta_2} \neq \lambda\ \lambda_1 = \frac{2\pi}{\beta_1}$

$\frac{\lambda_2}{\lambda_1} = \frac{\beta_1}{\beta_2} \rightarrow \lambda_2 = \frac{\beta_1}{\beta_2}\lambda_1$

$\frac{w/c}{w\sqrt{\varepsilon_r}/c} = \frac{\lambda_1}{\sqrt{\varepsilon_r}}$

$\boxed{2}$ a) $\because\ \eta_1 = \eta_3$ $\because\ SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$, $\Gamma = \frac{Z-\eta_1}{Z+\eta_1}$

$\because\ Z = \eta_2 \cdot \frac{\eta_3 + j\eta_2\tan(\beta_2 d)}{\eta_2 + j\eta_3\tan(\beta_2 d)}$ $\lambda\ \beta_2 d = \beta_2 \cdot 0.1$

$\lambda\ \beta_2 = (\omega\cdot\sqrt{\mu_r\cdot\varepsilon_r})/c = 24\pi \rightarrow \tan(\beta_2 d) = \tan(2.4\pi)$

$\lambda\ \eta_2 = \frac{120\pi\sqrt{\mu_r}}{\sqrt{\varepsilon_r}} = 80\pi$

$\therefore Z = 182.16755\angle -0.240183$ rad Ω

$\rightarrow \Gamma = 0.3684022\angle -2.850203869$

$= \boxed{0.3684022\angle 3.43298144 \text{ rad}}$

$\therefore SWR = 2.11659$

$|E|_{max}$ at $\theta_\Gamma + 2\beta_1 d = -2n\pi$ assume $n=0$

$\rightarrow d = \frac{-\theta_\Gamma}{2\beta_1}$ $\lambda\ \beta_1 = 4\pi$

$\rightarrow d_{max} = \frac{-3.43298144}{2\cdot4\pi} = -13.66$ cm

b) $SWR = 1 \rightarrow |\Gamma| = 0$

$\rightarrow d = n\frac{\lambda_2}{2} = n\cdot\frac{\pi}{\beta_2} = \frac{1}{24} = 4.1667$ cm

$\boxed{3}$ $\because$ $\frac{P_{rd}}{P_{inc}} = |\Gamma|^2 \Rightarrow |\Gamma| = \frac{1}{2}$ $\quad \eta_1 = \eta_3 = \eta_0$

$\because$ $\Gamma = \frac{Z - \eta_1}{Z + \eta_1} \wedge Z = \eta_2 \cdot \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}$

$d = \frac{\lambda_2}{4} \wedge \lambda_2 = \frac{2\pi}{\beta_2} \rightarrow d = \frac{\pi}{2\beta_2}$

$\rightarrow \tan(\beta_2 d) = \tan(\frac{\pi}{2}) = \infty$

ignoring $\eta_3 + \wedge \eta_2$

$\rightarrow Z = \eta_2 \cdot \frac{j\eta_2}{j\eta_3} = \frac{\eta_2^2}{\eta_3}$

$\because$ η_2 non-mag lossless $\rightarrow \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r2}}}$

$\rightarrow Z = \frac{\eta_0^2 / \varepsilon_r}{\cancel{\eta_0}} = \frac{\eta_0}{\varepsilon_r}$, $\boxed{\varepsilon_r > 1 \rightarrow \Gamma < 0}$

$\therefore \frac{-1}{2} = \frac{\frac{\eta_0}{\varepsilon_r} - \eta_0}{\frac{\eta_0}{\varepsilon_r} + \eta_0} \rightarrow \frac{1 - \varepsilon_r}{1 + \varepsilon_r} = \frac{-1}{2}$

$\rightarrow \varepsilon_r = \frac{1-\Gamma}{1+\Gamma} \rightarrow \boxed{\varepsilon_r = 3}$

---

1) $\because$ $P^{trans}_{m} = P_{inc}(1 - |\Gamma|^2)$

$\rightarrow |\Gamma|^2 = \quad \rightarrow |\Gamma| = \frac{2}{3}$

$\rightarrow SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 5$

2) $\because$ $\lambda_2 = 1.2$ m $\rightarrow \beta_2 = \frac{2\pi}{1.2} = \frac{5}{3}\pi$ rad/m

$\because$ $\beta_2 = \omega\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}$

$\because$ $\eta_2 < \eta_1 \rightarrow \Gamma = \frac{-2}{3} \rightarrow \eta_2 = \frac{1+\Gamma}{1-\Gamma} \cdot \eta_1 = 24\pi$

$\because$ $\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} \rightarrow \varepsilon_r = 25$

$\rightarrow \frac{5}{3}\pi = 2\pi f \cdot \sqrt{25} / c \rightarrow \boxed{f = 50 \text{ MHz}}$

$P_{avg} = \frac{|E_{x0}|^2}{2|\eta|} e^{-2\alpha z} \cos\theta_\eta$

1) $\because \eta_2=\sqrt{\eta_1\eta_3} \wedge d=(2n+1)\frac{\lambda_2}{4}$

$\because n_{glass}=2 \rightarrow \varepsilon_{r_3}=4 \rightarrow \eta_3 = 60\pi$

$\rightarrow \eta_2 = 60\sqrt{2}\pi \rightarrow \boxed{\varepsilon_{r_2}=2}$

~~$\therefore \beta_2 = \frac{w\cdot\sqrt{\varepsilon_{r_2}}}{c} \wedge w=w_{air}, w_{air}=\beta_{air}\cdot c \wedge \beta_{air}=\frac{2\pi}{\lambda_0}$~~

$\lambda_2=\frac{\lambda_0}{\sqrt{\varepsilon_r}} = 388.91nm$

$\therefore d = 1\cdot\frac{388.9n}{4} = 97.23\ nm$

2) $\because P_{ref}=|\Gamma|^2\cdot P_{inc} \wedge \Gamma=\frac{Z-\eta_1}{Z+\eta_1}$

$\because Z=\eta_2\cdot\frac{\eta_3+j\eta_2\tan(\beta_2 d)}{\eta_2+j\eta_3\tan(\beta_2 d)}, d=97.23nm$

$\because \beta_2=\frac{2\pi}{\lambda_2} \wedge \lambda_2=\frac{\lambda_0}{\sqrt{2}} \rightarrow \beta_2=0.32319\times10^8$

$\rightarrow Z = 188.5\angle 6.43761\times10^{-5}$ rad ≈ 0

$\rightarrow Z = 188.5 \approx \eta_3$

$\therefore \Gamma=\frac{-1}{3} \therefore P_{ref}=\frac{1}{9}\cdot P_{inc}$

power reflectivity $=\frac{P_{ref}}{P_{inc}}\times100 = |\Gamma|^2\times100=11.1\%$

$\boxed{2}$ ~~$\because P_{avg}=\frac{|E_0|^2}{2|\eta|}\cdot[1-e^{-2\alpha y}]\cdot S = 1.6mW$~~

~~$\wedge \beta=16\pi, \alpha=0 \rightarrow \frac{10^2}{2\eta}\cdot 300\times10^{-4}=1.6mW$~~

~~$\rightarrow \eta =$~~ $\beta=16\pi$

$P_{avg}=\iint \frac{1}{2}\frac{|E_0|^2}{|\eta|}\cdot\bar{a}_y\cdot\frac{3x+4y}{\sqrt{3^2+4^2}}\cdot ds$

$\rightarrow 1.6mW=\frac{|E_0|^2}{2|\eta|}\cdot\left[\cancel{\frac{3}{5}\bar{a}_x}+\frac{4}{5}\bar{a}_y\right]\cdot\bar{a}_y\cdot300\times10^{-4}$

$\rightarrow 1.6mW=\frac{10^2\cdot4}{10\eta}\cdot300\times10^{-4}$

$\rightarrow \boxed{\eta = 750}\ \Omega \rightarrow \varepsilon_r=4 \quad \mu_r=16$

$\because \beta=w\sqrt{\mu\varepsilon} \wedge \alpha=0 \rightarrow \frac{16\pi\cdot c}{w}=\sqrt{\mu_r\varepsilon_r} \wedge \frac{750}{120\pi}=\sqrt{\frac{\mu_r}{\varepsilon_r}}$

$\boxed{3}$ a) $\frac{100}{|\eta|}$, $|\eta| = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{2863.344} \angle \frac{0.633633}{2}$

$= 53.51 \angle 0.3168165 \text{ Rad}$

b) $H_0 = 1.86881$ A/m

$\boxed{4}$ $P_{avg}^{reflected} = P_{inc} \cdot |\Gamma|^2$.

Subject EM2 first exam Date 15/4/2021 No.

MOHAMMAD SANAD ALTAHER 130806

أنا محمد سند هيثم الطاهر أشهد أني قرأت و فهمت وطبقت تعليمات
هذا الامتحان، ولم أتلق أي مساعدة من أي شخص في حل هذا الامتحان
أنا لست بغشاش ولا كذاب.

Q1) $\because \bar{E} = \bar{a}_x E_{io} e^{-j\beta z}$

$R_s = 0.44\ \Omega$

$$P_{avg}^{inc} = \frac{|E_{io}|^2}{2|\eta_1|} \cdot \cos(\theta_\eta) \cdot e^{-2\alpha z}$$

$$\because R_s = \frac{1}{\sigma \delta_c} = 0.44\ \Omega$$

$$P_{avg}^{Transmitted} = \frac{|E_{to}|^2}{2|\eta_2|} \cdot \cos(\theta_\eta) \cdot \left[1 - e^{-2\alpha z}\right]$$

$\wedge$ $E_{to} = T E_{io}$ & $P_{avg}^{Transmitted} = P_{avg}^{inc}\left[1 - |\Gamma|^2\right]$

$\because \eta_1 = 120\pi$ $\wedge$ $\eta_2 = (1+j)\frac{\alpha}{\sigma}$

$\sigma = \frac{\alpha}{R_s}$ & $\boxed{\eta_2 = (1+j) R_s} = 0.62225 \angle 0.785$ rad

$$\therefore \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.99767 \angle 3.13925$$

$\rightarrow P_{avg}^{Transmitted} = 4.6546\ W/m^2$ at interface

at $2\delta_c$ $\rightarrow$ $4.6546 \cdot \left[0 + e^{-2 \cdot \frac{1}{\delta_c} \cdot 2\delta_c}\right]$

$$= 4.6546 \cdot [0.0183]$$

$$= \boxed{0.085252\ W/m^2}$$

Subject Date No.

Q2) $f = 1\,GHz$, $\mu_r = 4$, $\varepsilon_r = 9$, lossless

a)

$\because$ $\eta_1 = \eta_3$, $d = n\frac{\lambda_2}{2}$, $\lambda_2 = \frac{2\pi}{\beta_2}$

$\because$ $\beta_{lossless} = \omega \cdot \sqrt{\mu_r \cdot \varepsilon_r} / c$

$\rightarrow \beta_2 = 40\pi \rightarrow \boxed{\lambda_2 = \frac{1}{20}\ m}$

$\therefore d = 2.5\ cm$

b) $f = 1.5\ GHz$ $\because$ $\eta_1 = \eta_3$, $\Gamma = \frac{Z - \eta_1}{Z + \eta_1}$

, $\beta_2 \cdot d = \frac{\omega \cdot \sqrt{\mu_r \varepsilon_r}}{c} \cdot \frac{1}{40} = \frac{3}{2}\pi$

$\therefore \tan(\beta_2 d) = \infty$

$\rightarrow Z = \eta_2 \cdot \frac{\eta_2}{\eta_3} = \frac{\left(\eta_0 \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}}\right)^2}{\eta_0}$

$\therefore Z = \eta_0 \cdot \frac{\mu_r}{\varepsilon_r}$

$\rightarrow \Gamma = \frac{\frac{\mu_r}{\varepsilon_r} - 1}{\frac{\mu_r}{\varepsilon_r} + 1} = \boxed{-\frac{5}{13}}$

power reflectivity $= \frac{P_{Ref}}{P_{inc}} \times 100\% = |\Gamma|^2 \times 100\%$

$\therefore$ $\boxed{\text{power reflectivity} = 14.8\%}$

c) zero reflection $\rightarrow \Gamma = 0$

SWR = 1 , quarter-wave transformer: $\eta_2 = \sqrt{\eta_1 \eta_3}$

$\rightarrow \eta_2 = 120\pi = \eta_0 \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}} \rightarrow \sqrt{\frac{\mu_r}{\varepsilon_r}} = 1$

$\therefore \mu_r = \varepsilon_r = 9$

If $\mu_r = \varepsilon_r$ at all times for a lossless medium, the intrinsic impedance of that medium will be equal to the intrinsic impedance of the materials separated by it and hence the reflection coefficient will always equal zero.

NOTEBOOK

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Q3) $E_0 = 1000$ V/m , $f = 10$ GHz

$E_s = 200$ V/m

∵ $E_0 = 1000$ V/m , $E_s = 200$ V/m

$$E_s = E_0 \cdot e^{-2d \cdot \alpha} \rightarrow -2\alpha d = \ln\left(\frac{200}{1000}\right)$$

$$\therefore \boxed{d = \frac{-1.60944}{-2\alpha}}$$

∵ loss tangent $= \frac{\sigma}{\omega\varepsilon} = 0.05143 \ll 1$

∴ good dielectric simplified equations can be used

$$\alpha = \frac{\sigma}{2}\sqrt{\mu/\varepsilon} = \frac{0.1}{2} \cdot 120\pi \cdot \sqrt{\frac{1}{3.5}}$$

$$= \boxed{10.0755 \text{ Np/m}}$$

$$\therefore \boxed{d = 7.987 \text{ cm}}$$

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Q3) redome

$\frac{\sigma}{\omega\epsilon} << 1 \rightarrow$ good dielectric

$E_2 = T_1 \cdot E_1 \quad \wedge \quad E_4 = T_2 \cdot E_3$

$T = \frac{2\eta_2}{\eta_1 + \eta_2} \quad \eta_a = 120\pi \quad , \eta_s = \frac{\mu}{\epsilon} = 120\pi \cdot \sqrt{\frac{\mu_r}{\epsilon_r}}$

$\therefore \quad T_1 = \frac{2\eta_s}{\eta_a + \eta_s} \quad \wedge \quad T_2 = \frac{2\eta_a}{\eta_a + \eta_s}$

$= 0.69666 \qquad = 1.30337$

$\therefore \quad E_2 = 696.66 \text{ V/m} \qquad E_3 = \frac{200}{T_2} = \boxed{153.448 \text{ V/m}}$

$E_3 = E_2 \cdot e^{-2\alpha d} \quad , \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = 10.0955 \text{ N/m}$

$\therefore \quad \boxed{d = 7.51 \text{ cm}}$

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Q4) ∵ perfect conductor → total reflection

$\therefore E_{max} = 2 E_{i0} \quad \wedge \quad E_{min} = -2E_{i0}$

$\because P^{inc}_{avg} = \frac{|E_{i0}|^2}{2|\eta|} \rightarrow E_{i0} = \sqrt{0.6 \cdot 2\eta_0}$

$= 21.269$ V/m

∵ H min occurs at E max , $\beta_1 = \frac{\omega}{c} = \frac{\pi}{150}$

$E_{max} = 2|E_{i0}| = \boxed{42.538 \text{ V/m}}$

$E_{max} = 2|E_{i0}| \cdot \sin(\beta_2 \cdot 750)$

$\rightarrow \beta_2 \cdot 750 = \frac{9\pi}{2} \rightarrow \beta_2 = 0.01885$ rad/m

$\wedge \ \beta_2 = \omega\sqrt{\mu \varepsilon} \rightarrow \omega = \beta_2 \cdot c$

$\rightarrow \omega = 2\pi \cdot 0.9$ MHz

$\boxed{f = 900 \text{ kHz}}$

$\wedge \ H = 0$ A/m

Subject general form of plane waves Date 18/4/2021 No.

- a wave traveling in the +z direction: $\bar{E} = \bar{E}_0 e^{-j\beta z}$

$$\bar{E} = (E_{0x}\bar{a}_x + E_{0y}\bar{a}_y)e^{-j\beta z}$$

- generally: $\boxed{\bar{E}(x,y,z) = (E_{0x}\bar{a}_x + E_{0y}\bar{a}_y + E_{0z}\bar{a}_z)\, e^{-j(k_x x + k_y y + k_z z)}}$

$$\rightarrow \bar{E}(x,y,z) = \bar{E}_0 e^{-j\bar{k}\bar{R}} = \bar{E}_0 e^{-jk\bar{a}_k\cdot\bar{R}}$$

where $\bar{k}$: wave number vector, $\bar{k} = \bar{a}_x k_x + \bar{a}_y k_y + \bar{a}_z k_z$

$= k\cdot\bar{a}_k$ (direction of propagation)

and $\bar{R}$: position vector, $\bar{R} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$

- since $\bar{\nabla}^2\bar{E} - \mu\varepsilon\frac{\partial^2\bar{E}}{\partial t^2} = 0$ must always be satisfied

in phasor domain: $\frac{\partial^2 A}{\partial t^2} = j^2\omega^2 A_s$

$$\rightarrow \bar{\nabla}^2\bar{E} - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} = \nabla^2\bar{E}_s + (\omega^2\mu\varepsilon)\bar{E}_s = 0$$

$$\rightarrow \nabla^2 E_x + \omega^2\mu\varepsilon E_x = 0 = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + \omega^2\mu\varepsilon E_x = 0$$

where $E_x = E_{0x}e^{-j(k_x x + k_y y + k_z z)}$

$$\rightarrow \omega^2\mu\varepsilon E_x = \left[k_x^2 + k_y^2 + k_z^2\right]E_x$$

$$\therefore \beta^2 = \omega^2\mu\varepsilon = k_x^2 + k_y^2 + k_z^2 = k^2$$

- also, since $\rho_v = 0 \rightarrow \nabla\cdot\bar{E} = 0$

$$\rightarrow \nabla\cdot\left(\bar{E}_0\cdot e^{-j\bar{k}\cdot\bar{R}}\right) = 0$$

+ given the product rule of vector calculus: $\nabla(\Phi\Psi) = \Phi\nabla(\Psi) + \Psi\nabla(\Phi)$ (scalars)

$$\therefore \bar{\nabla}\cdot\left(\bar{E}_0 e^{-j\bar{k}\bar{R}}\right) = \underbrace{e^{-j\bar{k}\bar{R}}(\bar{\nabla}\cdot\underset{\text{constant}}{\underset{\downarrow}{\bar{E}_0}})}_{0} + \bar{E}_0\cdot\bar{\nabla}\left(e^{-j\bar{k}\bar{R}}\right) = 0$$

$$\rightarrow \bar{E}_0\cdot\left(\bar{\nabla}e^{-j\bar{k}\bar{R}}\right) = 0 \rightarrow \bar{E}_0\cdot\left(-j\bar{k}\,e^{-j\bar{k}\bar{R}}\right) = 0$$

$$\therefore \bar{E}_0\cdot\bar{k} = 0 \rightarrow \bar{E}_0 \perp \bar{k}$$

$$\boxed{\bar{E}_0\cdot\bar{a}_k = 0} \rightarrow \bar{E}_0 \perp \bar{a}_k$$

$\because \quad \bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t}$

$\rightarrow \bar{\nabla} \times \bar{E}_s = -\mu j \omega \bar{H}_s \rightarrow \bar{H}_s = \frac{j}{\mu \omega} (\bar{\nabla} \times \bar{E}_s)$

$\because$ identity : $\nabla \times (f \bar{A}) = (\nabla f) \times \bar{A} + f(\nabla \times A)$

$\rightarrow \bar{H}_s = \frac{j}{\mu \omega} \left[(\nabla e^{-j\bar{k}\cdot\bar{R}}) \times \bar{E}_0 + e^{-j\bar{k}\cdot\bar{R}} \underbrace{(\nabla \times \bar{E}_0)}_{0} \right]$

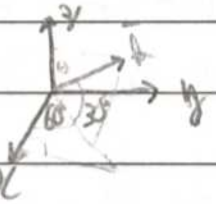
$\rightarrow \bar{H}_s = \frac{j}{\mu \omega} \left[(-\bar{k} \cdot j) \times \bar{E}_0 e^{-j\bar{k}\cdot\bar{R}} \right]$

$$\therefore \quad \boxed{\bar{H}_s = \frac{k}{\mu \omega} \left[\bar{a}_k \times \bar{E} \right] , \quad \frac{k}{\mu \omega} = \frac{1}{\eta}}$$

$\rightarrow \bar{H} = \frac{1}{\eta} (\bar{a}_k \times \bar{E}) \wedge \bar{E} = -\eta (\bar{a}_k \times \bar{H})$

(for TEM)

example from notes:



$\rightarrow \theta_k = 90^\circ - 30^\circ = 60^\circ \quad , \phi = 60^\circ$

θ : angle from the positive z axis

$\because$ E has no z components $\wedge$ $|E|_{(0,0,0)} = 10$

$\rightarrow \bar{E} = \left[E_{0x} \bar{a}_x + E_{0y} \bar{a}_y \right] e^{-j\bar{k}\bar{R}} = E_0 e^{-j\bar{k}\bar{R}}$

$\bar{k} = k \bar{a}_k = \beta \bar{a}_k \quad \because$ free space $\wedge$ $f = 12 MHz$

$\rightarrow \beta = 12 \cdot 2\pi \cdot \frac{10^6}{c} = 0.08 \pi$ rad/m

$\bar{a}_k = \bar{a}_x \cos(30) \cos(60) + \bar{a}_y \cos(30) \sin(60) + \bar{a}_z \sin(30)$

$= \frac{\sqrt{3}}{4} \bar{a}_x + \frac{3}{4} \bar{a}_y + 0.5 \bar{a}_z$

$\rightarrow \beta \bar{a}_k = 0.02\pi \left[\sqrt{3} \bar{a}_x + 3 \bar{a}_y + 2 \bar{a}_z \right]$

$\rightarrow \bar{k} = 0.02\pi \left[\sqrt{3} \bar{a}_x + 3 \bar{a}_y + 2 \bar{a}_z \right]$

$\because \bar{k} \cdot \bar{E} = 0 \rightarrow \pi 0.02 \sqrt{3} \cdot E_{0x} + 0.06\pi E_{0y} = 0$

$\wedge \quad E_{0x}^2 + E_{0y}^2 = 100$

$\therefore \quad E_{0x} = -\sqrt{3} E_{0y} \rightarrow 4 E_{0y}^2 = 100 \rightarrow E_{0y} = \mp 5$

$\wedge \quad E_{0x} = \mp(-5\sqrt{3}) \quad \therefore \bar{E} = 5 \left(-\sqrt{3} \bar{a}_x + \bar{a}_y \right) e^{-j 0.02\pi (\sqrt{3} x + 3y + 2z)}$

$\bar{H} = \frac{1}{\eta_0} \left(\bar{a}_k \times \bar{E} \right) = \frac{10}{4 \cdot 120\pi} \left[-\bar{a}_x - \sqrt{3} \bar{a}_y + 2\sqrt{3} a_z \right] e^{-j\bar{k}\bar{R}}$

$-10 \bar{a}_x - 10\sqrt{3} a_y + 20\sqrt{3} \bar{a}_z$

$= \frac{1}{48\pi} \left[-\bar{a}_x - \sqrt{3} \bar{a}_y + 2\sqrt{3} a_z \right] e^{-j\bar{k}\bar{R}}$

example 2 from notes:

$\because \bar{E} = 10 e^{-j(6x+8z)} \bar{a_y} \rightarrow \bar{k} = (6\bar{a_x} + 0\bar{a_y} + 8\bar{a_z})$

$\rightarrow k^2 = \beta^2 = W^2 \cdot \mu_0 \varepsilon_0 \rightarrow W = \sqrt{100 \cdot (3\times10^8)^2} = 3\times10^9$

$\rightarrow f = \frac{W}{2\pi} = \frac{3\times10^9}{2\pi} \quad \because \lambda = \frac{2\pi}{\beta} = 2\pi/10 = 0.2\pi (m)$

$\bar{E}(t) = 10\cos(wt - 6x - 8z)\bar{a_y}$

$H = \frac{1}{\eta_0}(\bar{a_k} \times \bar{E}) \quad \bar{a_k} = 0.6\bar{a_x} + 0.8\bar{a_z}$

$\bar{a_k} \times \bar{E} = -10 \cdot 0.8\bar{a_x} + 0.6 \cdot 10\bar{a_z}$

$= \frac{1}{12\pi}[-0.8\bar{a_x} + 0.6\bar{a_z}] e^{-j\bar{k}\cdot\bar{R}} = \frac{1}{12\pi}[-0.8\bar{a_x} + 0.6\bar{a_z}] e^{-j(6x+8z)}$

- $E(\bar{R},t) = E_o \cos(\bar{k}\cdot\bar{R} - wt) = E_o \cos(wt - \bar{k}\cdot\bar{R})$

since cosine is even, $\bar{R} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$ (position vector)

$\bar{k}$: propagation vector, $\bar{k} = k_x\bar{a}_x + k_y\bar{a}_y + k_z\bar{a}_z$

$k^2 = k_x^2 + k_y^2 + k_z^2 = w^2 \cdot \mu\epsilon = \beta^2$ (for lossless)

+ maxwell's equations become:

$$\bar{k}\times\bar{E} = w\mu\bar{H} \qquad \bar{k}\times\bar{H} = -w\epsilon\bar{E}$$

$$\bar{k}\cdot\bar{E} = 0 \qquad \bar{k}\cdot\bar{H} = 0$$

- $\bar{E}$, $\bar{H}$, and $\bar{k}$ are all orthogonal
- assuming $\bar{E}_i = E_{io}\cos(k_{ix}x + k_{iy}y + k_{iz}z - w_i t)$

$$\bar{E}_r = \bar{E}_{ro}\cos(k_{rx}x + k_{ry}y + k_{rz}z - w_r t)$$

$$\bar{E}_t = \bar{E}_{to}\cos(k_{tx}x + k_{ty}y + k_{tz}z - w_t t)$$

the tangential component of $\bar{E}$ must be continuous on the interface $(z=0)$:

$$E_i(z=0) + E_r(z=0) = E_t(z=0)$$

- the above boundary condition can only be satisfied if

$$\left.\begin{array}{l} w_i = w_r = w_t = w \\ \text{phase matching conditions} \left\{\begin{array}{l} k_{ix} = k_{rx} = k_{tx} = k_x \\ k_{iy} = k_{ry} = k_{ty} = k_y \end{array}\right. \end{array}\right\} \text{must be satisfied}$$

- the first phase matching condition requires that the frequency remains unchanged, whereas the second and third require the propagation vectors (k_i, k_r, k_t) all lie on the same plane

hence: $k_i\sin(\theta_i) = k_r\sin(\theta_r) = k_t\sin(\theta_t)$ (1)

$$k_{ix} = k_{rx} = k_{tx}$$

- since k_i and k_r lie in the same (lossless) medium therefore $k_i = k_r = \beta_1$, implying $\boxed{\theta_i = \theta_r}$

- since $k_i \sin(\theta_i) = k_t \sin(\theta_t) \rightarrow \beta_1 \sin(\theta_i) = \beta_2 \sin(\theta_t)$

$\rightarrow \omega\sqrt{\mu_0\mu_{r1}\varepsilon_0\varepsilon_{r1}}\ \sin(\theta_i) = \omega\sqrt{\mu_0\mu_{r2}\varepsilon_0\varepsilon_{r2}}\ \sin(\theta_t)$

$\rightarrow \sqrt{\mu_{r1}\varepsilon_{r1}}\cdot\sin(\theta_i) = \sqrt{\mu_{r2}\varepsilon_{r2}}\cdot\sin(\theta_t)$

(for non-mag) $\therefore$ $\boxed{n_1 \sin(\theta_i) = n_2 \sin(\theta_t)}$ Snell's law

where n: refractive index of the material

* parallel polarization:

assume $\bar{E}_{is} = E_{io}\left(\cos\theta_i\,\bar{a}_x - \sin\theta_i\,\bar{a}_z\right)e^{-j\beta_1(x\sin\theta_i+z\cos\theta_i)}$

$\rightarrow \bar{H}_{is} = \frac{1}{\eta_1}\left(\bar{a}_k \times \bar{E}_{is}\right)$, $\bar{a}_k = \sin\theta_i\,\bar{a}_x + \cos\theta_i\,\bar{a}_z$

$\rightarrow \bar{a}_k \times \bar{E}_{is} = 0\,\bar{a}_x + \overset{1}{\cancel{\left(\cos^2\theta_i+\sin^2\theta_i\right)}}\bar{a}_y + 0\,\bar{a}_z$

$\therefore \bar{H}_{is} = \frac{E_{io}}{\eta_1}\cdot e^{-j\beta_1(x\sin(\theta_i)+z\cos(\theta_i))}\ \bar{a}_y$

& $\bar{E}_{rs} = E_{ro}\left(\cos(\theta_r)\,\bar{a}_x + \sin(\theta_r)\,\bar{a}_z\right)e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$

$\rightarrow \bar{H}_{rs} = -\frac{E_{ro}}{\eta_1}e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\ \bar{a}_y$

$\therefore \bar{E}_{ts} = E_{to}\left(\cos\theta_t\,\bar{a}_x - \sin\theta_t\,\bar{a}_z\right)e^{-j\beta_2(x\sin\theta_t+z\cos\theta_t)}$

$\rightarrow \bar{H}_{ts} = \frac{E_{to}}{\eta_2}e^{-j\beta_2(x\sin\theta_t+z\cos\theta_t)}\ \bar{a}_y$

at $z=0$, $\theta_i = \theta_r$: tangential components

$E_{is}(z=0) + E_{rs}(z=0) = E_{ts}(z=0)$

$E_{io}\left(\cos\theta_i\,\bar{a}_x - \sin\theta_i\,\bar{a}_z\right)e^{-j\beta_1 x\sin\theta_i}$ +

$E_{ro}\left(\cos\theta_r\,\bar{a}_x + \sin\theta_r\,\bar{a}_z\right)e^{-j\beta_1 x\sin\theta_i}$ =

$E_{to}\left(\cos(\theta_t)\,\bar{a}_x - \sin\theta_t\,\bar{a}_z\right)e^{-j\beta_2 x\sin\theta_t}$

$E_{io}\cos\theta_i + E_{ro}\cos\theta_i = E_{to}\cos\theta_t$ x-components

$\rightarrow E_{to}\cos\theta_t = \left[E_{io}+E_{ro}\right]\cos(\theta_i)$ —— ①

& $H_{isx} + H_{rsx} = H_{tsx}$

$$\rightarrow \frac{1}{\eta_1}\left(E_{i0} - E_{r0}\right) = \frac{1}{\eta_2} \cdot E_{t0} \qquad \rightarrow E_{t0} = \frac{\eta_2}{\eta_1} E_{i0} - \frac{\eta_2}{\eta_1} E_{r0} \quad -②$$

sub. ② in ①:

$$\frac{\eta_2}{\eta_1} E_{i0} - \frac{\eta_2}{\eta_1} E_{r0} = \frac{\cos\theta_i}{\cos\theta_t} E_{i0} + \frac{\cos\theta_i}{\cos\theta_t} E_{r0}$$

$$\rightarrow \frac{E_{r0}}{E_{i0}} = \frac{\eta_2/\eta_1 - \cos(\theta_i)/\cos(\theta_t)}{\eta_2/\eta_1 + \cos(\theta_i)/\cos(\theta_t)}$$

$$\therefore \frac{E_{r0}}{E_{i0}} = \Gamma_{11} = \frac{\eta_2 \cos(\theta_t) - \eta_1 \cos(\theta_i)}{\eta_2 \cos(\theta_t) + \eta_1 \cos(\theta_i)}$$

& $$\tau_{11} = \frac{2\eta_2 \cos(\theta_i)}{\eta_2 \cos(\theta_t) + \eta_1 \cos(\theta_i)} = \frac{E_{t0}}{E_{i0}}$$

- the above equations for the parallel reflection and transmission coefficients are called fresnel's equations
  Γ_{11} & τ_{11} are known as fresnel's coefficients
- fresnel's equations reduce to the normal incidence equations if $\theta_i = \theta_r = \theta_t = 0$
- relation between reflection and transmission coefficients:

$$1 + \Gamma_{11} = \tau_{11}\left[\frac{\cos\theta_t}{\cos\theta_i}\right]$$

* brewster angle (θ_{B11}): the angle at which ($\Gamma_{11} = 0$) no reflection occurs ($E_{r0} = 0$)

- brewster angle is often called polarising angle, since only an arbitrarily polarized wave will be reflected with its only $\bar{E}$ component being perpendicular to the plane of incidence

- when $\eta_2 \cos(\theta_t) = \eta_1 \cos(\theta_i) \rightarrow \theta_i = \theta_{B\parallel}$

$\rightarrow \eta_2^2 \cos^2(\theta_t) = \eta_1^2 \cos^2(\theta_i) \rightarrow \eta_2^2 [1-\sin^2(\theta_t)] = \eta_1^2 [1-\sin^2(\theta_i)]$

from snell's law :

$$\frac{\eta_2^2}{\eta_1^2} \left[1 - \sin^2(\theta_{B\parallel}) \cdot \frac{\mu_1 \varepsilon_1}{\mu_2 \varepsilon_2}\right] = 1 - \sin^2(\theta_{B\parallel})$$

$$\rightarrow \sin^2(\theta_{B\parallel}) = \frac{1 - \mu_2\varepsilon_1/\mu_1\varepsilon_2}{1 - (\varepsilon_1/\varepsilon_2)^2}$$

for non-magnetic media : $\tan(\theta_{B\parallel}) = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{\eta_2}{\eta_1}$

* perpendicular polarization :

assume : $E_{is} = E_{io} e^{-j\beta_1 (x \sin\theta_i + z \cos\theta_i)} \bar{a}_y$

$\rightarrow \bar{H}_{is} = \frac{E_{io}}{\eta_1} (-\cos\theta_i \bar{a}_x + \sin\theta_i \bar{a}_z) e^{-j\beta_1 (x \sin\theta_i + z \cos\theta_i)}$

$\rightarrow \bar{E}_{rs} = E_{ro} e^{-j\beta_1 (x \sin(\theta_r) - z \cos(\theta_r))} \bar{a}_y$

$\wedge \bar{H}_{rs} = \frac{E_{ro}}{\eta_1} (\cos(\theta_r) \bar{a}_x + \sin\theta_r) \bar{a}_z) e^{-j\beta_1 (x \sin\theta_r - z \cos\theta_r)}$

$\therefore \bar{E}_{ts} = E_{to} e^{-j\beta_2 (x \sin\theta_t + z \cos\theta_t)} \bar{a}_y$

$\wedge \bar{H}_{ts} = \underset{E_{to}/\eta_2}{H_{to}} (-\cos\theta_t \bar{a}_x + \sin(\theta_t) \bar{a}_z) e^{-j\beta_2 (x \sin\theta_t + z \cos\theta_t)}$

- applying the boundary conditions gives :

$E_{io} + E_{ro} = E_{to} \wedge \frac{1}{\eta_1} (E_{io} - E_{ro}) \cos(\theta_i) = \frac{1}{\eta_2} E_{to} \cos(\theta_t)$

yielding $\frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos(\theta_i) - \eta_1 \cos(\theta_t)}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \Gamma_\perp$

$\wedge \frac{E_{to}}{E_{ro}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \tau_\perp$

where $1 + \Gamma_\perp = \tau_\perp$ (fresnel's equations for perpendicular polarization)

- for no reflection: $\eta_2 \cos(\Theta_{B\perp}) = \eta_1 \cos(\Theta_t)$

$\Gamma_\perp = 0$ when $\Theta_i = \Theta_{B\perp}$ (brewster angle)

$$\rightarrow \eta_2^2 \left[1 - \sin^2(\Theta_{B\perp})\right] = \eta_1^2 \left[1 - \sin(\Theta_t)\right]$$

$$\rightarrow \sin^2(\Theta_{B\perp}) = \frac{1 - \mu_1 \varepsilon_2 / \mu_2 \varepsilon_1}{1 - (\mu_1/\mu_2)^2}$$

- hence, for non-magnetic material, $\Theta_{B\perp}$ does not exist. (or if $\mu_1 = \mu_2$)
- if $\varepsilon_1 = \varepsilon_2 \rightarrow \sin(\Theta_{B\perp}) = \sqrt{\frac{\mu_2}{\mu_1 + \mu_2}}$, $\boxed{\tan(\Theta_{B\perp}) = \sqrt{\frac{\mu_2}{\mu_1}}}$

example 10.11:

a) $\because k^2 = 0.866^2 + 0.5^2 = w^2 \cdot \mu_0 \varepsilon_0 \rightarrow w = 3 \times 10^8$ Rad s^{-1}

$\because \lambda = \frac{2\pi}{\beta}$ ∧ $\beta = \sqrt{k^2} \rightarrow \lambda \approx 2\pi$ m

b) $H_s = \frac{1}{\eta_0} (\bar{a}_k \times \bar{E}_s)$, $\bar{a}_k = \frac{0.866 \bar{a}_y + 0.5 \bar{a}_z}{1}$

$\bar{a}_k \times \bar{E}_s = [0 \bar{a}_x + 50 \bar{a}_y - 86.6 a_z] e^{j \bar{k} \cdot \bar{R}}$

$\rightarrow \bar{H}_s = [132.63 \bar{a}_y - 229.71 \bar{a}_z] e^{j \bar{k} \bar{R}}$ mA/m

c) $P_{avg} = \frac{1}{2} \text{Re}\{\bar{E}_s \times \bar{H}_s^*\}$, $\bar{H}_s^* = [132.63 \bar{a}_y - 229.71 \bar{a}_z] e^{-j \bar{k} \bar{R}}$

$= \frac{E_0^2}{2\eta_0} \bar{a}_k = \frac{10^4}{240\pi} (0.866 \bar{a}_y + 0.5 \bar{a}_z)$

$= 11.49 \bar{a}_y + 6.631 \bar{a}_z$ W/m^2

practice exercise 10.11:

a) $\bar{E}_s = [10 \bar{a}_y + 5 \bar{a}_z] e^{j(2y - 4z)} \rightarrow k^2 = 20 \rightarrow w = 1.342$ GRad s^{-1}

$\rightarrow \lambda = 1.405$ m

b) $\bar{H}_s = \frac{1}{120\pi} [a_k \times \bar{E}_s]$, $\bar{a}_k = \frac{1}{\sqrt{20}} [2 \bar{a}_y - 4 \bar{a}_z]$

$\rightarrow \bar{a}_k \times \bar{E}_s = -50 \bar{a}_x \cdot \frac{1}{\sqrt{20}} \cdot e^{j(2y - 4z)}$

$\therefore \bar{H}_s = -29.66 e^{j(2y - 4z)} \bar{a}_x$ mA/m

c) $\frac{10^2 + 5^2}{2 \cdot 120\pi} \cdot \frac{1}{\sqrt{20}} [2 \bar{a}_y - 4 \bar{a}_z] = 0.07414 \bar{a}_y - 0.1483 \bar{a}_z$

example 10.12: $E = 8 e^{j(4x+3z)} \bar{a}_y$

a) $k_i = 4\bar{a}_x + 3\bar{a}_z \rightarrow k = 5 = W\sqrt{\mu_0 \varepsilon_0} \rightarrow W = 1.5\,G\,rad\,s^{-1}$

∴ k_i is in xz-plane and $\bar{E}$ is perpendicular to it

→ perpendicular polarization

b) ∵ $\bar{k}_i \cdot \bar{R} = x \sin(\theta_i) + z \cos(\theta_i)$

$$\rightarrow \frac{\sin\theta_i}{\cos\theta_i} = \frac{4}{3} \rightarrow \theta_i = \tan^{-1}\left(\frac{4}{3}\right) = 0.9273 \text{ rad}$$

Or dot product: $|\bar{V}| \cdot |\bar{W}| \cdot \cos\theta = \bar{V} \cdot \bar{W}$

∴ $\bar{a}_n \cdot \bar{a}_k = 1 \cdot 1 \cdot \cos\theta_i$

$$\rightarrow \theta_i = \cos^{-1}\left(\bar{a}_z \cdot \frac{4\bar{a}_x + 3\bar{a}_z}{5}\right) = \cos^{-1}\left(\frac{3}{5}\right) = 0.9273$$

c) ∵ perpendicular:

$$\Gamma_\perp = \frac{\eta_2 \cos(\theta_i) - \eta_1 \cos(\theta_t)}{\eta_2 \cos(\theta_i) + \eta_1 \cos(\theta_t)}$$

Snell's law: $n_1 \sin(\theta_i) = n_2 \sin(\theta_t)$

$n_1 = 1$, $n_2 = \sqrt{2.5}$

$$\rightarrow \theta_t = \sin^{-1}\left(\frac{\sin(0.9273)}{\sqrt{2.5}}\right) = 0.506 \text{ rad}$$

$\eta_1 = 120\pi \wedge \eta_2 = \frac{120\pi}{\sqrt{2.5}}$

$\rightarrow \Gamma_\perp = -0.389$ (-0.3931)

∴ $E_{r0} = -3.112 e^{j \bar{k}_r \cdot \bar{R}}$, $\bar{k}_r = 4\bar{a}_x - 3\bar{a}_z$

d) $\tau_\perp = 1 + \Gamma = 0.611 \rightarrow E_{t0} = 4.888$

$k_t^2 = \beta_2^2 = 62.5$, $k_{tx} = k_{ix} = 4$

$k_{tz} = k_t \cos(\theta_t) = \frac{5\sqrt{10}}{2} \cdot 0.8762 = \cancel{5}6.9189$

$\rightarrow \bar{E}_t = 4.888 e^{j(4x + 6.9189z)} \bar{a}_y$

$\rightarrow \bar{H}_t = \frac{4.888 \cdot \sqrt{2.5}}{120\pi} \cdot (\bar{a}_k \times \bar{a}_y) e^{-} \quad \bar{a}_k = \frac{4\bar{a}_x + 6.9189\bar{a}_z}{7.9914}$

$-0.86674\bar{a}_x + 0.5005\bar{a}_z$

$= [-0.01995\bar{a}_x + 0.01026\bar{a}_z] e^{j(4x + 6.9189z)}$

practice exercise 10.11: $\bar{E}_i = (10\bar{a}_y + 5\bar{a}_z)\, e^{j(-2y+4z)}$

a) $\because \bar{a}_k = \frac{-2a_y + 4a_z}{\sqrt{20}} \rightarrow \cos(\theta_i) = 0.89443$

$\rightarrow \theta_i = \theta_r = 26.56^\circ$

- from Snell's law $\sqrt{1} \cdot \sin(26.56) = \sqrt{4} \cdot \sin(\theta_t)$

$\rightarrow \theta_t = 12.92^\circ$

b) $\because$ ki is in yz-plane and so is $\bar{E}$

$\rightarrow$ parallel polarization $\rightarrow \Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$

$\therefore \Gamma_{\parallel} = -0.2946 \rightarrow \tau_{\parallel} = (1 + \Gamma_{\parallel}) \frac{\cos(\theta_i)}{\cos(\theta_t)}$

$= -0.295 \qquad = \boxed{0.6493}$

c) reflected $= \bar{E}_{ro} = -0.2946\,[10\bar{a}_y + 5\bar{a}_z] = -2.946\bar{a}_y - 1.473\bar{a}_z$

$\rightarrow \bar{E}_r = [-2.946\bar{a}_y - 1.473\bar{a}_z]\, e^{j(-2y-4z)}$

$\rightarrow E_1 = (10\bar{a}_y + 5\bar{a}_z)\underset{\cos(wt+2y-4z)}{e^{j(-2y+4z)}} - [2.946\bar{a}_y + 1.473\bar{a}_z]\underset{\cos(wt+2y+4z)}{e^{-j(2y+4z)}}$

d) $E_{to} = \tau E_{io} \Rightarrow E_{to} = 0.649 \cdot \sqrt{125} = 7.2339$

$\bar{k}_t = -2\bar{a}_y + 2\bar{a}_z \rightarrow E_{toy} = 7.2339 \cdot \cos(\theta_t) = 7.05$

$k_t = \beta = 8.9466069 \qquad ? = \beta \cos(\theta_t) \qquad E_{toz} = E_{to} \sin\theta_t$

$= 8.92$

$\rightarrow E_{to} = (7.05\bar{a}_y + 1.618\bar{a}_z)\cos(wt + 2y - 8.92z)$ V/m

e) $\theta_{B\parallel} = \tan^{-1}\frac{n_2}{n_1} = 63.4345^\circ$

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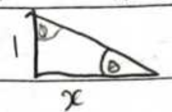
example from notes: $E_i = (50\bar{a}_x + 100 a_y - 50\sqrt{3}\,\bar{a}_z)$
$e^{-j\pi(\sqrt{3}x+z)}$

$\tan\theta = \frac{1}{x}$ $\tan\phi = \frac{x}{1}$



$\phi + \theta = 90°$

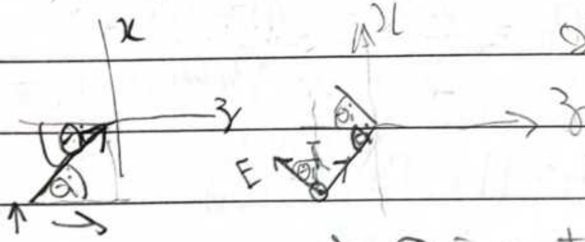
$\theta_i = \theta_{B\parallel}$ ∧ $\varepsilon_{r1} = 1$ ∧ material are non-magnetic

a) $\because \bar{k} = \pi\sqrt{3}\,\bar{a}_x + \pi\,\bar{a}_z \rightarrow k = \beta = 2\pi = \omega\sqrt{\mu_0\varepsilon_0}$

$\rightarrow f = 3\times10^{8}$ Hz

b) $\theta_r = \theta_i$ ∧ $\theta_i = \theta_{B\parallel} = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}}$

or $\theta_{B\parallel} = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$



$\because \tan(\theta_i) = \frac{k_x}{k_z}$

$\rightarrow \theta_i = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 60° = \theta_r$

or $\because \bar{k} = \pi\sqrt{3}\,\bar{a}_x + \pi\,\bar{a}_z = k\sin\theta_i\,\bar{a}_x + k\cos\theta_i\,\bar{a}_z$

$\rightarrow 2\pi\cdot\sin\theta_i = \sqrt{3}\pi \rightarrow \theta_i = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$

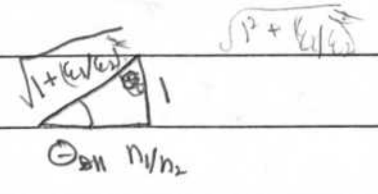
c) $\because \beta_1\sin(\theta_i) = \beta_2\sin(\theta_t)$

or $n_1\sin(\theta_i) = n_2\sin(\theta_t)$, $n_1 = 1$ ∧ $n_2 = ?$

$\because 60° = \theta_{B\parallel} \rightarrow \tan(60) = \frac{n_2}{n_1} \rightarrow n_2 = \sqrt{3}$

$\therefore \theta_t = 30°$

or from $\theta_i = \theta_{B\parallel} \rightarrow \theta_t = 90° - \theta_{B\parallel} = 30°$



d) $\bar{E}_r$ ∧ $\bar{E}_t$

$\because \bar{E}_i = \bar{E}_{i\parallel} + \bar{E}_{i\perp}$ $\because \theta_i = \theta_{B\parallel} \rightarrow$ perpendicular components reflected

$\rightarrow \bar{E}_{r\parallel} = 0$

$\bar{E}_{i\parallel} = (50\,\bar{a}_x - 50\sqrt{3}\,\bar{a}_z)\,e^{-j\pi(\sqrt{3}x+z)}$

$\bar{E}_{i\perp} = 100e^{-j\pi(\sqrt{3}x+z)} \rightarrow \Gamma_\perp = -0.5$

$\rightarrow E_{r\perp} = -50e^{-j\pi(\sqrt{3}x-z)}\,\bar{a}_y$

$\because$ $1+\Gamma_{\perp}=T_{\perp} \rightarrow T_{\perp}=0.5$ λ $1+\Gamma_{\parallel}=T_{\parallel}\frac{\cos\theta_t}{\cos\theta_i}$

$\rightarrow T_{\parallel}=\frac{\cos\theta_i}{\cos\theta_t}=\frac{\sqrt{3}}{3}$

$E_{t\perp} = 50\, e^{-j(\bar{k}\cdot\bar{R})}$ $k_x = k_2 \cdot \sin\theta_t$ $k_2 = \beta_2 = \beta_1 \cdot \sqrt{\varepsilon_r}$

$k_x = 5.4414$ λ $k_z = 9.4248$

$\rightarrow E_{t\perp} = 50 e^{-j(5.4414x+3\pi z)}\, \bar{a}_y$

λ $E_{t\parallel} = E_{io} \cdot T_{\parallel}$ E_{io}: $\because$ $E_{xi} = E_{io} \cdot \cos\theta_i \rightarrow E_{io} = \frac{50}{0.5} = 100$

$\therefore E_{io} = 100$ & $E_{t\parallel} = \frac{100}{\sqrt{3}} \cdot [\cos\theta_t\, \bar{a}_x - \sin\theta_t\, \bar{a}_z]\, e^{-j\bar{k}\cdot\bar{R}}$

$\rightarrow E_{t\parallel} = [50\, \bar{a}_x - \frac{50}{\sqrt{3}}\, \bar{a}_z]\, e^{-j(5.4414x+3\pi z)}$

Or) $E_{io} = \sqrt{50^2 + \sqrt{3}^2\, 50^2} = 100$

Subject Critical angle and polarization Date 28/4/2021 No.

* Critical angle θ_c (total reflection)

- the critical angle is the angle of incidence that gives $\theta_t = \frac{\pi}{2}$

if $\mu_1 = \mu_2 \rightarrow$ Snell's law : $n_1 \sin\theta_i = n_2 \sin\theta_t$

+ for $\varepsilon_2 > \varepsilon_1 \rightarrow \sin\theta_t = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \sin\theta_i < \sin\theta_i$

$\therefore$ $\theta_t < \theta_i$ (closer to normal) hence, θ_c does not exist

+ for $\varepsilon_2 < \varepsilon_1 \rightarrow \sin\theta_t = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin\theta_i > \sin\theta_i \therefore \theta_t > \theta_i$

for $\theta_t = \pi/2 \rightarrow \theta_i = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

- if $\theta_i = \theta_c$ then $\theta_t = \frac{\pi}{2}$, the refracted wave will be along the interface

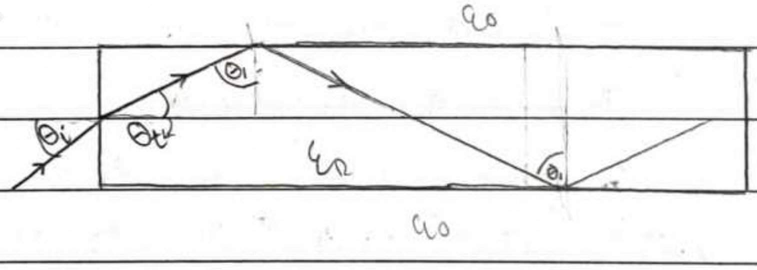
- $\sin(\theta_c) = \frac{n_2}{n_1} = \tan(\theta_{B\parallel})$

- if $\theta_i = \theta_c \rightarrow \theta_t = \frac{\pi}{2}$

$\therefore \Gamma_\perp = 1 , \tau_\perp = 1 + \Gamma_\perp = 2 \wedge \Gamma_\parallel = -1 , \tau_\parallel = \frac{2n_2}{n_1}$

- if $\theta_i > \theta_c$, then all fresnels coefficients will be complex $\Gamma_\parallel , \tau_\parallel , \Gamma_\perp , \tau_\perp$ will all be complex, but $\boxed{|\Gamma_\parallel| = |\Gamma_\perp| = 1}$ in this case, the wave will propagate along the interface (in the x-direction) but it will attenuate in the z-direction. such a wave is called a surface wave.

* optical fiber:



for total internal reflection, $\theta_1 > \theta_c$ for fiber to air

$\rightarrow \sin(\theta_1) \geq \sin(\theta_c) \wedge \theta_1 = 90 - \theta_t$

$\rightarrow \cos(\theta_t) \geq \sin(\theta_c) \rightarrow 1 - \sin^2(\theta_t) \geq \sin^2(\theta_c)$

$\wedge \sin(\theta_t) = \sqrt{\frac{\varepsilon_0}{\varepsilon_r \varepsilon_0}} \cdot \sin\theta_i \wedge \sin(\theta_c) = \frac{1}{\sqrt{\varepsilon_r}}$

$\therefore \frac{1}{\sqrt{\varepsilon_r}} \leq 1 - \frac{\sin^2\theta_i}{\varepsilon_r} \rightarrow \varepsilon_r \geq 1 + \sin^2(\theta_i)$

$\rightarrow \boxed{\varepsilon_r \geq 2}$

example from notes:

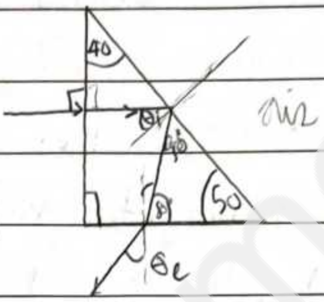
a) no power transmitted across hypotenuse → $\Theta_{ih} \geq \theta_{ch}$

$\because \Theta_{ih} = 90 - 50^\circ = 40^\circ$

$\Theta_c = \sin^{-1}\left(\frac{n_{air}}{n_{med}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{\varepsilon_r}}\right)$

$\to 1/\sqrt{\varepsilon_r} \leq 0.6428$

$\to 1 \leq (0.6428)^2 \varepsilon_r \to \boxed{\varepsilon_r \geq 2.4203}$



b) $\Theta_{ie} = 10^\circ$

$\because n_{med} \sin 10 = n_{air} \sin(\Theta_e)$

$\to \Theta_e = 15.67^\circ$ & $\boxed{\varepsilon_r = 2.42}$

example from notes: $\bar{E}_i = 5(\bar{a}_y + \sqrt{3}\,\bar{a}_z)\, e^{j6(\sqrt{3}y - z)}$

$\because$ incident on a perfect conductor → $T_{\parallel}$ & $T_{\perp} = 0$

for f: $\because \bar{k} = 6\sqrt{3}\,\bar{a}_y - 6\bar{a}_z \to |\bar{k}| = \beta = 12$

& $\beta = \omega\sqrt{\mu_0 \varepsilon_0} \to f = \frac{18}{\pi} \times 10^8$

* polarization of waves:

+ polarization of an antenna: polarization of the wave radiated by the antenna

- polarization of a plane wave is a trace of the electric field's tip as a function of time at a fixed location
- in far field, the wave appears as a plane wave (the phase fronts are planes, $\bar{E} \perp \bar{H} \perp \bar{a}_r$

+ in the following parts, assume: (travelling in -z direction)

$$\bar{E}(z,t) = E_x(z,t)\,\bar{a}_x + E_y(z,t)\,\bar{a}_y$$

$$E_x(z,t) = Re\{E_{x0}\, e^{j\omega t}\, e^{jkz}\, e^{j\phi_x}\}$$

$$= E_{x0}\cos(\omega t + kz + \phi_x)$$

& $E_y(z,t) = E_{y0}\cos(\omega t + kz + \phi_y)$

1) linear polarization: (LP)

conditions: $\Delta\phi = \phi_y - \phi_x = \pm n\pi$, $n=0,1,2$ -

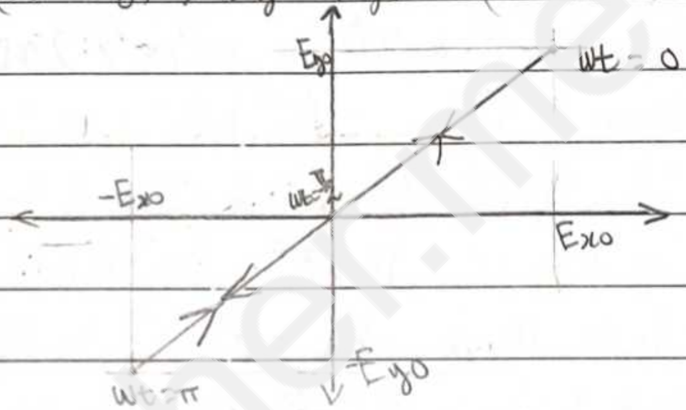
$\boxed{or}$ $E_{x0}=0$ or $E_{y0}=0$

e.g.: if $n=0 \rightarrow \phi_y=\phi_x$ (assume 0)

$\rightarrow E_x = E_{x0}\cos(wt+kz)$, $E_y = E_{y0}\cos(wt+kz)$

Varying t gives:

- if $E_x=0$, polarization will be along the vertical axis and linear
- if $E_y=0$, the polarization will be linear along the horizontal axis.



2) circular polarization: (CP)

if $E_{x0} = E_{y0}$ $\boxed{\&}$ $\Delta\phi = \pm(2n+1)\frac{\pi}{2}$, $n=0,1,2$

example: $\Delta\phi = -\frac{\pi}{2} \rightarrow \phi_x=0, \phi_y=-\frac{\pi}{2}$

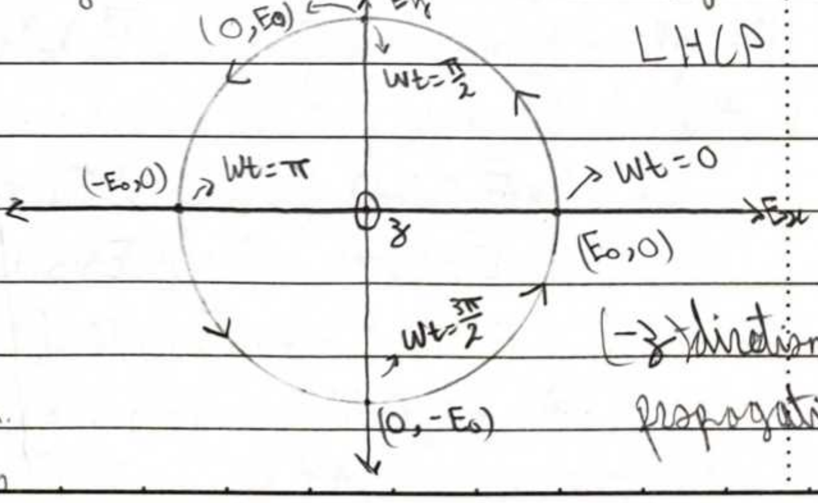
$\therefore E_x(z) = E_0 e^{jkz}$, $E_y(z) = E_0 e^{jkz} e^{-j\frac{\pi}{2}}$

$\Rightarrow E_x(z,t) = E_0\cos(wt+kz)$

$\wedge\ E_y(z,t) = E_0\sin(wt+kz)$

Set $z=0$ & vary t $\rightarrow E_x(t) = E_0\cos(wt) \wedge E_y(t) = E_0\sin(w$

- to determine whether it is left-hand circular polarization or right hand circular polarization use your right hand to curl your fingers along the direction of rotation. if your thumb points in the direction of propagation then it is RHCP, else it is LHCP



LHCP

(-z) direction of propagation

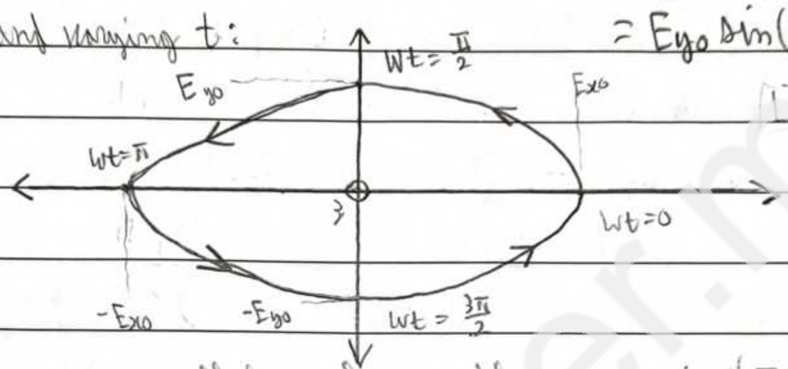
3) elliptical polarization: (EP)

either : ① $E_{0x} \neq E_{0y}$ & $\Delta\phi = \pm (2n+1)\frac{\pi}{2}$, $n = 0, 1, 2, \dots$

or : ② $\Delta\phi \neq \frac{n\pi}{2}$, $n = 0, 1, 2, 3 \dots$

example on ①: $E_x = E_{x0} \cos(wt + kz)$, $E_y = E_{y0} \cos(wt + kz - \frac{\pi}{2})$

setting $z = 0$ and varying t: $= E_{y0} \sin(wt + kz)$

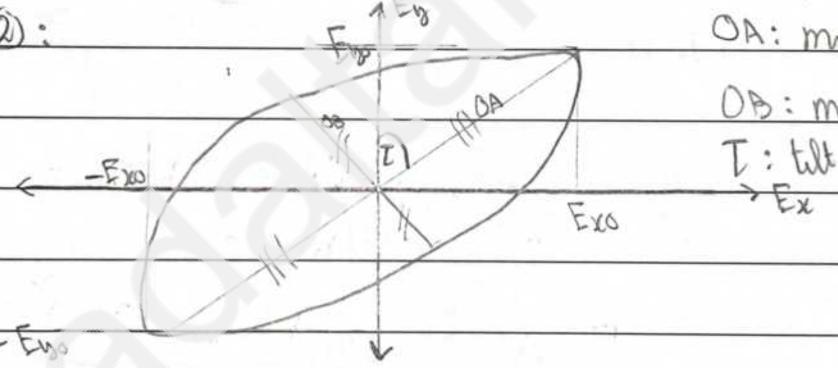


$\boxed{LHEP}$

$E_{x0} > E_{y0}$

- the major axis will be along the x-axis ($E_{x0} > E_{y0}$)
  if ($E_{y0} > E_{x0}$) then the y-axis will be the major-axis

example on ②:



OA: major axis

OB: minor axis

τ : tilt angle

example from notes: $\bar{E} = (3\bar{a}_x + j4\bar{a}_y)\, e^{-0.2z}\, e^{-j0.5z}$

$\rightarrow E_x = 3e^{-0.2z} \cdot \cos(wt - \frac{z}{2})$

& $E_y = \text{Re}\{j4\, e^{-0.02z}\, e^{-j0.5z}\, e^{jwt}\} = -4\, e^{-0.02z} \sin(wt - 0.5z)$

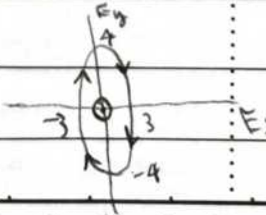
set $z = 0 \rightarrow E_x(t) = 3\cos(wt)$ & $E_y(t) = -4\sin(wt)$

∴ Elliptical polarization at $wt = 0 \rightarrow (3, 0)$ | at $wt = \frac{3\pi}{2} \rightarrow (-3, 0)$

at $wt = \frac{\pi}{2} \rightarrow (0, -4)$

direction of propagation: $(+z)$

∴ $\boxed{LHEP}$



example from notes: $P_{tot} = 40 kW$, $f = 137.5 MHz$ (isotropic)

$\bar{E} = E_0 (-j\bar{a}_x + \bar{a}_y) e^{-j\beta z}$ V/m

a) polarization of wave: CP or LP since $|E_{x0}| = |E_{y0}|$

check $\Delta\phi$, if $\Delta\phi = n\pi$, $n = 0, 1, 2 ..$ then LP

$E_x = Re\{e^{-j\frac{\pi}{2}} e^{-j\beta z} \cdot e^{j\omega t}\} = \cos(\omega t - \beta z - \frac{\pi}{2}) \overset{z=0}{=} \sin(\omega t)$

$\wedge E_y = \cos(\omega t) \rightarrow \Delta\phi = -\frac{\pi}{2} \rightarrow$ CP

at $\omega t = 0 \rightarrow (0, E_0)$, $\omega t = \frac{\pi}{2} \rightarrow (E_0, 0)$

traveling in +z direction → LHCP

b) $\beta = \beta_{freespace} = \omega\sqrt{\mu_0 \epsilon_0} = 2.88$ rad/m

∵ isotropic means the power is distributed across the surface evenly

$\rightarrow P_{avg} = \frac{\boxed{|E_{0x}|^2 + |E_{0y}|^2} \rightarrow 2|E_0|^2}{2\eta_0} = \frac{P_{tot}}{4\pi R^2} = \frac{40k}{4\pi \cdot 1000^2} = \frac{|E_0|^2}{\eta_0}$

$\rightarrow P_{avg} = 3.183\ nW/m^2 \cdot 120\pi = E_0^2$

$\rightarrow \boxed{E_0 = 1.095\ mV/m}$

$P_{avg} = \frac{|E_{x0}|^2 + |E_{y0}|^2 + |E_{z0}|^2}{2\eta}$ density

Subject quiz three practice Date 30/4/2021 No.

$\boxed{1}$ not CP as amplitudes of components differ.

$E_z = 100\cos(wt - \beta x)$ at $x = 0$ → in phase

$E_y = -150\cos(wt - \beta x)$ ∴ linear polarization

power density : $\frac{(100)^2 + (150)^2}{240\pi} = 43.1$ W/m² in direction of propagation

as $\bar{P}_{avg} = \frac{1}{2}Re\{E \times \bar{H}^*\}$

$\bar{H} = \frac{1}{120\pi}\cdot[a_x \times \bar{E}] = \frac{1}{\eta}(-150\,\bar{a}_z - 100\,\bar{a}_y)\,e^{-j\beta x}$ A/m

$= (-0.2652\,\bar{a}_z\ 0.2652\,\bar{a}_y)\,e^{-j\beta x}$ A/m

$E \times H^*$ $\qquad \bar{H}^* = -(0.2652\,\bar{a}_z + 0.2652\,\bar{a}_y)\,e^{j\beta x}$

$$\begin{vmatrix} a_x & a_y & a_z \\ 0 & -150e^{-j\beta x} & 100e^{-j\beta x} \\ 0 & \frac{-100}{\eta_0}e^{j\beta x} & \frac{-150}{\eta_0}e^{j\beta x} \end{vmatrix}$$

$\rightarrow \bar{a}_x\left(\frac{-150\cdot -150}{\eta_0} - \frac{-100\cdot 100}{\eta_0}\right) = \frac{32500}{2\eta_0} = 43.1\,\bar{a}_x$

$\boxed{4}$ $\mu_1 = \mu_2$ $\qquad \Gamma(\theta_i = 0) = 0.1$ $\qquad$ normal incidence

a) $\eta_2\cos\theta_t = \eta_1\cos\theta_i$ or $\frac{n_2}{n_1} = \tan(\theta_{B\parallel})$

∵ $\Gamma = 0.1 = \frac{\eta_1 - \eta_2}{\eta_2 + \eta_1}$ ∧ $T = \frac{2\eta_2}{\eta_1 + \eta_2} = 1.1$

∴ $2\eta_2 = 1.1\,\eta_1 + 1.1\,\eta_2$ →

∧ $\eta_2 = 0.1\,\eta_2 + 0\eta_1 + \eta_1 \rightarrow 0.9\,\eta_2 = 1.1\,\eta_1$

∧ $\frac{\eta_2}{\eta_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{n_1}{n_2} = \frac{1.1}{0.9} \rightarrow \frac{n_2}{n_1} = \frac{9}{11}$

∴ $\theta_{B\parallel} = 39.3°$

b) $|\Gamma_\perp| = 1$, ∵ $\epsilon_2 < \epsilon_1$ take $\theta_i > \theta_c \rightarrow \theta_i > \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$\rightarrow \theta_i > 54.903°$

∴ $|\Gamma| = 1$ for $54.903° \leq \theta_i \leq 90°$

$\cos\theta_t = 0 \Rightarrow \theta_t = 90^\circ$

or $\cos(\theta_i) = 0 \Rightarrow \theta_i = 40^\circ$

$\boxed{II}$ $E_x = \sqrt{5}\cos(\omega t - \beta z + 26.56^\circ)$

$E_y = 2\sqrt{5}\cos(\omega t - \beta z - 153.43^\circ) =$

$\Rightarrow \Delta\phi = -153.43 - 26.56 \simeq -180^\circ$

$\therefore$ linear polarization

$\because P_{tot} = P_{avg} \cdot S_{area}$

$\Rightarrow P_{avg} = \frac{150k}{4\pi R^2} \wedge P_{avg} = \frac{|E_0|^2}{2\eta_0} = \frac{E_{x0}^2 + E_{y0}^2}{2\eta_0}$

$\Rightarrow P_{avg} = \frac{5m^2 + 20m^2}{240\pi} = \frac{150k}{4\pi R^2}$

$R^2(25\times10^{-6}) = \frac{240\pi}{4\pi}\cdot 150k$

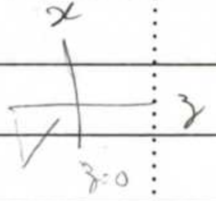
$\Rightarrow R^2 = \frac{60\cdot150k}{25\times10^{-6}} = 3.6\times10^{11}$

$\Rightarrow R = 600\,km$

$\boxed{3}$ $\because \bar{E}_i = 10\,\bar{a}_y\,e^{-j\sqrt{2}\pi(x+z)}$

$\Rightarrow \bar{k} = \sqrt{2}\pi\,\bar{a}_x + \sqrt{2}\pi\,\bar{a}_z$

$\wedge \beta = |k| = 2\pi = \frac{2\pi\cdot200\times10^6\cdot\sqrt{\varepsilon_{r1}}}{c}$



$\Rightarrow \varepsilon_{r1} = 2.25$

$\because k_x = |k|\cdot\sin\theta_i$

$\Rightarrow 2\pi\cdot\sin\theta_i = \sqrt{2}\pi$

$\therefore \theta_i = 45^\circ$

Snell's law: $n_1\sin\theta_i = n_2\sin\theta_t$

$\Rightarrow \theta_t = \sin^{-1}\left(\frac{\sqrt{\varepsilon_{r1}}}{\sqrt{\varepsilon_{r2}}}\cdot\sin\theta_i\right)$

$= 90^\circ$

$\therefore \Gamma_{\parallel} = 1, \; \tau_{\parallel} = \frac{2\eta_2}{\eta_1} = \frac{2\cdot n_1}{n_2} = 2\sqrt{2}$

$\Gamma_\perp = 1 \quad \wedge \quad \tau_\perp = 2$

$\therefore \quad E_{t0\parallel} = 2\sqrt{2} \cdot 10$

$\wedge \quad E_{t0\perp} =$

$\boxed{4}$ $\mu_1 = \mu_2$, $\Gamma_\parallel(\theta_i = 0) = -0.2 \rightarrow \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.2$

$1.2\,\eta_2 = 0.8\,\eta_1 \rightarrow \frac{\eta_2}{\eta_1} = \frac{n_1}{n_2} = \frac{0.8}{1.2} = \frac{2}{3}$

for $\Gamma_\parallel = 0 \rightarrow \eta_2 \cos\theta_t = \eta_1 \cos\theta_i$

$\rightarrow \tan(\theta_{B\parallel}) = \frac{n_2}{n_1} \rightarrow \theta_{B\parallel} = \theta_i = 56.31^\circ$

for $\Gamma_\parallel = 0.5$

$2[\eta_2 \cos(\theta_t) - \eta_1 \cos(\theta_i)] = \eta_2 \cos(\theta_t) + \eta_1 \cos(\theta_i)$

$\rightarrow 3\,\eta_1 \cos(\theta_i) = \eta_2 \cos(\theta_t)$

$\rightarrow 4.5 \cos(\theta_i) = \cos(\theta_t)$

$\rightarrow 20.25\,[1 - \sin^2(\theta_i)] = 1 - \sin^2(\theta_t)$

$\sin^2(\theta_t) = (\frac{n_1}{n_2})^2 \sin^2(\theta_i)$

$\rightarrow 20.25 - 20.25\sin^2\theta_i = 1 - \frac{4}{9}\sin^2(\theta_i)$

$\rightarrow 19.25 = \frac{713}{36}\sin^2(\theta_i)$

$\rightarrow \sin(\theta_i) = 0.985895$

$\rightarrow \theta_i = 80.3985$

$\boxed{5}$ $\bar{E}_i = 8\cos(\omega t - 4x - 3z)\,\bar{a}_y$

$\rightarrow \bar{\beta}_i = 4\bar{a}_x + 3\bar{a}_z \rightarrow |\beta| = 5 = \beta = \omega/c$

$\rightarrow \omega = 1.5$ G rad s^{-1}

$\therefore \quad 4 = |\beta| \cdot \sin\theta_i \rightarrow \theta_i = 53.130^\circ$

non-magn snell's law: $\frac{1}{n_2}\sin\theta_i = \sin\theta_t$

$\rightarrow \theta_t = 30.4^\circ$

$\therefore \quad \tau_\parallel = 0.66255 \rightarrow \tau_\perp = 0.6111$

∴ $E_{to\parallel} = 8 \cdot 0.6255\bar{a}_y = 5$

∧ $E_{to\perp} = 8 \cdot 0.611 = 4.8889$ perpendicular polarization

→

$E_t = 4.888\,\bar{a}_y \cos(\omega t - \bar{k}\cdot\bar{R})$

∵ $\beta_2 = \frac{\omega \cdot \sqrt{\epsilon_r}}{c} = 7.906$

→ $k_x = 7.906 \cdot \sin\theta_t$ ∧ $k_y = 7.906 \cdot \cos\theta_t$

→ $E_t = 4.889 \cos(1.5\times10^9 t - 4x - 6.82z)\,\bar{a}_y$

→ $\bar{H}_t = \frac{1}{\eta_2}(\bar{a}_k \times \bar{E}_t)$

$\bar{a}_k = \frac{4\bar{a}_x + 6.82\bar{a}_z}{7.9065} = 0.5059\bar{a}_x + 0.86258\bar{a}_z$

→ $\bar{a}_k \times \bar{E}_t = -4.21715\bar{a}_x + 2.49374\bar{a}_z$

→ $\bar{H}_t = \frac{\sqrt{\epsilon_r}}{120\pi}(\bar{a}_k \times \bar{E}_t) = -17.68\bar{a}_x + 10.375\bar{a}_z$

→ $\bar{H}_t = (-17.68\bar{a}_x + 10.375\bar{a}_z)\cos(1.5\times10^9 t - 4x - 6.82z)$ mA/m

Subject 11.2 & 11.3: transmission line parameters and equations Date 30/4/2021 No.

R: resistance per unit length (Ω/m)

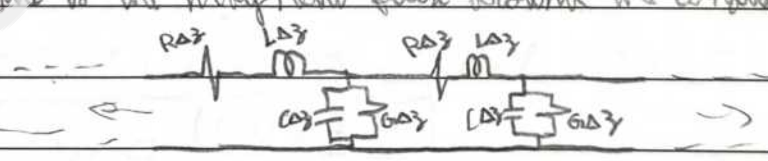
L: inductance per unit length (H/m)

C: capacitance per unit length (F/m)

G: conductance per unit length (S/m)

- the above parameters are not discrete or lumped, instead they are distributed along the line.
- only TEM wave propagation inside transmission lines will be studied (direction of propagation is along the line)
- $G \neq \frac{1}{R}$, R is the ac resistance per unit length due to the conductors, whereas G is the conductance per unit length due to the dielectric separating the conductor.
- for each line $LC = \mu\varepsilon$, $\frac{G}{C} = \frac{\sigma}{\varepsilon}$
- G is due to the lossy dielectric between the conductors. if $\sigma_d = 0$ then $G = 0$
- if $\sigma_c = \infty$ then $R = 0$ (perfect conductor)
- C is the capacitance between the two conductors, L is the external inductance due to the magnetic flux around the conductors.

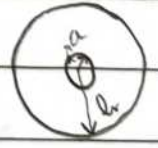
* transmission line:



* coaxial T.L.: ($\bar{E} \times \bar{H}$) poynting vector points in direction of propagation along T.L.

inner cylinder radius: a

outer cylinder radius: b



$$R = \frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a} + \frac{1}{b}\right] \ (\Omega/m)$$

$$L = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) \ (H/m)$$

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} \ (F/m)$$

$\varepsilon = \varepsilon_0\varepsilon_r$ dielectric

$$G = \frac{2\pi\sigma_d}{\ln(b/a)} \ (S/m)$$

* parallel plate T.L : (planar line)

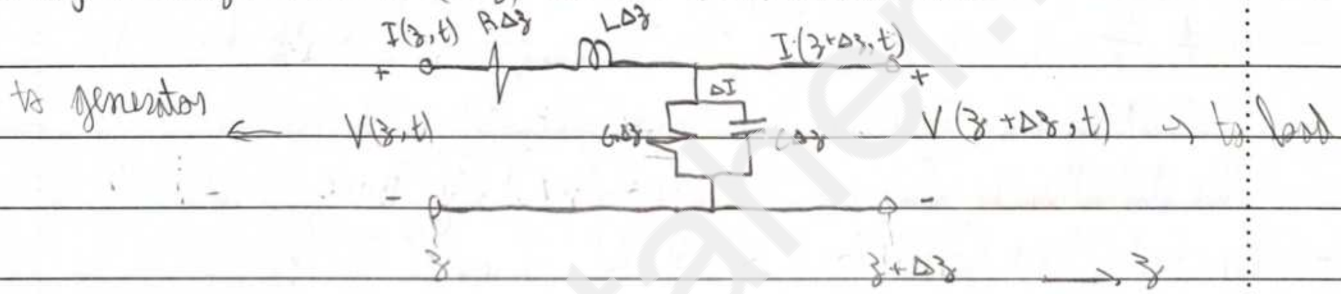
W : width of plates , d : distance between plates

$W >> d$

$R = \frac{2}{W\delta d}$ (Ω/m) , $L = \frac{\mu d}{W}$ (H/m) , $C = \frac{\varepsilon W}{d}$ (F/m)

$G = \frac{\sigma_d W}{d}$ (S/m)

- Circuit theory will be used in this chapter : $V = -\int \bar{E} \cdot d\bar{l}$, $I = \oint \bar{H} \cdot d\bar{l}$
- taking a single increment (Δz) in the transmission line:



* taking KVL :

$$V(z,t) = I(z,t)\,R\Delta z + \frac{dI(z,t)}{dt} L\Delta z + V(z+\Delta z,t)$$

rearrange :

$$-\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = R\,I(z,t) + L\frac{dI(z,t)}{dt}$$

take the limit as Δz approaches zero

$$\boxed{\frac{-dV(z,t)}{dz} = R \cdot I(z,t) + L \cdot \frac{dI(z,t)}{dt}}$$

* taking KCL gives : $I(z,t) - I(z+\Delta z,t) = \Delta I = V(z+\Delta z,t) \cdot G\Delta z + C\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t}$

$$\rightarrow -\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = V(z+\Delta z,t) \cdot G + C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

taking $\Delta z \rightarrow 0$

$$\rightarrow \boxed{-\frac{dI(z,t)}{dz} = V(z,t) \cdot G + C\frac{dV(z,t)}{dt}}$$

- assuming harmonic time dependence and taking phasors:

$$\boxed{\frac{-dV_s}{dz} = (R+j\omega L)\, I_s} \quad \lambda \quad \boxed{\frac{-dI_s}{dt} = (G+j\omega C)\, V_s}$$

- differentiating with respect to z:

$$\frac{+d^2V_s}{dz^2} = (R+j\omega L)\frac{dI_s}{dz} = (R+j\omega L)(G+j\omega C)\, V_s$$

$$\therefore \quad \boxed{\frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0} \quad \lambda \quad \boxed{\frac{d^2I_s}{dz^2} - \gamma^2 I_s = 0}$$

where γ is the propagation constant for T.L

$\rightarrow \gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{ZY}$

α: attenuation constant, β: phase constant

- define Z: series impedance per unit length. $Z = R + j\omega L$
- define Y: shunt admittance per unit length: $L = G + j\omega C$
- wave length: $\lambda = \frac{2\pi}{\beta}$, phase velocity: $u_p = \frac{\omega}{\beta} = f\lambda$

Subject quiz practice continued Date 30/4/2021 No.

$\boxed{4}$ $\because \Gamma(\theta_i = 0) = 0.1 \rightarrow \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.1 \rightarrow 0.9\eta_2 = 1.1\eta_1$

$\rightarrow \frac{n_1}{n_2} = \frac{1.1}{0.9}$

a) $\Gamma_{\parallel} = 0$ at $\theta_i = \theta_{B\parallel} = \tan^{-1}\left(\frac{n_2}{n_1}\right) = 39.289^\circ$

b) $|\Gamma_{\perp}| = 1$ if $\theta_i > \theta_c$ for $\varepsilon_2 < \varepsilon_1 \rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$\rightarrow \theta_i \geqslant 54.9^\circ$ but smaller than 90°

$\boxed{5}$ $\because$ perpendicular $\bar{E}$

$\bar{k}_1 = 4\bar{a}_x + 3\bar{a}_z \rightarrow 4 = |k| \sin\theta_i$

$\rightarrow \theta_i = 53.13^\circ$ $\because$ non-mag $\wedge$ $\varepsilon_{r2} = 2.5$

$\rightarrow \theta_t = \sin^{-1}\left(\frac{1}{\sqrt{2.5}} \sin(\theta_i)\right) = 30.395^\circ$

$\therefore \tau_{\perp} = 0.61105 \rightarrow E_{t0} = 8\tau_{\perp} = 4.888$

$\rightarrow \bar{E}_t = 4.888 \cos(\omega t - \bar{k}_2 \cdot \bar{R})\, \bar{a}_y$

$\because \bar{k}_1 = 4\bar{a}_x + 3\bar{a}_z \rightarrow \beta_1 = 5 = \omega / c$

$\rightarrow \beta_2 = \frac{5c \cdot \sqrt{\varepsilon_{r}}}{c} = 5 \cdot \sqrt{\varepsilon_{r2}} = 7.90569$

$\rightarrow k_{x2} = |k| \sin\theta_t \approx 4 \wedge k_{z2} = |k| \cos(\theta_t) = 6.819$

$\wedge\ \bar{a}_{k} = \frac{4\bar{a}_x + 6.819\bar{a}_z}{7.90569} = 0.50596\,\bar{a}_x + 0.82254\,\bar{a}_z$

$\rightarrow (\bar{a}_{k} \times \bar{E}_t) = -4.2161\,\bar{a}_x + 2.4731\,\bar{a}_z$

$\rightarrow \bar{H}_t = \frac{\sqrt{\varepsilon_r}}{\eta_0}(\bar{a}_{k} \times \bar{E}_t)$

$= (-17.68\,\bar{a}_x + 10.37\,\bar{a}_z) \cos(1.5\times10^{9} t - 4x - 6.823z)$

$\boxed{3}$ perpendicular:

$\because \bar{k} = \sqrt{2}\pi\,\bar{a}_x + \sqrt{2}\pi\,\bar{a}_z \rightarrow \beta = 2\pi = 2\pi f \sqrt{\mu_0 \varepsilon}$

$\rightarrow c = f\sqrt{\varepsilon_{r1}} \rightarrow \varepsilon_{r1} = 2.25$

$\wedge \because |k| \cdot \sin(\theta_i) = k_x = \sqrt{2}\pi \rightarrow \theta_i = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\rightarrow \theta_i = 45^\circ \rightarrow \theta_t = \sin^{-1}\left(\frac{n_1}{n_2}\sin(\theta_i)\right) = 90^\circ$

$\rightarrow \tau_{\perp} = 2$ $\beta_2 = \sqrt{2}\pi \rightarrow k_{2z} = 0 \wedge k_{2x} = \sqrt{2}\pi$

$\rightarrow E_{t0} = 20\,\bar{a}_y$

$\rightarrow \bar{E}_t = 20\, e^{-j\sqrt{2}\pi x}\, \bar{a}_y$

MOHAMMAD SANAD ALTAHER 130806

Subject quiz #3 Date 30/4/2021 No.

أنا محمد سند الطاهر أشهد أني قرأت وفهمت وطبقت تعليمات هذا الإمتحان، ولم

أتلق أي مساعدة من أي شخص في حل هذا الإمتحان. أنا لست بغشاش ولا كذاب.

$\varepsilon_{r1} > \varepsilon_{r2}$, $\varepsilon_{r1} = 4$, $\varepsilon_{r2} = 1$

$E_2 = -100 e^{-j10x} \bar{a}_z \rightarrow \beta_2 = 10 \rightarrow w = 3 \times 10^9 \text{ rad s}^{-1}$

$\bar{k}_2 = 10 \bar{a}_x \rightarrow \theta_t = 90^\circ \therefore \theta_i = \theta_c$

$\rightarrow \theta_c = \sin^{-1}\left(\frac{1}{4}\right) = 14.478^\circ$

$\because$ parallel polarization : $T_{\parallel} = \frac{2\eta_2}{\eta_1}$, $\eta_2 = \eta_0 \wedge \eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}}$

$\rightarrow |E_{i0}| = 1/4 \cdot |E_{t0}| = \boxed{25}$

$\bar{E}_i = E_{i0}[\sin\theta_i \bar{a}_x - \cos\theta_i \bar{a}_z] e^{-j\bar{k}\cdot\bar{R}}$

$\bar{k} = \beta_1 \cdot \sin\theta_i \bar{a}_x + \beta_1 \cos\theta_i \bar{a}_z \wedge \boxed{\beta_1 = 20 \text{ rad/m}}$

$\rightarrow \boxed{\bar{k} = 5 \bar{a}_x + 19.365 \bar{a}_z}$

$\rightarrow \bar{a}_k = 0.25 \bar{a}_x + 0.9682 \bar{a}_z$

$\therefore \bar{H}_i = \frac{1}{\eta_1}(\bar{a}_k \times \bar{E}_i)$

$\rightarrow \bar{a}_k \times \bar{E}_i = 12.103 \bar{a}_y$

$\rightarrow \boxed{\bar{H}_i = \frac{1}{60\pi} \cdot (12.103 \bar{a}_y) e^{-j(5x + 19.365z)}}$

$\rightarrow \bar{H}_i = 64.21 e^{-j(5x + 19.365z)} \bar{a}_y$

Subject 11.3: transmission line equations continued Date 1/5/2021 No.

+ the solutions to the wave equations:

$$\frac{d^2 V_s}{dz^2} - \gamma^2 V_s = 0$$

$$\wedge \quad \frac{d^2 I_s}{dz^2} - \gamma^2 I_s = 0$$

are:

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

where $V_0^+ e^{-\gamma z}$ is traveling in the $+z$ direction

and $V_0^- e^{\gamma z}$ is traveling in the $-z$ direction

$\wedge$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

where $I_0^+ e^{-\gamma z}$ and $I_0^- e^{\gamma z}$ are traveling in the positive and negative z directions respectively.

* characteristic impedance (Z_0): the ratio of the positively traveling voltage wave to the positively traveling current wave at any point on the line

$$\underbrace{Z_0}_{\text{characteristic impedance}} = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \underbrace{R_0}_{\text{characteristic resistance}} + j\underbrace{X_0}_{\text{characteristic reactance}}$$

Characteristic admittance: $\frac{1}{Z_0}$

* lossless line: $\boxed{R = G = 0}$

- conductor is perfect ($\sigma_c = \infty$) and dielectric separating conductors is lossless ($\sigma_d = 0$)

$$\rightarrow \gamma = j\beta = j\omega\sqrt{LC}$$

$$\wedge \quad Z_0 = \sqrt{L/C} \rightarrow X_0 = 0 \wedge R_0 = \sqrt{\frac{L}{C}} = Z_0$$

* distortionless line : $\frac{R}{L} = \frac{G}{C}$

- attenuation constant is frequency independent, whereas phase constant is linearly dependent on frequency.

$$\gamma^2 = (R + j\omega L)(G + j\omega C) = RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)$$

$$\rightarrow \gamma = \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta$$

$$\rightarrow \boxed{\alpha = \sqrt{RG}} \quad \wedge \quad \boxed{\beta = \omega\sqrt{LC}}$$

$$\wedge \quad Z_0 = \sqrt{\frac{R(1 + j\omega L/R)}{G(1 + j\omega C/G)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = \boxed{R_0} + \cancel{jX_0}$$

- u & Z_0 for distortionless lines are the same for lossless

example 11.1:

$\because \sigma_{air} \simeq 0 \rightarrow$ lossless $\wedge$ $\beta = 3$ at $\omega = 2\pi \cdot 100\,MHz$

$\wedge \quad Z_0 = 70\,\Omega$

$\because \beta = \omega\sqrt{LC} \quad \wedge \quad Z_0 = \sqrt{\frac{L}{C}}$

$\rightarrow \sqrt{LC} = 4.7746 \times 10^{-9} \rightarrow C \cdot 70 = \sqrt{LC}$

$\rightarrow C = 68.21\,pF/m \rightarrow L = 334.22\,nH/m$

practice exercise 11.1:

$\because Z_0$ real $\rightarrow$ distortionless $\rightarrow \alpha = \sqrt{RG} \quad \wedge \quad \beta = \omega\sqrt{LC}$

$\wedge \quad Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \rightarrow G \cdot Z_0 = \alpha \rightarrow G = 0.5\,mS/m$

$\rightarrow R = 80^2 \cdot G = 3.2\,\Omega/m$

$\wedge \quad C Z_0 = \sqrt{LC} \quad \wedge \quad \sqrt{LC} = \frac{\beta}{\omega}$

$\rightarrow C = 5.968\,pF/m$

$\wedge \quad L = 38.19\,nH/m$

example 11.2:

$\because u = \frac{w}{\beta} = 0.6 \cdot 3\times10^8 \rightarrow \beta = \frac{10}{9}\pi = w\sqrt{LC}$

$\rightarrow \frac{1}{\sqrt{LC}} = 0.6\cdot3\times10^8$

distortionless $\rightarrow \alpha = \sqrt{RG} \wedge Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$

$\because \sqrt{LC} = 5.55\times10^{-9} \rightarrow Z_0\cdot C = 5.55\times10^{-9} \rightarrow C = 92.59$ pF/m

$\wedge L = 333.34$ nH/m

$\because \alpha = \sqrt{RG} \rightarrow G\cdot Z_0 = \alpha \rightarrow G = 0.33$ mS/m

$\wedge \frac{Z_0}{R} = \frac{1}{\alpha} \rightarrow Z_0\cdot\alpha = R = 1.2\ \Omega/m$

$\wedge \lambda = \frac{2\pi}{\beta} = \frac{2\pi\cdot9}{10\pi} = 1.8$ m

practice exercise 11.2:

distortionless : $Z_0 = \frac{R}{G}$ X

use exact : $Z_0 = \sqrt{\frac{R+jwL}{G+jwC}} = \sqrt{5005.69}\ \angle\frac{-2.7336^\circ}{2}$

$\rightarrow Z_0 = 70.75 \angle -1.3668^\circ$

$\gamma = \sqrt{(R+jwL)(G+jwC)} = 8.8908\times10^{-3}\angle 88.63^\circ$

$= \boxed{2.125677\times10^{-4} + 8.888\times10^{-3}\, j}$

$u = \frac{w}{\beta} = \frac{2000\pi}{8.888\times10^{-3}} = 707$ km/s

- for an infinite length line, no reflection occurs

$V(z) = V_0^+ e^{-\gamma z} \wedge I(z) = I_0^+ e^{-\gamma z}$

$\rightarrow Z_0 = \frac{V(z)}{I(z)}$

+ in general : $V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$

$\wedge \boxed{I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}}$

+ for a coaxial line : $\boxed{Z_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right)}$ | for a lossless parallel plate : $\boxed{\eta\frac{d}{w}}$

example from notes:

$\because$ air-filled $\rightarrow \sigma_d = 0 \rightarrow G = 0$ λ Z_0 real $\rightarrow R = 0$

$\therefore$ lossless $\rightarrow Z_0 = \sqrt{\frac{L}{C}}$ $\because LC = \sqrt{\mu_0 \epsilon_0}$

$\rightarrow Z_0 = \frac{\sqrt{LC}}{C} \rightarrow C = \frac{1}{Z_0 \cdot c} = 47.62$ pF/m

λ $L = 4900 \cdot C = 0.233$ μF/m

$\because \beta = w\sqrt{LC} \rightarrow \beta = \frac{w}{c} = \frac{2}{3}\pi$ rad m^{-1}

example from notes:

$\because Z_0$ real λ distortionless $\rightarrow \frac{R}{L} = \frac{G}{C}$

λ $Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$ λ $\alpha = \sqrt{RG} = 0.01$ dB/m

$1\,Np = 20\log_{10}(e)\,dB \rightarrow 0.01\,dB = 1.1513$ mNp/m

$\rightarrow G \cdot Z_0 = \alpha \rightarrow G = 23.03$ μS/m

λ $R = \alpha^2 / G^2 = 57.56$ mΩ/m

$Z_0^2 \cdot C = L \rightarrow L = 250$ nH/m

$\because u_p = \frac{w}{\beta}$ λ $\beta = w\sqrt{LC} \rightarrow u_p = \frac{1}{\sqrt{LC}} = 2 \times 10^8$ m/s

in 1 km $V = V_0 \cdot e^{-\alpha 1000} \rightarrow \% = 31.6\%$

11.4: input impedance, SWR, and power 2/5/2021

- given an initial voltage and current at ($z=0$)

$$V_0 = V(z=0) \wedge I_0 = I(z=0)$$

solving the wave equations: $V_0 = V_0^+ + V_0^-$

$$I_0 = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

$$\rightarrow V_0^+ = \frac{1}{2}(V_0 + Z_0 I_0) \wedge V_0^- = \frac{1}{2}(V_0 - Z_0 I_0)$$

- given voltage and current at ($z=l$), $V(z=L) = V_L \wedge I(z=L) = I_L$

$$\rightarrow V_0^+ = \frac{1}{2}(V_L + Z_0 I_L) e^{\gamma L} \wedge V_0^- = \frac{1}{2}(V_L - Z_0 I_L) e^{-\gamma L}$$

- the voltage and current at any point:

$$\because V_L = I_L \cdot Z_L \rightarrow V_0^+ = \frac{1}{2} I_L (Z_L + Z_0) e^{\gamma L}$$

$$\wedge V_0^- = \frac{1}{2} I_L (Z_L - Z_0) e^{-\gamma L}$$

$$\rightarrow V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$= \frac{1}{2} I_L \left[(Z_L + Z_0) e^{\gamma(L-z)} + (Z_L - Z_0) e^{-\gamma(L-z)} \right]$$

expand then apply $e^{\theta} + e^{-\theta} = 2\cosh\theta$

and $e^{\theta} - e^{-\theta} = 2\sinh\theta$

and define $L - z = z'$ (distance from load)

$$\rightarrow \boxed{V(z') = I_L \left[Z_L \cosh(\gamma z') + Z_0 \sinh(\gamma z') \right]}$$

$$\rightarrow I(z) = \frac{1}{Z_0}\left(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z} \right)$$

$$\rightarrow \boxed{I(z') = \frac{I_L}{Z_0} \left[Z_L \sinh(\gamma z') + Z_0 \cosh(\gamma z') \right]}$$

- input impedance at a distance z' from the load:

$$Z(z') = \frac{V(z')}{I(z')} = Z_0 \frac{Z_L \cosh(\gamma z') + Z_0 \sinh(\gamma z')}{Z_L \sinh(\gamma z') + Z_0 \cosh(\gamma z')}$$

where $\tanh = \frac{\sinh}{\cosh}$

$$\boxed{Z(z') = Z_0 \frac{Z_L + Z_0 \tanh(\gamma z')}{Z_0 + Z_L \tanh(\gamma z')}} \quad \boxed{\text{lossy}}$$

- for a lossless line: $\alpha = 0$ & $\gamma = j\beta$

$\because$ $\tanh(j\theta) = j\tan(\theta)$

$\cosh(j\theta) = e^{j\theta} + e^{-j\theta} = \cos\theta$

$$\therefore \boxed{Z(z') = Z_0 \frac{Z_L + jZ_0 \tan(\beta z')}{Z_0 + jZ_L \tan(\beta z')}} \text{ lossless}$$

* electrical length (frequency dependent): $\beta l = \frac{2\pi}{\lambda} \cdot l$

* Voltage reflection coefficient: $\Gamma(z) = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}}$

$$\rightarrow \Gamma(z) = \frac{\frac{1}{2} I_L (Z_L - Z_0) e^{-\gamma L}}{\frac{1}{2} I_L (Z_L + Z_0) e^{\gamma L}} \cdot e^{2\gamma z}$$

$$\therefore \boxed{\Gamma(z') = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma z'}}$$

- taking $z' = 0 \rightarrow z = L$

$$\rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma_L| e^{j\theta_{\Gamma_L}}$$ (since Z_0 & Z_L are complex)

$$\therefore \Gamma(z') = \Gamma_L e^{-2\gamma z'} = |\Gamma_L| e^{-2\alpha z'} \cdot e^{-j(2\beta z' - \theta_{\Gamma_L})}$$

$$\rightarrow \theta_{\Gamma} = \theta_{\Gamma_L} - 2\beta z'$$

- for a lossless line, $\alpha = 0 \rightarrow |\Gamma(z)| = |\Gamma_L|$, constant
- current reflection coefficient at any point is the negative of the voltage reflection coefficient at that point $[-\Gamma(z')]$

$$\therefore V(z) = V_0^+ e^{-\gamma z} + V_0^+ e^{-\gamma z} \cdot \Gamma(z)$$

$$= V_0^+ e^{-\gamma z} [1 + \Gamma(z)] = \boxed{V_0^+ e^{-\gamma z} [1 + \Gamma_L e^{-2\gamma z'}]}$$

$$\text{\&} \quad \boxed{I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} \left[1 - \Gamma_L e^{-\gamma(L - z)}\right]}$$

$$\therefore Z(z) = \frac{V(z)}{I(z)} = \boxed{Z_0 \cdot \frac{1 + \Gamma(z)}{1 - \Gamma(z)}}$$

$$\because\ Z(z) = Z_0 \cdot \frac{1+\Gamma(z)}{1-\Gamma(z)} \rightarrow \boxed{\Gamma(z) = \frac{Z(z) - Z_0}{Z(z) + Z_0}}$$

- if $\Gamma(z) = 0 \wedge \Gamma_L = 0 \rightarrow \boxed{Z(z) = Z_0}$

$$\therefore\ V(z) = V_0^+ e^{-\gamma z} = V_0 e^{-\gamma z} \wedge I(z) = I_0^+ e^{-\gamma z} = I_0 e^{-\gamma z}$$

- matched load, transmission line appears of infinite length.
- matching allows for maximum power transfer.

* lossless line case: $\alpha = 0$

$$\rightarrow V(z) = V_0^+ e^{-j\beta z}\left[1 + \Gamma_L e^{-j2\beta z'}\right]$$

where $V(z)$ is a standing wave.

- standing wave ratio: $SWR = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$

$$\because\ |\Gamma| = |\Gamma_L| \rightarrow SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \wedge \boxed{1 \leq SWR < \infty}$$

$$\because\ V(z) = V_0^+ e^{-j\beta z}\left[1 + |\Gamma| e^{j\theta_r} e^{-j2\beta z'}\right]$$

$$\wedge\ I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z}\left[1 - |\Gamma| e^{j\theta_r} e^{-j2\beta z'}\right]$$

→ V_{max} & I_{min} occur at the same point. V_{min} & I_{max} occur at the same point.

$$V_{max} = |V_0^+| \cdot [1 + |\Gamma|] \wedge I_{min} = \frac{|V_0^+|}{|Z_0|} \cdot [1 - |\Gamma|]$$

1- at $V(z) = V_{max} \rightarrow \theta_\Gamma - 2\beta z' = -2n\pi$, $n = 0, 1, 2, \ldots$

$$\therefore\ z'_{max} = \frac{\theta_{\Gamma_L} + 2n\pi}{2\beta},\ \beta = \frac{2\pi}{\lambda}$$

- define the maximum input impedance along the line: real quantity

$$Z_{max} = \frac{V_{max}}{I_{min}} = Z_0 \frac{1+|\Gamma|}{1-|\Gamma|} = \boxed{Z_0 (SWR)}$$

2- at $V(z) = V_{min} \rightarrow \boxed{\theta_r - 2\beta z' = -(2n+1)\pi}$ $n = 0, 1, 2, 3 \ldots$

$$\therefore\ z_{min} = \frac{\theta_r + (2n+1)\pi}{2\beta}$$

$$V_{min} = |V_0^+| \cdot (1 - |\Gamma|)$$

$$I_{max} = \frac{|V_0^+|}{Z_0}(1 + |\Gamma|)$$

- define the minimum input impedance: real quantity

$$Z_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0}{SWR}$$

- the distance between two successive maxima (or minima) can be found as : $d = z'_{max}|_{n=1} - z'_{max}|_{n=0} = \frac{\lambda}{2}\ m$

+ special loads:

A- shorted-line ($Z_L = 0$) :

$$\Rightarrow Z(z') = jZ_0 \tan(\beta z')\ ,\ SWR = \infty\ \wedge\ \Gamma_L = -1$$

- $Z(z')$ is pure imaginary (pure reactance). hence, the transmission line is either capacitive or inductive depending on z'.

B - open - circuit ($Z_L = \infty$):

$$Z(z') = Z_0 \cdot \frac{Z_L + jZ_0 \tan(\beta z')}{Z_0 + jZ_L \tan(\beta z')} \quad \text{as } Z_L \rightarrow \infty$$

$$\rightarrow \boxed{Z(z') = \frac{Z_0}{j\tan(\beta z')}} = \boxed{-j\cot(\beta z') \cdot Z_0}$$

$$\wedge\ \boxed{\Gamma_L = 1}\ ,\ SWR = \infty\ ,\ \theta_{\Gamma_L} = 0$$

- the current in the transmission line does not equal 0:

$$\because |V(z)| = |V_0^+| \cdot |1 + |\Gamma_L| e^{j(\theta_{\Gamma_L} - 2\beta z')}|$$

$$\rightarrow |V(z')| = |V_0^+| \cdot |1 + |\Gamma_L| e^{-j2\beta z'}| = |V_0^+| \cdot |1 + e^{-j2\beta z'}|$$

$$\Rightarrow \boxed{|V(z')| = 2|V_0^+| \cdot |\cos(\beta z')| = |V_L| \cdot |\cos(\beta z')|}$$

$$\therefore |I(z)| = \frac{2|V_0^+|}{Z_0} \cdot |\sin(\beta z')|$$

- as the current must equal to zero at $z' = 0$

- due to fringing, an ideal open circuit is not possible in practice.

+ relation between Z_{oc} ∧ Z_{sc} : $\boxed{Z_{oc} \cdot Z_{sc} = Z_0^2}$

$$\wedge\ \tan(\beta z') = \sqrt{\frac{-Z_{sc}}{Z_{oc}}}$$

C - matched load: $Z_L = Z_0 \rightarrow \Gamma_L = 0$, $SWR = 1$

$\rightarrow V(z) = V_0^+ e^{-j\beta z}$, $I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z}$

- no standing wave, no reflection, as if the line is of infinite length
- input impedance at any point along the line is equal to the characteristic impedance $\rightarrow$ $(Z(z) = Z_0)$
- all of the incident power is delivered to the load (max power transfer)

D - Resistive load, $Z_L = R_L$ (lossless line)

$\rightarrow \Gamma_L = \frac{R_L - R_0}{R_L + R_0}$ = pure real number

1) if $\theta_{\Gamma_L} = 0 \rightarrow R_L > R_0$ λ $\Gamma_L > 0$

V_{max} & I_{min} occur when: $\theta_\Gamma - 2\beta z'_{max} = -2n\pi$, $n = 0, 1, 2$

$\rightarrow z'_{max} = \frac{n\pi}{\beta} = \frac{n}{2}\lambda = \boxed{n\frac{\lambda}{2}}$, $n = 0, 1, 2, 3 \ldots$

- $V(z)$ is max at $z' = 0$, min at $\frac{\lambda}{4}$, max again at $\frac{\lambda}{2}$ …
- Where $V(z)$ is max, $I(z)$ is min
- $\boxed{SWR = \frac{R_L}{R_0}}$

2) if $R_L < R_0 \rightarrow \theta_{\Gamma_L} = \pi$

- in this case, $V(z)$ starts at min (at $z' = 0$)

$\therefore z'_{min} = \frac{n\lambda}{2}$, $n = 0, 1, 2 \ldots$

λ $\boxed{SWR = \frac{R_0}{R_L}}$

E - Reactive load, $\boxed{Z_L = jX_L}$

$\Gamma_L = \frac{jX_L - R_0}{jX_L + R_0} \rightarrow |\Gamma_L| = 1$

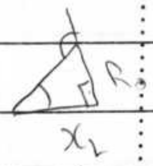
$\tan\left(\frac{3}{4}\right)$: first quadrant

$\tan\left(\frac{-3}{-4}\right)$: third quadrant $\theta_{\Gamma_L} = \tan^{-1}\left(\underbrace{\frac{X_L}{-R_0}}_{2^{nd}\text{ quadrant}}\right) - \tan^{-1}\left(\underbrace{\frac{X_L}{R_0}}_{1^{st}\text{ quadrant}}\right)$

$\tan\left(\frac{3}{-4}\right)$: second quadrant

$\tan\left(\frac{-3}{4}\right)$: fourth quadrant $\because \tan^{-1}(x) + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$

$\rightarrow \tan^{-1}\left(\frac{x_L}{-R_0}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{-R_0}{x_L}\right)$



λ tan is an odd function

$$\therefore \theta_{\Gamma_L} = \frac{\pi}{2} + \tan^{-1}\left(\frac{R_0}{X_L}\right) - \frac{\pi}{2} + \tan^{-1}\left(\frac{R_0}{X_L}\right) = \boxed{2\tan^{-1}\left(\frac{R_0}{X_L}\right)}$$

∧ $|V(z')| = 2|V_0^+| \cdot \left|\cos\left(\beta z' - \frac{\theta_{\Gamma_L}}{2}\right)\right|$

- has the same maximum value as the open circuit voltage.
- case E's voltage magnitude is the same as that for the open-circuited line but with a phase shift of $\frac{\theta_{\Gamma_L}}{2}$

+ the value of θ_{Γ_L} depends on the type of load:

1- inductive load → $X_L > 0$

$\therefore \quad 0 < \theta_{\Gamma_L} < \pi$ (first or second quadrant)

V_{max} at $\theta_{\Gamma_L} - 2\beta z'_{max} = -2n\pi \rightarrow z'_{max} = \frac{\theta_{\Gamma_L} + 2n\pi}{2\beta}$

for $n = 0, 1, 2 \ldots$

V_{min} at $\theta_{\Gamma_L} - 2\beta z'_{min} = -(2n+1)\pi \quad n = 0, 1, 2 \ldots$

$\rightarrow \quad z'_{min} = \frac{\theta_{\Gamma_L} + (2n+1)\pi}{2\beta}$

- hits a max voltage value first

2- capacitive load: $X_L < 0$

$\rightarrow \pi < \theta_{\Gamma_L} < 2\pi$

- hits a min voltage value first

F - quarter-wave section: $l = (2n+1)\frac{\lambda}{4}, \quad n = 0, 1, 2 \ldots$

$\rightarrow \beta l = \frac{2\pi}{\lambda} \cdot l = \frac{\pi}{2}(2n+1), \; n = 0, 1, 2 \ldots$

$\therefore \tan(\beta l) = \pm\infty$

$$\rightarrow \quad \boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$

- if $Z_L = \infty$ (open-circuited) : $Z_{in} = 0$
- if $Z_L = 0$ (short-circuited) : $Z_{in} = \infty$

G - half-wave section: $l = n\frac{\lambda}{2}, \quad n = 0, 1, 2$

$\rightarrow \beta l = n\pi \rightarrow \tan(\beta l) = 0 \rightarrow \boxed{Z_{in} = Z_L}$

* power transfer:

$$\therefore P_{avg} = \frac{1}{2} Re\{V(z) \cdot I^*(z)\} = \frac{1}{2} Re\left\{\left[\frac{V(z)}{Z_L}\right]^* \cdot V(z)\right\}$$

$$= \boxed{\frac{1}{2}\left|\frac{V_L}{Z_L}\right|^2 \cdot R_L = \frac{1}{2}|I_L|^2 \cdot R_L} \quad \text{at } z = L$$

- the above calculations are for the power delivered to the load
- for a lossless line: $V(z) = V_0^+ e^{-j\beta_0 z}[1 + \Gamma(z)]$

∧ $P_{avg}(z) = P_{avg, load}$ = constant

define: incident voltage: $V_{inc} = V_0^+ e^{-j\beta_0 z}$

∧ reflected voltage: $V_{ref} = V_0^+ \Gamma(z) e^{-j\beta_0 z}$

$\rightarrow V(z) = V_{inc} + V_{ref}$

∧ incident & reflected currents:

$I_{inc} = \frac{V_{inc}}{Z_0}$ ∧ $I_{ref} = \frac{-V_{ref}}{Z_0}$

$\rightarrow I(z) = I_{inc} + I_{ref}$

incident power: $P_{inc} = \frac{1}{2} Re\{V_{inc} I_{inc}^*\} = \boxed{\frac{1}{2}\frac{|V_0^+|^2}{Z_0}}$

reflected power: $P_{ref} = \frac{1}{2} Re\{V_{ref} I_{ref}^*\} = \boxed{\frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot |\Gamma|^2}$

$\therefore \boxed{P_{ref} = |\Gamma|^2 \cdot P_{inc}}$

- hence, the power dissipated in the load is

$\therefore P_{inc} - P_{ref} = P_{inc} \cdot [1 - |\Gamma|^2]$

$\therefore \boxed{P_{avg}(z) = \frac{|V_0^+|^2}{2 Z_0} \cdot [1 - |\Gamma|^2]}$

note that: $P_{ref} = \frac{1}{2}\frac{|V_0^-|^2}{Z_0}$

- the three cases that give pure standing waves are:

1- open-circuited load (A)

2 - short-circuited load (B)

3 - pure reactive load (E)

example 11.3: $W = 10^6$ rad/m, $\alpha = 8$ dB/m, $\beta = 1$ rad/m

$Z_0 = 60 + j40\ \Omega$, $V_s = 10\angle 0°$ V ∧ $Z_g = 40\ \Omega$

$Z_L = 20 + j50\ \Omega$

$\rightarrow \alpha = \frac{8}{20\log_{10}e}$ Np/m $= 0.921$ Np/m, lossy line

a) input impedance: $Z(z') = Z_0 \cdot \frac{Z_L + Z_0 \tanh(\gamma z')}{Z_0 + Z_L \tanh(\gamma z')}$

input impedance $\rightarrow z' = l = 2m \rightarrow \gamma z' = 2\alpha + j3$

$\because \tanh = \frac{\sinh}{\cosh}$ ∧ $\boxed{\sinh(x+y) = \sinh(x)\cdot\cosh(y) + \cosh(x)\cdot\sinh(y)}$

∧ $\boxed{\cosh(x+y) = \cosh(x)\cdot\cosh(y) + \sinh(x)\sinh(y)}$

$$\rightarrow \tanh(2\alpha + j3) = \frac{\sinh(2\alpha)\cdot\cos(2) + j\cosh(2\alpha)\cdot\sin(2)}{\cosh(2\alpha)\cos(2) + j\sinh(2\alpha)\sin(2)}$$

$$= \frac{-1.27979 + j2.94051}{-1.34574 + j2.79638} = 1.033812\angle -0.03803$$

$$\therefore Z_{in} = (60 + j40)\cdot\frac{(20+j50) + ((60+j40)\tanh(\gamma z'))}{((60+j40) + (20+j50)\tanh(\gamma z'))}$$

$$= \boxed{60.26 + j38.79\ \Omega}$$

or using

$$Z(z') = Z_0\,\frac{1 + \Gamma(z')}{1 - \Gamma(z')}$$

∧ $\Gamma(z') = \Gamma_L e^{-2\gamma z'}$

∧ $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.1586 + 0.30345j$

$\rightarrow \Gamma(z') = 0.34239 \cdot e^{-2\alpha \cdot l}\angle(2.0524 - 2\beta z')$ rad

$\rightarrow \Gamma(z') = 8.6016189 \times 10^{-3} \angle -1.9496$ rad

$\rightarrow Z(z') = Z_0 \cdot [0.99346 - 0.015895j]$

$$= \boxed{60.25 + 38.99j\ \Omega}$$ ✓

b) $I(z) = \frac{V_g}{Z_{in}+Z_g} = \frac{10\angle 0^\circ}{100.25 + 38.79j} = 86.76 - 33.571j$ mA

$= 93.03\angle -0.36419$ rad mA

-21.15°.

c) $I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$

~~$\because V_0^+ = \frac{I_L}{2}(Z_L + Z_0) e^{\gamma L} \wedge V_0^- = \frac{I_L}{2}(Z_L - Z_0) e^{-\gamma L}$~~

~~$\rightarrow V_0^+$~~

$\because V_0^+ = \frac{1}{2}(V_0 + Z_0 I_0) \wedge V_0^- = \frac{1}{2}(V_0 - Z_0 I_0)$

$= I_0 Z_{in} \rightarrow V_0^+ = \frac{I_0}{2}(Z_{in} + Z_0) \wedge V_0^- = \frac{I_0}{2}(Z_{in} - Z_0)$

$\rightarrow V_0^+ = 6.689\angle 0.211$ rad $\wedge V_0^- = 0.059472\angle 1.40535$ rad

$\therefore I(z/2) = I(1) = \frac{V_0^+}{Z_0} \cdot e^{-\gamma} - \frac{V_0^-}{Z_0} \cdot e^{\gamma}$

$\rightarrow I(1) = (0.09273\angle -0.377) \cdot e^{-\alpha} \cdot e^{-j\beta} - [(7.9644 \times 10^{-4})\angle 0.81935$ rad$]$

$\rightarrow I(1) = [0.0369177\angle -1.397] - [(3.17298 \times 10^{-4})\angle -0.18264$ rad$]$

$= 36.8\angle -1.3850$ rad mA

$= 36.8\angle 280.6^\circ$ mA

practice exercise 11.3: $Z_g = 0 \wedge$ matched load $\rightarrow Z_L = Z_0$

a) $Z(z) = Z_0 = 30 + j60$

b) $\because$ matched load $\rightarrow$ no backward wave

$\rightarrow I(z = 40) = \frac{V_0^+}{Z_0} e^{-\gamma \cdot 40}$ $\rightarrow V_{in} = \frac{1}{2} V_g$

$\rightarrow I(40) = V_g \cdot \frac{1}{Z_0} = 0.22361\angle -1.10715$ rad

c) $\because V(z' = l) = 7.5\text{V} \angle 0^\circ$

$\wedge V(z' = 0) = 5\angle -48^\circ$

$\rightarrow 7.5\angle 0^\circ = V_0^+ e^{-\gamma z}$ at $z' = l, z = 0$

$\rightarrow \boxed{V_0^+ = 7.5\text{V}}$

$\wedge$ at $z' = 0, z = l \rightarrow 5\angle -48^\circ = 7.5 e^{-\alpha z} \cdot e^{-j\beta z}$

$\rightarrow 5 = 7.5 e^{-\alpha \cdot 40} \rightarrow \ln(\frac{5}{7.5}) = -\alpha \cdot 40 \rightarrow \boxed{\alpha = 0.01014}$ Np/m

$\wedge -48^\circ = -\frac{4}{15}\pi = -\beta \cdot 40 \rightarrow \beta = -0.020944$ rad/m

$\rightarrow \boxed{\gamma = 0.01014 + j0.020944 /m}$

example from notes: $l = 100\ m$, $f = 3\times10^9\ Hz$, air-filled T.L.

$Z_0 = 50\ \Omega$, $Z_g = 2Z_0$, $V_{in} = 5V \land V_g = 10V$

$\because V_{in} = V_g \cdot \frac{Z_{in}}{Z_g+Z_{in}} \rightarrow \frac{V_{in}}{V_g} Z_g = \left(1-\frac{V_{in}}{V_g}\right) Z_{in}$

$\rightarrow Z_{in} = \frac{V_g}{V_g - V_{in}} \cdot \frac{V_{in}}{V_g} \cdot Z_g = 100\ \Omega = Z_g$

$\because l = 100\ m \land \lambda = \frac{2\pi}{\beta} \land \beta = 2\pi f\sqrt{\mu_0\varepsilon_0}$ $\because$ air-filled

$\rightarrow \lambda = \frac{c}{f} = 0.1\ m$

Cases F&G: $l = (2n+1)\frac{\lambda}{4}$ or $l = n\frac{\lambda}{2}$ X X

Resistive load → $l = 1000\lambda$ →

$\because Z(z) = Z_0 \frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)} \land \beta = \frac{2\pi}{\lambda} \rightarrow \beta\cdot l = 2\pi\cdot\frac{1000\lambda}{\lambda}$

$\beta l = 2000\pi$

$\rightarrow \tan(\beta l) = 0 \rightarrow \boxed{Z(z) = Z_L} = 100\Omega$

$\because l = n\frac{\lambda}{2}$ where $n = 2000$ → half-wave section

- if $Z_g = Z_0$, then the circuit is said to be source matched

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2006

[1] a) ∵ $\Gamma = 0 \rightarrow Z = \eta_1$ ∧ $\eta_1 = \eta_3$

∴ $d = n\frac{\lambda_2}{2}$ half-wave section

∧ $\lambda_2 = \frac{2\pi}{\beta}$ ∧ $\beta = 2\pi f\sqrt{\mu_0 \varepsilon_r \varepsilon_0} \rightarrow \lambda_2 = 9.375\ mm$

$\rightarrow \boxed{d = 4.6875\ mm}$

b) at 10 GHz, $\Gamma = 0$

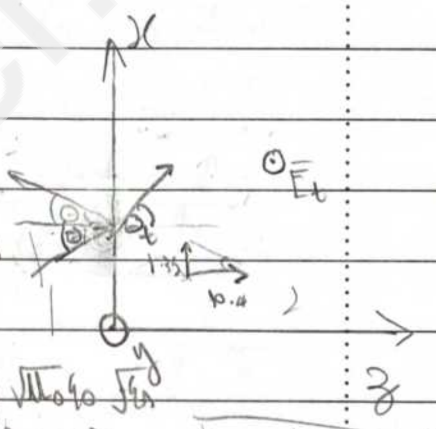
at 5 GHz, $\Gamma = \frac{Z - \eta_1}{Z + \eta_1}$ ∧ $Z = \eta_2 \cdot \frac{\eta_3 + j\eta_2 \tan(\beta_2 \lambda)}{\eta_2 + j\eta_3 \tan(\beta_2 \lambda)}$

$\rightarrow Z = \eta_2 \cdot \frac{\eta_2}{\eta_3} \rightarrow Z = 46.875\pi$

$\rightarrow \Gamma = -0.4382$

[3] ~~Parallel~~ polarization $\boxed{\text{perpendicular}}$

$\rightarrow \theta_t = 0.1290859\ rad$



∵ $\varepsilon_1 = \varepsilon_0$ ∧ both non-magnetic bodies

$\rightarrow \bar{k}_2 = 1.35\,\bar{a}_x + 10.4\,\bar{a}_z$

$\rightarrow |k_2| = \beta_2 = 10.48725 = w\sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_{r_2}}$

$\rightarrow \sqrt{\varepsilon_{r_2}} = \frac{\beta_2 \cdot c}{w} = 5.0073 \rightarrow \boxed{\varepsilon_{r_2} = 25.0730}$

∵ $n_1 \cdot \sin\theta_i = n_2 \cdot \sin\theta_t$

$\rightarrow \sin\theta_i = \sqrt{25} \cdot \sin\theta_t \rightarrow \theta_i = 0.70047\ rad$

∵ $\cancel{\Gamma_{\parallel}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$, $\cancel{\eta_2} = \frac{120\pi}{5.0073}$

∧ $\cancel{\Gamma_{\parallel}} = 0.31724$ ∴ $E_{io} = \frac{0.2676}{\cancel{\Gamma}} = \cancel{0.8435\ V/m}$

∵ E_t has only a y-component

$\rightarrow$ perpendicular polarization

∴ $T_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \boxed{0.26684}$

∴ $E_{io} = \frac{E_{to}}{T_{\perp}} = 1.0028\ V/m$

∴ $E_i = 1.0028\,\bar{a}_y\,e^{-jk_1 R}$

∵ $|k_1| = \beta_1 = \frac{200\pi \times 10^6}{3 \times 10^8} = 2.0944 \Rightarrow \bar{k} = |k| \sin\theta_i \, \bar{a}_x + k \cos\theta_i \, \bar{a}_z$

$\Rightarrow k_x = 1.35 \wedge k_z = 1.60125$

$\Rightarrow \bar{E}_i = \bar{a}_y \, e^{-j(1.35x + 1.60125z)}$

$\boxed{2004}$

$\boxed{1}$ ∵ H_i has one y component ∴ E_i has x & z components → parallel polarization

∴ $E_i = -\eta(a_k \times H_i)$

$\bar{k}_1 = 10\bar{a}_x + 10\sqrt{8}\,\bar{a}_z \rightarrow a_k = 0.33\bar{a}_x + 0.33\sqrt{8}\,\bar{a}_z$

$\rightarrow \bar{a}_k \times \bar{H}_i = -0.2 \cdot 0.33\sqrt{8}\,\bar{a}_x + 0.066\bar{a}_z$

∴ $E_i = (0.066\sqrt{8}\,\bar{a}_x - 0.066\bar{a}_z) \cdot \eta \, \cos(3\times10^9 t - 10x - 10\sqrt{8}z)$ A/

∵ $|k_1| = 30 = \beta_1 = \frac{\omega\sqrt{\epsilon_r}}{c} \rightarrow \sqrt{\epsilon_r} = 3 \Rightarrow \boxed{9} = \epsilon_r$

∴ $\eta = 40\pi \rightarrow E_i = (213.26\,\bar{a}_x - 24\pi\bar{a}_z)\cos(3\times10^9 t - 10x - 10$

∵ $10 = 30 \cdot \sin(\theta_i) \rightarrow \theta_i = 19.47°$

$\rightarrow n_1 \sin(\theta_i) = n_2 \sin(\theta_t)$

$\rightarrow \theta_t = \sin^{-1}(3\sin(\theta_i)) = \frac{\pi}{2}$

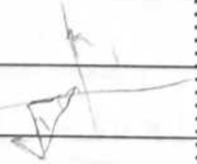
∴ $\theta_i = \theta_c$

$\wedge \; E_{t0} = T_{\parallel} E_{i0} = \frac{2\eta_2}{\eta_1} = \frac{240\pi}{40\pi} = 6$

$\rightarrow E_{t0} = -226.2\,\bar{a}_z \cos(3\times10^9 t - \bar{k}\bar{R})$

$|\bar{k}| = \beta = 10 \rightarrow \bar{k} = 10\,\bar{a}_x$

$\rightarrow \boxed{E_t = -226.2\,\bar{a}_z(\cos 3\times10^9 t - 10x)}$



2) find $\theta_i \wedge \theta_t \rightarrow \bar{a}_{kt} = \bar{a}_x \wedge E_{t0} = E_{3t0}(-\bar{a}_z)$

$\rightarrow E_{t0} = T_{\parallel} \cdot |E_{i0}| \wedge |E_{i0}| = \eta_1 \cdot H_{i0} = 40\pi \cdot 0.2$

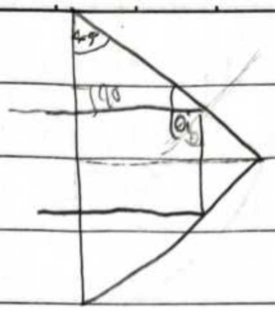
$\rightarrow E_{t0} = 8\pi \cdot 6 = 48\pi$ ∴ $\bar{E}_t = -48\pi\,\bar{a}_z \cos(\omega - \bar{k}_2\bar{R})$

$k_2 = \beta_2 \cdot \sin\theta_t \, \bar{a}_x + \cancel{\beta_2 \cos\theta_t \, \bar{a}_z}$ $\beta_2 = \frac{\omega}{c} = 10$

$\rightarrow k_2 = 10\,\bar{a}_x$

2000

3 a) $\varepsilon_1 > \varepsilon_2$



$\theta_i = 45° = \theta_c$

$\therefore$ power transmitted $= 0$ & $T = 0$

$\wedge$ $\theta_t = \frac{\pi}{2}$ & $n_1 \sin\theta_i = n_2 \cdot \sin\frac{\pi}{2} \rightarrow n_1 = \frac{1}{\sin\theta_i}$

$\rightarrow n_1 = \sqrt{\varepsilon_{r1}} \rightarrow \boxed{\varepsilon_{r1} \geq 2}$

b) $\varepsilon_{r1} = 1.5$ $\wedge$ perpendicular polarization $\theta_t = 60°$

$$\rightarrow E_{to\,i1} = T\,E_{io} = \frac{2 \cdot \frac{120\pi}{\sqrt{1.5}}}{120\pi\left(1 + \frac{1}{\sqrt{2.5}}\right)} = 0.89898$$

$E_{to\,r1} = \Gamma_\perp \cdot T_1 E_{io}$ $\wedge$ $\Gamma_\perp = 0.26795$

$\rightarrow E_{to\,r2} = \Gamma_\perp^2\, T_1 E_{io}$

$E_{ro} = \frac{E_{o\,r2}}{T_1} \rightarrow E_{ro} = \Gamma_\perp^2 \cdot E_{io}$

$\rightarrow \frac{E_{ro}}{E_{io}} = \Gamma_\perp^2 = 0.07199$ ✗

or $E_{ro} = E_{o\,r2} \cdot T_2$ $\wedge$ T_2 from prism to air

while T_1 is from air to prism

$\rightarrow \boxed{T_2 = T_1}$ $\rightarrow$ $E_{ro} = \Gamma_\perp^2 \cdot E_{io}$

✗ ✗ ✗

$T_1 \neq T_2$ T from air to medium does not equal reciprocal of T from medium to air

$\therefore E_{ro} = E_{io} \cdot T_1 \cdot T_2 \cdot \Gamma_\perp^2$

$T_1 = \frac{2\eta_2}{\eta_2 + \eta_0} = 0.898979$ $\wedge$ $T_2 = \frac{2\eta_0}{\eta_2 + \eta_0} = \boxed{1.10102}$

$\therefore \frac{E_{ro}}{E_{io}} = \boxed{0.07106}$

1999/2000

1) a) $E_{xi} = E_0 \cdot \frac{-1}{2} = E_0 \cdot \cos\theta_i \rightarrow \cos\theta_i = \frac{-1}{2}$

$\rightarrow \theta_i = 110^\circ$ or 60°

(or) $k_x = |k| \sin(\theta_i)$ $|k| = 20\pi$

$\rightarrow k_x = 10\sqrt{3}\,\pi = 20\pi \cdot \sin\theta_i \rightarrow \sin\theta_i = \frac{\sqrt{3}}{2}$

$\rightarrow \boxed{\theta_i = 60^\circ} = \theta_{B\parallel}$

$\therefore$ $n_1 \sin\theta_i = n_2 \sin\theta_t \rightarrow 1 \cdot \sin 60 = \sqrt{3} \sin\theta_t$

$\rightarrow \boxed{\theta_t = 30^\circ}$

b) $\therefore$ parallel polarization $\rightarrow \Gamma_{\parallel} = 0 \rightarrow E_{r0} = 0$

c) $\overline{E}_{t0} = E_{i0} \cdot T_{\parallel}$ $\wedge$ $T_{\parallel} = [\Gamma_{\parallel} + 1] \cdot \left(\frac{\cos(\theta_i)}{\cos(\theta_t)}\right)$

$\rightarrow T_{\parallel} = 0.57735$

$\rightarrow \overline{E}_{t0} = 0.57735 \cdot E_0 \left(-\cos(\theta_t)\,\overline{a}_x + \sin\theta_t\,\overline{a}_z\right)$

$= E_0(-0.5\,\overline{a}_x + 0.2887\,\overline{a}_z) \cos(\omega t - \overline{k}_2 \overline{R})$

$\wedge$ $\overline{k}_2 = \beta_2 \sin(\theta_t)\,\overline{a}_x + \beta_2 \cos(\theta_t)\,\overline{a}_z$

$\wedge$ $\beta_2 = \frac{\omega \sqrt{\varepsilon_{r2}}}{c} = 108.83$ rad/m

$\rightarrow k_{2x} = 54.415$ $\wedge$ $k_{2z} = 94.25$

$\rightarrow \overline{E}_t = E_0(-0.5\,\overline{a}_x + 0.2887\,\overline{a}_z) \cos(\omega t - 54.415x - 94.25z)$

d) $\Gamma_{\parallel} = 1 \rightarrow \theta_t > \frac{\pi}{2}$ impossible $\therefore$ $\boxed{\varepsilon_2 > \varepsilon_1}$

$\boxed{1999}$

3) a) ∵ H_t has one component ∴ parallel polarization

∵ $|k_x| = \beta_2 = 3.49 \approx \pi = \frac{\pi 10^8}{c} \cdot \sqrt{\epsilon_r}$

$\to \epsilon_r = \frac{9}{4}$

∵ $k_{xt} = \pi \cdot \sin\theta_t \to \theta_t = 19.525^\circ$

$\to n_1 \sin\theta_i = n_2 \sin\theta_t \to \theta_i = \sin^{-1}\left(\frac{3}{2} \cdot 0.3342\right)$

$\to \boxed{\theta_i = 30.1^\circ}$

b) $E_t = -\eta(\bar{a}_k \times \bar{H})$ & $\bar{a}_k \times \bar{H} = -\frac{2.96}{\pi} \cdot 0.062\, \bar{a}_x + \frac{1.05}{\pi} \cdot 0.062\, \bar{a}_z$

$\to \frac{-120\pi \cdot 2}{3} \cdot E_{x0}$

∴ $E_t =$

or from magnitudes $E_{t0} = \eta \cdot H = \frac{120\pi \cdot 2}{3} \cdot 0.062 = \frac{124}{25}\pi$

& $E_{i0} = \frac{E_{t0}}{\tau_{\parallel}}$ & $\tau_{\parallel} = 0.77927$

$\to E_{i0} = 20.166$

$\to H_{i0} = 53.5$

∴ $\bar{H}_i = 0.0534\, \bar{a}_y \cos(\pi 10^8 t - \bar{k} \cdot \bar{R})$

& $\bar{k} = \beta_1 \sin\theta_i\, \bar{a}_x + \beta_1 \cos\theta_i\, \bar{a}_z$

$\to \bar{k} \cdot \bar{R} = \boxed{1.05x + 1.812z}$

MOHAMMAD SANAD ALTAHER 130806

أنا محمد سند الطاهر أشهد أني قرأت وفهمت وطبقت تعليمات هذا الإمتحان،
ولم أتلق أي مساعدة من أي شخص في حل هذا الإمتحان. أنا لست بغشاش
أو كذاب.

Q1) $\mu_{r1} = \varepsilon_{r1} = 9$, $\mu_{r2} = \varepsilon_{r2}$, $f = 600\,MHz$

perpendicular polarization, since $\bar{H}_2$ has $\bar{z}$ component

$$\rightarrow \bar{H}_2 = H_0(-\cos(\theta_t)\,\bar{a}_x + \sin(\theta_t)\,\bar{a}_z) \rightarrow \theta_t = \frac{\pi}{2}$$

$$\because \bar{\beta}_2 = \beta_2 \sin\theta_t\, \bar{a}_x + \beta_2 \cos\theta_t\, \bar{a}_z$$

$$\rightarrow \beta_2 = 16\pi = \frac{\omega\sqrt{\mu_{r2}\varepsilon_{r2}}}{c} \rightarrow \boxed{\sqrt{\mu_{r2}\varepsilon_{r2}} = \varepsilon_{r2}}$$

$$\therefore \mu_{r2} = \varepsilon_{r2} = 4$$

$$\beta_1 = \frac{\omega}{c}\cdot 9 = 36\pi \rightarrow \bar{\beta}_1 = 36\pi(\sin\theta_i\,\bar{a}_x + \cos\theta_i\,\bar{a}_z)$$

$\because$ Snell's law: $n_1\sin(\theta_i) = n_2\sin(\theta_t)$

$$n_x = c\sqrt{\mu_x\varepsilon_x} \rightarrow n_1 = 9 \wedge n_2 = 4$$

$$\therefore \theta_i = \sin^{-1}\left(\frac{4}{9}\sin(\theta_t)\right)$$

$$\rightarrow \boxed{\theta_i = 26.39^\circ}$$

$$\because E_{t0} = \frac{5}{3\pi}\cdot\eta_2 = \frac{5}{3\pi}\cdot\frac{120\pi\cdot 2}{2} = \boxed{200\,V/m}$$

$$\rightarrow E_{i0} = E_{t0}/\tau_\perp \wedge \tau_\perp = \frac{2\eta_2}{\eta_1} = 2$$

$$\therefore E_{i0} = 100\,V/m$$

$$\bar{\beta}_1 = 50.27\,\bar{a}_x + 101.311\,\bar{a}_z$$

$$\therefore \boxed{E_i(x,z,t) = 100\,\bar{a}_y\cos(1.2\pi\times10^8 t - 50.27x - 101.311z)\ V/m}$$

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Q2) a) ∵ $\bar{H} = (j\bar{a}_x - 2\bar{a}_y)\, e^{j\beta z}$ → traveling in −z-direction

$\bar{E} = -\eta(\bar{a}_k \times \bar{H})$, $\bar{a}_k \times \bar{H} = -2\bar{a}_x + j\bar{a}_y$

→ $\bar{E} = \eta_0 \cdot (2\bar{a}_x - j\bar{a}_y) \cdot e^{j\beta z}$

→ $E_x = \eta_0 \cdot 2\cos(wt + \beta z)$

& $E_y = \eta_0 \cdot \cos(wt + \beta z - 90^\circ)$

$= \eta_0 \cdot \sin(wt + \beta z)$

∴ $\Delta\phi = \frac{\pi}{2}$ & $E_{0x} \neq E_{0y}$ → E.P

setting $z = 0$ and varying t: L.H.E.P

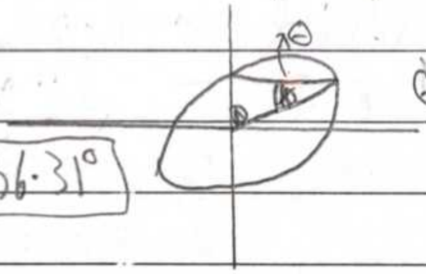
∵ the direction of propagation is −z but the ellipse rotates counter clockwise.

b) $\bar{E}(z=0) = \bar{a}_x E_{x0} \cos(wt) + \bar{a}_y E_{y0} \cos(wt + \phi)$

& $E_{x0} = E_{y0}$

max = 3 units

→ $\Theta = 33.7^\circ$ → $\boxed{\phi = 56.31^\circ}$



$\Theta = \tan^{-1}\left(\frac{2}{3}\right)$

for a maximum hypotenuse equal to three units and a maximum x and y components equal to 2 units each, the angle between the major axes can be found via pythagorean rule and the arctan of the larger side divided by the smaller $(\tan^{-1}(\frac{3}{2}))$

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Q3) $\Gamma_{\perp} = 0$ for $\theta_i = 48.9^\circ$

& $\Gamma_{\perp}(\theta_i = 0) = 0.2 \rightarrow \frac{\eta_2 - \eta_0}{\eta_2 + \eta_0} = 0.2$

$$\therefore \frac{\sqrt{\frac{\mu_r}{\varepsilon_r}} - 1}{\sqrt{\frac{\mu_r}{\varepsilon_r}} + 1} = 0.2 \rightarrow \boxed{1.2 = 0.8 \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}}} - ①$$

$$\therefore \theta_{B\perp} = \sin^{-1}\left(\left(\frac{1 - \varepsilon_r/\mu_r}{1 - 1/\mu_r^2}\right)^{1/2}\right) = 48.9^\circ$$

$$\therefore \boxed{\frac{\mu_r - \varepsilon_r}{\mu_r - \frac{1}{\mu_r}} = 0.567858} - ②$$

from ① $\boxed{\varepsilon_r = \frac{4}{9}\mu_r}$ sub in ②

$$\rightarrow \left(\frac{5}{9}\mu_r^2\right) / \left(\mu_r^2 - 1\right) = 0.567858$$

$$\rightarrow \left(\frac{5}{9} - 0.567858\right)\mu_r^2 = -0.567858$$

$$\therefore \boxed{\mu_r = 6.79} \; \& \; \boxed{\varepsilon_r = 3.02}$$

$\theta_{B\parallel} = \sin^{-1}\left(\left(\frac{1 - \mu_r/\varepsilon_r}{1 - (1/\varepsilon_r^2)}\right)^{1/2}\right) \Rightarrow \theta_{B\parallel}$ does not exist

$\theta_c =$

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example from notes: $\because V(z'=0) = V_{max} = 5V = V_L$

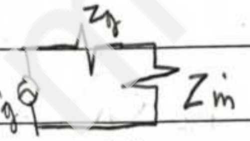
$\& \ Z_g = 5Z_0 \ \& \ Z_L = Z_0/5$

transmission line is lossless → Z_L resistive

$\rightarrow SWR = \frac{Z_0}{Z_0/5} = \boxed{5}$ $\because Z_L < Z_0$

~~assuming $V_L = V_{max}$ & V_{max} at $(2n+1)\frac{\lambda}{4}$~~

$\therefore Z_{in_{max}} = \frac{Z_0^2}{Z_L} = 5Z_0$



$\rightarrow V_{max} = 5 = V_g \cdot \frac{Z_{in}}{5Z_0 + Z_{in}} \rightarrow V_g = 10V$

+ in a pure resistive load;

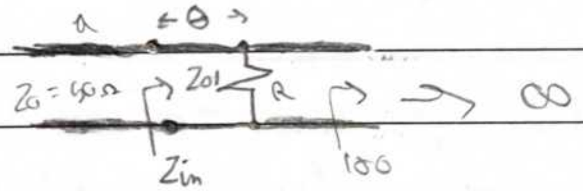
- if $R_L > R_0$, V will start at a maximum
- if $R_L < R_0$, V will start at a minimum

$\because V(z'=0) = V_{max} \rightarrow l = (2n+1)\frac{\lambda}{4}$ → quarter-wave trans

from (F): $Z_{in} = \frac{Z_0^2}{Z_L} \rightarrow Z_{in} = 5Z_0$

$\therefore V_{max} = V_g \cdot \frac{Z_{in}}{Z_{in} + Z_g} \rightarrow V_g = 10V$

example from notes:



Z_0: feeding line

given $Z_{01} = \sqrt{5000}\ \Omega$, $\theta = 90°$ at 3 GHz & θ is electrical length

& $R = 100\ \Omega$. lossless T.L.s → pure real characteristic impedance

∴ find SWR at 6 GHz and 9 GHz

$\because \theta = 90° = \beta \cdot l \ \& \ \beta = 2 \cdot 3\pi X$ $\because \beta = \frac{2\pi}{\lambda}$

$\therefore 90° \cdot \frac{\pi}{180} = \frac{2\pi}{\lambda} \cdot l \rightarrow \frac{\pi}{2}\lambda = 2\pi l \rightarrow$

$l = \frac{\lambda}{4}$ → case 6

$\theta \propto f$

∵ infinite length :  & $Z_1 = 50\Omega = 100 // 100$

∴ at 6 GHz , $l = \frac{\lambda}{2}$ case ⑦

→ $Z_{in} = Z_L = 50\ \Omega$ ∴ matched load → SWR = 1

at 9 GHz : $\frac{3}{2}\pi\lambda = 2\pi l$ → $\frac{3}{4}\lambda = l$ → case 6

∴ $Z_{in} = \frac{Z_{01}^2}{50} = \frac{|\sqrt{5000}|^2}{50} = 100\ \Omega$

∴ SWR $= \frac{100}{50} = \boxed{2}$

example from notes: $P_L = 1\text{kW}$, $V_{max} = 250$ rms V

a) max SWR:

∵ $V_{max} = 250 \cdot \sqrt{2}$ ∵ rms given

& $V_{max} = |V_0^+|(1 + |\Gamma|) \leq 353.5534\ V$

∵ $P_{load} = (1 - |\Gamma|^2)\frac{|V_0^+|^2}{2Z_0} = 1\text{kW}$

assuming $V_{max} = 353.5534$ → $|V_0^+| = \frac{353.5534}{1 + |\Gamma|}$

& $\frac{100\text{kW}}{1 - |\Gamma|^2} = \frac{(353.5534)^2}{(1 + |\Gamma|)^2}$

$1 - |\Gamma|^2 = (1 - |\Gamma|)(1 + |\Gamma|)$ → $100\text{kW} : (353.5534)^2 \frac{1 - |\Gamma|}{1 + |\Gamma|}$

→ SWR $= \frac{(353.5534)^2}{100\text{kW}} = \frac{125k}{100k} = 1.25$

∴ SWR ≤ 1.25

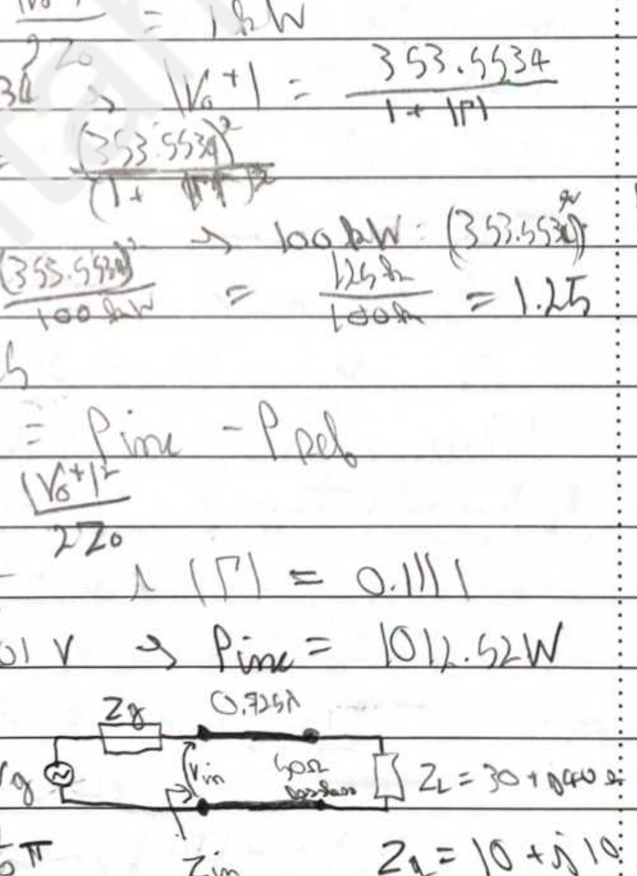
b) P_{inc} : ∵ $P_{load} = P_{inc} - P_{ref}$

& $P_{inc} = \frac{|V_0^+|^2}{2Z_0}$

∵ $|V_0^+| = \frac{250 \cdot \sqrt{2}}{1 + |\Gamma|}$ & $|\Gamma| = 0.1111$

→ $|V_0^+| = 318.201\ V$ → $P_{inc} = 1012.52\text{W}$

example from notes:



∵ $l = 0.725\lambda$

& $\beta l = \frac{2\pi}{\lambda} \cdot 0.725\lambda = \frac{29}{20}\pi$

$Z_g = 10 + j10$

cannot apply any cases

& $V_g = 100V$

∵ T.L is lossless; must use exact expression or find P_L

① $Z_{in} = Z_0 \frac{Z_L + jZ_0\tan(\beta_0 z')}{Z_0 + jZ_L\tan(\beta_0 z')}$, $\tan(\beta_0 z') = \tan(\frac{24}{20}\pi)$

$\therefore Z_{in} = 39.8519 - 60.5353j\ \Omega$

$\therefore P_{load} = P_{avg}$ ∵ TL lossless , $V_{in} = \frac{Z_{in}}{Z_g + Z_{in}} \cdot V_g$

→ $V_{in} = 100.165 \angle -0.2204$ rad V

& $I_{in} = V_g / Z_g + Z_{in} = 1.5564 \angle 0.6827$ rad A

→ $P_{load} = \frac{1}{2} Re\{V_{in} \cdot I_{in}^*\} = \frac{1}{2} Re\{100.165 \cdot 1.5565 \angle -0.2204 - 0.6827\}$

→ $2P_{load} = 96.5342$ W → $P_{load} = 48.27$

② by first calculating Γ:

∵ $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.5j = \frac{1}{2}\angle \frac{\pi}{2}$ rad

→ $\Gamma(z') = 0.5j\, e^{-2\gamma z'}$ lossless: $\gamma = j\beta_0$

must have frequency to solve with β_0 & $\beta_0 = \frac{20}{24}\pi / 0.72\lambda$

no need for frequency: $e^{-2\gamma z'} = e^{-j2\cdot 2\pi \cdot 0.72\lambda}$

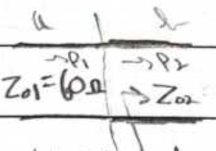
→ $\Gamma(z') = 0.5\, e^{-j(2.4\pi - \frac{\pi}{2})} = 0.5\, e^{-j2.4\pi}$

∴ $Z_{in} = Z_0 \frac{1+\Gamma(z')}{1-\Gamma(z')}$ → $Z_{in} = 39.852 - 60.535j$

∴ $P_L = P_{in} = \frac{1}{2}|I_{in}|^2 \cdot Re\{Z_{in}\}$

& $|I_{in}| = 1.5564$ → $P_{in} = 48.27$ W

example from notes:

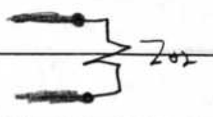


matched load in b

find Z_{02}

∵ b is matched load 12mW 9 mW → 3 mW reflected

→



& |V| is at min at the interface

→ $Z_{02} < Z_{01}$

∵ $P_2 = P_1[1 - |\Gamma|^2]$ → $|\Gamma|^2 = 1 - \frac{9}{12} = 0.25$

∴ $|\Gamma| = 0.5$ ∵ $Z_{02} < Z_{01}$ → $\Gamma = -0.5$

& Z_{02} & Z_{01} are lossless → $-0.5 = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$

→ $Z_{02} = \frac{-0.5 Z_{01}}{-1.5} = 20\ \Omega$

Subject 11.5 : Smith chart Date 11/5/2021 No.

- given $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$ for a lossless transmission line ($Z_0 = R_0$)

$\rightarrow \Gamma = \Gamma_r + j\Gamma_i$, sub r : real ∧ sub i : imaginary

define $z_L = \frac{Z_L}{Z_0}$, normalized impedance

$$z_L = r + jx$$

$$\rightarrow \Gamma = \Gamma_r + j\Gamma_i = \frac{z_L - 1}{z_L + 1}$$

$$\therefore r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} \quad \wedge \quad x = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$\therefore \left[\Gamma_r - \frac{r}{1+r}\right]^2 + \Gamma_i^2 = \left[\frac{1}{1+r}\right]^2$$

circle equations in complex plane

$$\wedge \quad \left[\Gamma_r - 1\right]^2 + \left[\Gamma_i - \frac{1}{x}\right]^2 = \left[\frac{1}{x}\right]^2$$

- normalized resistance circle ($r = \frac{R_L}{R_0}$) has a center at $(\Gamma_r, \Gamma_i) = \left(\frac{r}{1+r}, 0\right)$ and a radius : $\frac{1}{1+r}$

- normalized reactance circle ($x = \frac{X_L}{R_0}$) is centered at $(\Gamma_r, \Gamma_i) = \left(1, \frac{1}{x}\right)$ with a radius of $\left(\frac{1}{x}\right)$
- Γ_r is on the real axis, whereas Γ_i is on the imaginary axis. Since the maximum magnitude of Γ is 1, therefore all r & x-circles are bounded inside the unit circle of $r = 0$

example 11.4 : $\therefore Z_0 = 50\,\Omega \wedge Z_L = 60 + j40$

$\rightarrow r = \frac{6}{5} \wedge x = \frac{4}{5}$

$0.6\ell = \frac{w}{\beta_0} \rightarrow 0.6\ell = \frac{w \cdot \lambda}{2\pi} \rightarrow \lambda = 90$

$\beta\lambda =$

example 11.4:

a) normalize: $Z_{in} \rightarrow z_L = 1.2 + 0.8j$ Ω

after drawing gamma constant circle:

find $\frac{OP}{OQ} = \frac{2.2}{6.2} = 0.3548$

from protractor: $\theta_\Gamma = 56^\circ \rightarrow \Gamma_L = 0.3548 \angle 56^\circ$

b) to find SWR, create a circle centered at the origin with a magnitude of $|\Gamma| \approx 2.1$

c) express the length in terms of λ:

$u_p = 0.6c = \frac{w}{\beta} \rightarrow \beta = \frac{2\pi}{\lambda} = \frac{w}{0.6c}$

$\rightarrow \lambda = \frac{2\pi \cdot 0.6c}{w} = \frac{0.6c}{f} = 90$ m

$\therefore l = \frac{1}{3}\lambda \rightarrow z_{in} = 0.48 + 0.035j$

$\rightarrow Z_{in} = 50 z_{in} = 24 + 1.75j$

practice exercise 11.4:

$z_L = 1.15 - 0.5j$ Ω $\rightarrow Z_L = 80.5 - 35j$ Ω

$\lambda \; |\Gamma| = \frac{5.9}{26.5} = 0.2226 \rightarrow \Gamma = 0.2226 \angle 300^\circ$

$z_{in} = 0.7 + 0.35j = 49 - 17.5j$

first $V_{min} \rightarrow 0.166\lambda$

example 11.5: $z_L = 1.333 + 2j$ Ω

a) $|\Gamma| = \frac{16.7}{26.5} \rightarrow \boxed{\Gamma = 0.645 \angle 40^\circ}$

b) SWR = 5

c) $y_l = 0.21 - 0.35j \rightarrow Y_l = \frac{1}{Z_0}(y_l)$

$Y_l = 2.8 - 4.79j$ mS

d) $z_{in} = 0.3 + 0.65j = 22.5 + 48.75j$

e) V_{max} : 0.055λ & V_{min} : 0.305λ only & 2nd V_{max} at 0.555λ

f) $Z_{in} = (1.9 - 2.2j)75 = 142.5 - 165j$

practice exercise 11.5:

a) $z_L = 1 + 1j$

$SWR = 2.6$, $|\Gamma| = \frac{11.35}{25.45} = 0.446\angle 63.8°$

$z_{in} = 2 - 1j \rightarrow 0.287\lambda$

$\rightarrow l = 0.125\lambda = \frac{\lambda}{8}$

or $l = \frac{\lambda}{8}(1 + 4n)$

b) $r = 0.38 \rightarrow z_{min} = 22.8\,\Omega$

$r = 2.6 \rightarrow z_{max} = 156\,\Omega$

V_{max} : $0.25\lambda - 0.162\lambda = 0.088\lambda$

Subject 11.6: same applications of transmission lines. Date 18/5/2021 No.

example from notes:

$$z_{min} = 0.05\,m = 0.125\cdot\lambda$$

$\therefore$ at load, $z_L = 0.63 + j0.77\ \Omega$

$$\rightarrow Z_L = 31.5 + 38.5j\ \Omega$$

$$|\Gamma_L| = \frac{12}{25.5} = 0.4706 \ \wedge\ \theta_{\Gamma_L} = -90^\circ \rightarrow \Gamma_L = -0.471j$$

- in this section, transmission lines' use for load matching and impedance measurement is considered

1) quarter wave transformer (matching):

- if the load is mismatched, then a reflected wave exists
- reflections are considered losses, hence ($\Gamma = 0$) is desirable for maximum power transfer ($Z_0 = Z_L$)

$$\because Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}\right] \wedge\ l = \frac{\lambda}{4} \rightarrow \beta l = \frac{\pi}{2}$$

$$\rightarrow Z_{in} = \frac{Z_0^2}{Z_L} \rightarrow \frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L}$$

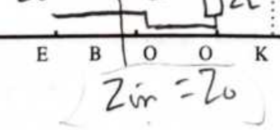
$$\therefore z_{in} = \frac{1}{z_L} \rightarrow \boxed{z_L = Y_{in}}$$

- thus by adding $\lambda/4$ on the smith chart, we obtain the input admittance corresponding to a given load.

$$Z_0' = \sqrt{Z_0 Z_L}\ ,\ Z_0' : \text{characteristic impedance of } \tfrac{\lambda}{4} \text{ section}$$

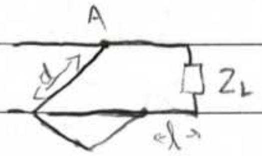
where all the above variables are real (Resistive)

- note that the standing wave is only eliminated at the specified frequency. changing the frequency will produce a standing wave
- hence the quarter wave transformer is a narrow-band frequency sensitive device



2) single stub tuner (matching):

- stubs are usually connected in parallel. however, series stubs are possible as well.



- parallel stub is usually short circuited

∴ $Z_{in} = Z_0 \rightarrow z_{in} = 1 = y_{in}$ at point A

assuming $y_{in} = 1 + jb + y_s \rightarrow y_s = -jb$

- steps to solving with a smith chart:

1) locate the load admittance and draw the constant gamma circle around it.
2) find the two points of intersection of the $r=1$ circle with the gamma circle drawn in (1)
3) choose either point as your z_{in} (depending on whether the input impedance should be inductive or capacitive) then measure its distance in terms of λ from the load
4) add $\frac{\lambda}{4}$ to the point selected in 3 then measure the distance from $z = 0 + j0$.
   the measured distance is the length of the stub.

- choose the shorter stub or the one closer to the load

example 11.6: $\frac{\lambda}{2} = 8cm$, $Z = 50\Omega$

∴ all values repeat every $\frac{\lambda}{2} \rightarrow \lambda = 16cm$

∴ air line $\rightarrow u_p = c \rightarrow f = \frac{u_p}{\lambda} = 1.875 GHz$

load is located at the sc minimum 16, 24

if 16 cm taken $\rightarrow l = 16cm - 11cm = 5cm$

$5cm = 0.3125\lambda$

$\rightarrow z_L = 1.4 + 0.79j \rightarrow Z_L = 70 + 39.5j\ \Omega$

minimum : 19.75, 25 DC

$\rightarrow$ minima : 19.75, 28.5 loaded

practice exercise 11.6 : maxima at 23, 33.5 — loaded

maxima at 25, 39.5 — DC

$\rightarrow l = 2cm$, $\frac{\lambda}{2} = 10.5\ cm \rightarrow \lambda = 21$

$\rightarrow l = \frac{2.75}{21}\lambda = 0.09524$

$\rightarrow z_L = 1.05 + 0.6j \rightarrow Z_L = 52.5 + 30j$ X

$z_L = 0.65 - 0.45j \rightarrow Z_L = 32.5 -$

example 11.9:

$y_{in} = 1 \rightarrow y_B = 1 + j b$

$y_L = 2.5 - 3.333j$ X

$\rightarrow l = 0.034\lambda \rightarrow d = 0.435\lambda$

$y_L = \frac{Z_0}{Z_L} = \frac{100}{80 + j30} = 1.6 - 1.2j$ 1/Ω

- the first point of intersection with the $g = 1$ circle is at distance 0.035λ

at both points $y_s = \pm 1j \rightarrow y_s = \mp 1j$

$l_2 = 0.359\lambda$

practice exercise 11.7 : $y_L = 0.4573 + 0.36584j$ S

$l_1 = 0.094\lambda$ or $l_2 = 0.271\lambda$

$\rightarrow d = ~~0.088~~\lambda$ or $d = 0.411\lambda$

0.125λ or $\lambda = 0.375\lambda$

$\therefore y_s = \pm j \rightarrow Y_s = j13.33$ mS

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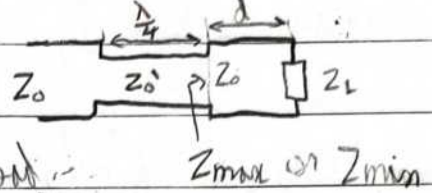
* quarter - wave transformer to match complex load:



- the load is placed at a distance that gives a resistive load.

$$\Rightarrow Z_0' = \sqrt{Z_{max} \cdot Z_0} \quad \text{or} \quad \sqrt{Z_{min} Z_0}$$

* procedure for solving single stub tuner questions:

1) locate z_L on the smith chart,
2) draw the constant $|\Gamma|$ - circle & locate y_L
3) locate the points of intersection of the $|\Gamma|$-circle and the $g=1$ circle.
4) find the distance from the load to the stub.
5) find the length of the stub. (from $y_s = -j l_s$)

example: $z_L = 0.4 + 0.3j \rightarrow y_L = \frac{100}{Z_L} = 1.6 - 1.2j$

first point $\rightarrow d = 0.032 \lambda \Rightarrow y_B = 1 - 1.05j$

$\Rightarrow y_s = 1.05j$

$\rightarrow l = 0.378 \lambda$

second point $\rightarrow d = 0.359 \lambda$

$\rightarrow l = ~~0.086 \lambda~~ \; 0.121 \lambda$

example: $z_L = 0.2 - 0.4j \rightarrow y_L = 1 + 2j$

point A: stub at distance 0 from load

$\Rightarrow l = 0.074 \lambda$

point B: stub at 0.1245λ from load

$\Rightarrow l = 0.426 \lambda$

- any stub can be represented by a reactive element

- If the stub has a different Z_0, its location will not change but its length will. take $\pm j b_s \cdot \left(\frac{Y_0}{Y_0'}\right)$

example:

$\because Z_L = 100 + j50 \wedge Z_0 = 50\,\Omega$, air-filled

$\rightarrow z_L = 2 + j1 \rightarrow d = 0.448\lambda$

$\rightarrow jx = -1j \rightarrow \frac{1}{j\omega C} = -1j \cdot 50 \rightarrow C = 1.06\,pF$

$\wedge \because \lambda = \frac{c}{f \cdot \sqrt{\mu_r \varepsilon_r}} \rightarrow \lambda = 10\,cm$

$\therefore d = 4.48\,cm$

Subject Quiz 4 practice Date 22/5/2021 No.

Chapter 11 part 1 examples:

example: air-filled & lossless, $Z_0 = 70\,\Omega$

$f = 100\,MHz$ find L, C, and β

∵ lossless airfilled → $Z_0 = \sqrt{\frac{L}{C}}$

∵ $\sqrt{LC} = \sqrt{\mu\varepsilon} = \frac{1}{c}$ → $Z_0 = \frac{\sqrt{LC}}{C} = \frac{1}{cC}$

→ $70 \cdot c = 1/C$ → $C = 47.62\,pF/m$

→ $L = \frac{1}{C \cdot c^2} = 0.233\,\mu H/m$

∵ $\beta = \omega\sqrt{\mu\varepsilon}$ → $\frac{\omega}{c}$ → $\beta = \frac{2}{3}\pi$ Rad/m

example: $R = 30\,\Omega/km$, $G = 0$, $L = 0.1\,H/km$ & $C = 20\,\mu F/km$

$f = 1\,kHz$

a) ∵ $R \neq G (=0)$ & $\frac{R}{L} \neq \frac{G}{C} (=0)$

↳ exact $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$ → $Z_0^2 = 5000 - 238.732j$

∴ $Z_0 = \sqrt{5005.696}\angle -0.0477$ Rad $= 70.751\angle -0.0238$ Rad

b) $\gamma = \sqrt{(R + j\omega L)(j\omega C)} = \sqrt[2]{7.90468\times10^{-5}}\angle \frac{3.0938834}{2}$ Rad

→ $\gamma = 8.8908\times10^{-3}\angle 1.5469$ Rad

$= 2.12\times10^{-4} + j8.88\times10^{-3}$ /m

c) $u_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{LC}} = 7.07\times10^{5}$ m/s ✗

$\boxed{u_p = \frac{\omega}{\beta}}$ $\frac{2\pi k}{8.888\times10^{-3}} = \boxed{7.067\times10^{5}\ m/s}$

example: $50\,\Omega$ distortionless line, $\alpha = 0.01\,dB/m$

$C = 0.1\,nF/m$, find R, L, G & u_p

∵ distortionless → $\frac{R}{L} = \frac{G}{C}$ & $Z_0 = \sqrt{\frac{L}{C}}$

& $\alpha = \sqrt{RG}$ $R = G\frac{L}{C}$ $\frac{R}{G} = \frac{L}{C} = Z_0^2$

∴ $2500 = \frac{R}{G} \rightarrow R = 2500G$

$\wedge$ $R/\sqrt{2500} = \alpha \rightarrow R = 50\alpha$

$\alpha = 0.01\ dB/m = 1.1513 \times 10^{-3}\ Np/m$

$\rightarrow R = 0.0576\ \Omega$

∵ $Z_0 = 50 \rightarrow L = 2500 \cdot 0.1\ nF/m = 0.25\ \mu F/m$

$\wedge$ $G = 23.04\ \mu S/m$

∵ $u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$ for distortionless $\rightarrow u_p = 2 \times 10^8\ m/s$

∵ $\frac{V_l}{V_0} = e^{-\alpha l} = 31.6\%$

distortion/lossless $\rightarrow$ distortionless

example: $f = 500\ MHz$, $(Z_0 = 80\Omega)$, $\alpha = 0.04\ Np/m$

$\beta = 1.5\ rad/m$, find $R, L, G, \& C$:

∵ distortionless: $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$, $\beta = \omega\sqrt{LC}$

$\rightarrow \frac{(1.5)^2}{(1000\pi M)^2} \cdot \frac{1}{C} = L \wedge Z_0^2 \cdot C = L$

$\rightarrow \frac{1.5}{1000\pi M} = Z_0 \cdot C \rightarrow C = 5.968\ pF/m$

$\rightarrow L = 38.197\ nH/m$

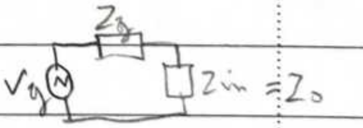
∵ $\alpha = \sqrt{RG} \rightarrow G = \frac{(0.04)^2}{R}$

$\wedge$ $G = \frac{R}{Z_0^2} \rightarrow \frac{R}{Z_0} = 0.04 \rightarrow R = 3.2\ \Omega/m$

$\wedge$ $G = 0.5\ mS/m$

samples 1:

$\boxed{1}$ 50Ω, $Z_g = Z_0$ $\wedge$ $V_g = 10\ V$



a) ∵ Z_0 real but lossy $\rightarrow$ distortionless

∴ $\frac{R}{L} = \frac{G}{C}$, $L = \frac{\mu}{2\pi} \ln \frac{b}{a} = 0.219722\ \mu H/$

∵ $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}} \rightarrow C = 87.88\ pF/m$

$\cancel{G = \frac{2\pi\sigma}{\ln(b/a)}} =$

$R = \frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right] \wedge \delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma_c}}$

$\rightarrow \delta = 6.6084 \times 10^{-6}$ m

$\rightarrow R = 1.10727\ \Omega$

$\because Z_0 = \sqrt{\frac{R}{G}} \rightarrow G = 4.4291 \times 10^{-4}$ S/m

b) $\because \beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{LC} \rightarrow \epsilon = \frac{LC}{\mu}$

$\rightarrow \epsilon_r = 1.737824$

c) $\because G = \frac{2\pi\sigma_d}{\ln(\frac{b}{a})} \rightarrow \sigma_d = \frac{G\ln\frac{b}{a}}{2\pi}$

$\rightarrow \sigma_d = 7.7443 \times 10^{-5}$ S/m

d) $\rightarrow V(t,z) = 5e^{-\gamma z}$ $\alpha = \sqrt{RG} = 0.022145$

$\beta = 2.76097$ rad/m

$\rightarrow V(t,z) = 5e^{-0.022144z} \cdot \cos(\omega t - 2.76097z)$ V/m

2) distortionless $\rightarrow \frac{G}{\omega C}$ = loss tangent

$\rightarrow \frac{\sigma_d}{\omega\epsilon_d}$ $\because Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$

$\frac{L}{C} = \frac{\mu d^2}{\epsilon w^2} \rightarrow Z_0 = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}$

$\because \eta = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0}}$

$\rightarrow 72 = \frac{\eta_0}{\pi} \cdot \frac{1}{\sqrt{\epsilon_r}}$ X

$\rightarrow \frac{120}{72} = \sqrt{\epsilon_r} \rightarrow \epsilon_r = 2.7778$ X

$\because \tan\delta = \frac{G}{\omega C} = \frac{R}{\omega L}$ $\frac{R}{L} = \frac{2}{\delta\sigma_c\mu d}$

$\rightarrow \frac{2\sqrt{\pi \cdot f \cdot \mu_0 \cdot \sigma_c}}{\omega \cdot \frac{0.1}{\pi} \cdot \mu_0 \cdot \sigma_c} = 1.1986 \times 10^{-4}$

$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$ $\because Z_0 = \sqrt{\frac{L}{C}} \rightarrow C = 77.16$ pF/m

$R = 0.09038\ \Omega/m \wedge L = 0.4\ \mu H/m$

$G = 17.434\ \mu S/m$

$\rightarrow \beta = 10.4919$ Rad/m ∧ $\alpha = 1.2553\times10^{-3}$ Np/m

3) $Z_0 = \sqrt{\frac{L}{C}}$ ✗ ∧ $\gamma = j\beta$ ✗ not lossless

$L = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = 0.26954\times10^{-6}$ H/m ✓

∧ $C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} = 93.4426$ pF/m ✓

$\therefore Z_0 = 53.5084\ \Omega$ ✗

∧ $\gamma = j\omega\sqrt{LC} = j31.416\ m^{-1}$ $\boxed{j\beta}$

$G = 0 \because \sigma_d = 0$ ∧ $R = \frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right]$

$\rightarrow R = 4.0829\ \Omega/m$

$\therefore \gamma = \sqrt{(R+j\omega L)(j\omega C)} = \sqrt{986.9489}\angle\frac{3.139164}{2}$ Rad

$= 31.416\angle 1.5696$ Rad

$\rightarrow \gamma = 0.03958 + 31.416j$

$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{2863.1565}\angle\frac{-2.43288\times10^{-3}}{2}$

$= 53.5085\angle -1.21644\times10^{-3}$

$\rightarrow Z_0 = 53.5085 - 0.065j\ \Omega$

examples form notes:

1) $Z_{in} = Z_0\frac{Z_L + Z_0\tanh(\gamma\ell)}{Z_0 + Z_L\tanh(\gamma\ell)}$, $\alpha = 8$ dB/m $= 0.921$ Np/m

$\tanh(\gamma\ell) = \tanh(1.842 + j2)$

$\because \tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$

$\rightarrow \tanh(\gamma\ell) = \frac{\tanh(1.842) + j\tan(2)}{1 + j\tanh(1.842)\cdot\tan(2)} = 1.03263 - 0.039291j$

$\rightarrow Z_{in} = 60.25 + j38.8\ \Omega$

or $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = 0.3424\angle 2.0525$ Rad

$\rightarrow \Gamma(z) = 8.602\times10^{-3}\angle 2.0525 - 2\beta l$

$\rightarrow \Gamma(z) = 8.602\times10^{-3}\angle -1.9496$ Rad

$\rightarrow (1-\Gamma(z))\,Z(l) = [1+\Gamma(z)]\,Z_0$

$\rightarrow Z(l) = 60.25 + 38.98\,j$

$\because \rightarrow V_{in} = 10\,\dfrac{Z_{in}}{Z_{in}+Z_g} = 6.6687\angle 0.203$

$\rightarrow I_{in} = 93\angle -0.37$ Rad mA

$\because I(z) = \dfrac{V_0^+}{Z_0}e^{-\gamma z} - \dfrac{V_0^-}{Z_0}e^{\gamma z}$

$V_0^+ \wedge V_0^- = \frac{1}{2}(V_0 \pm Z_0 I_0)$

example : matched load and Line

$\rightarrow Z_{in} = Z_0 \rightarrow V_{in} = \frac{1}{2}V_g = 7.5\angle 0^\circ$ V

$I_{in} = \dfrac{7.5\angle 0^\circ}{30 + j60} = 0.05 - 0.1j$ A

$\because V_L = V_{in}\cdot e^{-\gamma l} \rightarrow e^{-\alpha\cdot l}\cdot e^{-j\beta\cdot l} = \frac{2}{3}e^{j48^\circ}$

$\rightarrow \alpha = 0.01014$ Np/m

$\wedge\ \beta = 0.020944$ Rad/m

example : $\because V_{in} = V_g\cdot\dfrac{Z_{in}}{Z_{in}+Z_g}$

$\rightarrow Z_{in} = 2Z_0 = Z_g \rightarrow$ Resistive

$\wedge\ Z_{in} = Z_0\cdot\dfrac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$, $\beta l = \dfrac{100\cdot\pi f}{c}$ 2000π

$\rightarrow Z_{in} = Z_L = 2Z_0 = 100\,\Omega$

or from $\lambda = 10$ cm $\wedge\ l = 1000\lambda$ integer multiple

$\rightarrow$ half wave section $\rightarrow Z_{in} = Z_l$

example: $\because P_L = [1-|\Gamma|^2] \cdot \frac{|V_0^+|^2}{2Z_0}$

~~$\because V_0^+ \le 250 \cdot \sqrt{2} \rightarrow 1k \cdot \frac{2\cdot 50}{2 \cdot (250)^2} = 1-|\Gamma|^2$~~

~~$\rightarrow |\Gamma|^2 = 0.2 \rightarrow \Gamma \le \sqrt{0.4472136}$~~

~~$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} \le 2.$~~

$V_{max} = |V_0^+| \cdot (1+|\Gamma|) \le 250\sqrt{2}$

$\because P_L = 1000 \rightarrow |V_0^+|^2 = \frac{1000 \cdot 2Z_0}{1-|\Gamma|^2}$

$1-|\Gamma|^2 = (1+|\Gamma|)(1-|\Gamma|)$

$\rightarrow \frac{1\cancel{2}000\, Z_0}{\cancel{(1+|\Gamma|)}(1-|\Gamma|)} \cdot \cancel{(1+|\Gamma|)}^{\cancel{2}} \le 250^2 \cdot \cancel{2}$

$\rightarrow \frac{1+|\Gamma|}{1-|\Gamma|} \le \frac{250^2}{50k}$

$\rightarrow SWR \le 1.25$

$\because P_L = P_{inc}[1-|\Gamma|^2] \rightarrow P_{inc} = \frac{1k}{1-|\Gamma|^2}$

assuming $SWR = 1.25 \rightarrow |\Gamma| = 0.111$

$\rightarrow P_{inc} = 1012.5\,W$

example: $P_L = [1-|\Gamma|^2] \frac{|V_0^+|^2}{2Z_0}$

$|\Gamma| = |\Gamma_L| \because$ lossless $= 0.5$

$\because Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$ $\wedge\ \beta l = \frac{2\pi}{\lambda} \cdot 0.8$ ~~λ~~

$\rightarrow Z_{in} = 39.852 - 50.535\,\Omega$

$\rightarrow V_{in} = 100.16538 \angle -0.2204\ Rad$

∧ $I_{in} = \frac{V_{in}}{Z_{in}} = 1.20956 + 0.98188j$ A

$P_{inc} = P_L$ ∵ lossless → $\frac{1}{2}Re[V_{in} \cdot I_{in}^{*}] = P_{avg}$

→ $P_{in} = \frac{1}{2}[96.533] = 48.2665$ W

example: ∵ $V_{in} = 5V$ ∧ Z_L, Z_g, Z_0 pure real

∧ $Z_L < Z_0$ → Z_{max} (input) $= \frac{Z_0^2}{Z_0/5} = 5Z_0$

∵ $V_{in} = V_g \cdot \frac{Z_{in}}{Z_{in} + Z_g}$ & $Z_{in} = Z_g \Rightarrow V_g = 10V$

example: assuming Z_{02} is ∞ length →

∧ V starts at min ∧ lossless lines & resistive

∵ starts min → $Z_{02} < Z_{01}$ ∧ $\Gamma_L < 0$

∵ $P_{ref} = 3mW = |\Gamma|^2 \cdot P_{inc} \rightarrow |\Gamma|^2 = \frac{1}{4}$

∴ $\Gamma = \frac{-1}{2} \rightarrow SWR = 3$

∵ $SWR = \frac{R_0}{R_L} \rightarrow R_L = Z_L = 20\Omega$

example ∵ ∞ length → $Z_L = 100\Omega$ ∧ $R = 100\Omega$

∧ $Z_{01} = \sqrt{5000}$ ∧ $\theta = 90°$ at 3GHz

at $\theta = 90°$ → $Z_{in} = \frac{Z_0^2}{Z_L} = \frac{5000}{50} = 100$

→ $SWR = \frac{100}{5} = 2$

at 6GHz, $\theta = 180°$ → $Z_{in} = Z_L = 50\Omega$

→ $SWR = 1$

at 9GHz, $\theta = 270°$ → $Z_{in} = 100$ → $SWR = 2$

Samples 4 continued:

1) $l = 1.25\lambda$ → quarter wave section

∴ $Z_{in} = \frac{Z_0^2}{Z_L}$ → $Z_{in} = 10\,\Omega$ starts at min

∵ $V_{max} = V_g \frac{Z_{in}}{Z_{in}+Z_g}$ → $Z_g = 15\,\Omega$ X

∵ $40 = Z_0 = \sqrt{\frac{L}{C}} = \frac{d}{w}\sqrt{\frac{\mu}{\varepsilon}}$

∵ $\lambda = 10\,cm = \frac{u_p}{f}$ → $u_p = \frac{3\times10^8}{3}$

→ $\sqrt{\mu_r \varepsilon_r} = 3$. Assuming non-mag → $\varepsilon_r = 9$

→ $\frac{d}{w}\cdot\frac{120\pi}{3} = 40$ → $\frac{w}{d} = \pi$

∵ $SWR = \frac{160}{40} = 4$ → $V_{min} = 1V$

∴ $1V = 10V \cdot \frac{Z_{in}}{Z_{in}+Z_g}$ → $Z_g = 40\,\Omega$

2) matched source ∧ matched load → $Z_{in} = Z_0 \wedge V_{in} = 5V$

a) ∵ $Z_0 = 63.5\,\Omega$ → distortionless $\frac{R}{L} = \frac{G}{C}$

$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$ $L = \frac{\mu_0}{2\pi}\cdot\ln\left(\frac{b}{a}\right) = 0.26832\,\mu H/$

∧ $C = \frac{2\pi\varepsilon}{\ln\frac{b}{a}}$

→ $Z_0 = \sqrt{\frac{L\cdot\ln\left(\frac{b}{a}\right)}{2\pi\varepsilon_0\varepsilon_r}}$ → $\varepsilon_r = 2.264$

$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\ln\frac{b}{a}\left[\frac{1}{a}+\frac{1}{b}\right]}{\delta\sigma_c\sigma_d}} = 9.624\times10^{-5}\,S/m$

b) ∵ $\frac{R}{\omega L} = \frac{\sigma_d}{\omega\varepsilon}$ → $\sigma_d = \frac{\varepsilon R}{L} =$ 9.62×1

$R = 1.29\,\Omega/m$

b) $\beta l = \omega\sqrt{\mu\varepsilon}\cdot 5 = 15.759$ Rad

c) $P_{avg} = \left[1-|\Gamma|^2\right]\frac{|V_0^+|^2}{2Z_0}$ X lossless

∵ matched load → $V_0^+ = V_{in}$ ∧ $V_{in} = 5V$ ✓

→ $P_{avg} = \left[1-|\Gamma|^2\right]\cdot\frac{25}{107}$ lossless X

$|\Gamma|^2 = 0$ → $P_{avg} = 0.2336W$ X

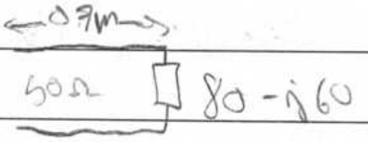
$P_{avg} = \frac{|V_0^+|^2}{2Z_0}\cdot e^{-2\alpha z} = \frac{25}{107}\cdot e^{-70\cdot\alpha}$

$G = \frac{R}{Z_0} = \frac{1.19}{535}$

$\alpha = \sqrt{RG} = 0.02411$ Np/m

→ $P_{avg} = 0.18359$ W

past seconds:



2) $Z_0 = 50\Omega$, lossless

a) ∵ $u_p = 0.8c = f\cdot\lambda$ → $\lambda = \frac{0.8c}{f} = 0.8m$

$l = \frac{0.9}{0.8}\lambda = X$

$\Gamma(z') = \frac{80-j60-50}{130-j60}e^{-2\gamma z'} = 0.4685e^{-j80.6789}\,e^{-2\gamma z'}$

$|\Gamma_L| = |\Gamma|$ lossless → $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 2.963$

b) $Z_{in} = 50\cdot\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l} = 80 - j60$

$l = 0.9m$ ∧ $\beta = \frac{2\pi}{\lambda}$ → $\beta l = \frac{9}{4}\pi$

→ $Z_{in} = 50\cdot\frac{Z_L - j50}{50 - jZ_L} = 80 - j60$

→ $Z_L = j50 + (1.6-1.2j)\cdot(50 - jZ_L)$

→ $Z_L = j50 + 80 - 60j - j1.6Z_L - 1.2Z_L$

→ $2.2Z_L + j1.6Z_L = 80 - 10j$ → $Z_L =$

$Z_L = 21.622 - 20.99j \rightarrow \Gamma_L = 0.46852\angle -2.246 \text{ Rad}$

$\beta l = \frac{11}{2}\pi \rightarrow Z_{in} = \frac{Z_0^2}{Z_L} = 80 - j60\ \Omega$

$\rightarrow Z_L = 20 + 15j\ \Omega$

$\therefore \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.4685\angle 2.46685 \text{ Rad}$

(a) $\because \Gamma(z) = \Gamma_L e^{-j2\beta z'}$

$\Gamma(z = \frac{l}{2}) = \Gamma_L e^{-j2\beta \cdot 2.2} = 0.4685\angle -0.69$

$\rightarrow \Gamma_L = 0.4685\angle(-0.6947 + \pi)$

b) $Z_{in} = Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}} \wedge \Gamma_{in} = 0.4685\angle 2.46685 \text{ Rad}$

$\rightarrow Z_{in} = 20 + 15j\ \Omega$

2) $\alpha = 0.0666 \text{ Np/m} \wedge \beta = 1.885 \text{ rad/m}$

$Z_0 = 50\ \Omega \rightarrow$ distortionless $\frac{R}{L} = \frac{G}{C}$

$\alpha = \sqrt{RG} \wedge \beta = w\sqrt{LC}$

$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}} \rightarrow R = 2500\,G \wedge L = 2500\,C$

$\therefore \alpha = 50\,G \rightarrow G = 1.332\times10^{-3} \text{ S/m}$

$\wedge R = 3.33\ \Omega/\text{m}$

$\wedge \beta = w\,50\,C \rightarrow C = 0.1333 \text{ nF/m}$

$\wedge L = 0.333\ \mu\text{H/m}$

a) $\because C = \frac{\varepsilon_r \varepsilon_0 \cdot W}{d}$ $L \cdot C = \mu\varepsilon \wedge \mu = \mu_0$

$\rightarrow \varepsilon_r = 3.995 \approx 4$

b) $\sigma_c \because R = \frac{2}{w\delta\sigma_c} \wedge \delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}}$

$\rightarrow R = \frac{2\sqrt{\pi f \mu_0}}{w\sqrt{\sigma_c}} \rightarrow \sqrt{\sigma_c} = 800.52$

$\rightarrow \sigma_c = 0.641 \text{ MS/m}$

c) ∵ $C = \frac{\varepsilon w}{d} \rightarrow d = 2.663\ mm$

d) ∵ $G = \frac{\sigma w}{d} \rightarrow \sigma_d = 3.534 \times 10^{-4}\ S/m$

3) 50 Ω lossless , $\lambda = 1m \rightarrow l = \frac{3}{4}\lambda$

a) quarterwave section ∴ $Z_{in} = \frac{Z_0^2}{Z_L} = \boxed{30-40j\ \Omega}$

b) Imin at Zmax = $z'_{max} = \frac{\theta_{\Gamma_L} + 2n\pi}{2\beta}$

$n=0 \rightarrow \frac{\theta_{\Gamma_L}\cdot\lambda}{4\pi}$

, $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.5\angle\frac{\pi}{2}$

$\rightarrow z'_{max} = \frac{1}{8}$

c) $z'_{min} = \frac{[\pi/2 + \pi]\lambda}{4\pi} = \frac{1.5\lambda}{4} = \frac{3}{8}\lambda$

$Z_{min} = \frac{Z_0}{SWR}$, $SWR = \frac{1.5}{0.5} = 3$

$\rightarrow Z_{min} = 16.69\ \Omega$

d) ∵ lossless → = V_{in} , $V_{in} = 96-28j\ V$

, $P_{avg} = \frac{|V_0^+|^2}{2\cdot 50}\left[1-|\Gamma|^2\right] =$

∵ $V_0^+ = \frac{1}{2}\left(V_{in} + \frac{V_{in}}{Z_{in}}\cdot Z_0\right) = 88+16j$

$\rightarrow P_{avg} = \frac{40^2\cdot 5}{2\cdot 50}\cdot 0.75 = 60W$

Or $P_{avg} = |I_{in}|^2\cdot\frac{1}{2}R_i$ $R_i = 30\Omega$

$\rightarrow 2\cdot R_i = 60$

Subject Quiz 4 Date 23/5/2021 No.

MOHAMMAD SANAD ALTAHER 130806

أنا محمد سند الطاهر أشهد أني قرأت و فهمت تعليمات هذا الامتحان
القصير وتقيد بها ، والله على ما أقول شهيد.

lossless , $Z_0 = 75\,\Omega$, $Z_g = Z_0$, $V_g = 500\,V$

load: resistive (R_L) , $V_{max} = 300\,V$ (not rms)

$\therefore V_{max} = |V_0^+| \cdot [1 + |\Gamma|]$ $\wedge$ lossless $\rightarrow |\Gamma| = |\Gamma_L|$

$$\wedge \quad SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{V_{max}}{V_{min}}$$

$$|\Gamma_L| = \frac{R_L - 75}{R_L + 75} \quad , \quad V_{min} = |V_0^+| \cdot [1 - |\Gamma|]$$

$$\rightarrow \frac{300}{|V_0^+| \cdot [1 - |\Gamma|]} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\frac{300}{|V_0^+|} = 1 + \frac{R_L - 75}{R_L + 75}$$

$$|V_0^+| = \frac{1}{2}(V_0 + I_0 Z_0)$$

$$\rightarrow \frac{600}{V_0\left(1 + \frac{75}{Z_{in}}\right)} = 1 + \frac{R_L - 75}{R_L + 75}$$

$$\therefore V_0 = 500 \cdot \frac{Z_{in}}{Z_{in} + 75} \quad \rightarrow \frac{600}{500} \cdot \frac{\cancel{Z_{in} + 75}}{\cancel{Z_{in} + 75}} = 1 + \frac{R_L - 75}{R_L + 7}$$

$\rightarrow R_L =$

N O T E B O O K

Subject

Chapter 12: Waveguides

Date 29/5/2021 No.

- Transmission lines can only support TEM waves, whereas waveguides can support many wave configurations (except TEM).
- TLs become inefficient at microwave frequencies due to skin effect and dielectric losses.
- Waveguides for sub-microwave frequencies are excessively large.
- Waveguides can function as high-pass filters since all frequencies below a specific cutoff frequency (f_c) will not be passed.
- Rectangular, hollow, and lossless waveguides are assumed.

12.2: Rectangular waveguides:

30/5/2021

- Recall the following equations for a lossless medium:

$$\nabla^2 \overline{E}_s + k^2 \overline{E}_s = 0 \quad \wedge \quad \nabla^2 \overline{H}_s + k^2 \overline{H}_s = 0$$

where $k = \omega\sqrt{\mu\varepsilon}$, and time factor $e^{j\omega t}$ is assumed

- Since ∇^2 is the vector laplacian, then $\nabla^2 \overline{E}_s + k^2 E_s = 0$ comprises three equations: $\nabla^2 E_{xs} + k^2 E_{xs} = 0$, $\nabla^2 E_{ys} + k^2 E_{ys} = 0$

$$\wedge \underbrace{\nabla^2 E_{zs} + k^2 E_{zs} = 0}_{\frac{\partial^2 E_{zs}}{\partial x^2} + \frac{\partial^2 E_{zs}}{\partial y^2} + \frac{\partial^2 E_{zs}}{\partial z^2} + k^2 E_{zs} = 0} \quad \text{(same for } H_s\text{)}$$

let $E_{zs}(x, y, z) = X(x)\,Y(y)\,Z(z)$

$$\rightarrow \frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2$$

each term in the above equation is constant since the variables are independent.

$$\rightarrow -k_x^2 - k_y^2 + \gamma^2 = -k^2$$

s.t. $X'' + k_x^2 X = 0$, $Y'' + k_y^2 Y = 0$, $Z'' - \gamma^2 Z = 0$

separation constants

solving the differential equations gives:

$$X(x) = C_1 \cos(k_x x) + C_2 \sin(k_x x)$$

$$Y(y) = C_3 \cos(k_y y) + C_4 \sin(k_y y)$$

$$Z(z) = C_5 e^{\gamma z} + C_6 e^{-\gamma z}$$

$$\therefore E_{zs}(x,y,z) = \left[C_1 \cos(k_x x) + C_2 \sin(k_x x)\right] \cdot \left[C_3 \cos(k_y y) + C_4 \sin(k_y y)\right] \cdot \left[C_5 e^{\gamma z} + C_6 e^{-\gamma z}\right]$$

- assuming the wave travels in the + z - direction, then the constant C_5 must be zero for the wave to be finite at infinity z.

$$\therefore E_{zs}(x,y,z) = \left[A_1 \cos(k_x x) + A_2 \sin(k_x x)\right] \cdot \left[A_3 \cos(k_y y) + A_4 \sin(k_y y)\right] e^{-\gamma z}$$

$$\& \; H_{zs}(x,y,z) = \left[B_1 \cos(k_x x) + B_2 \sin(k_x x)\right] \cdot \left[B_3 \cos(k_y y) + A_4 \sin(k_y y)\right] e^{-\gamma z}$$

- using faraday's law and Ampere's circuit law:

$$\nabla \times E_s = -j\omega\mu H_s \qquad \nabla \times H_s = j\omega\varepsilon E_s$$

gives: gives:

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \; (1) \qquad \frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_z \; (4)$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \; (2) \qquad -\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \; (5)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \; (3) \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \; (6)$$

express the above equations in terms of E_z & H_z:

$$h^2 = \gamma^2 + k^2 = \gamma^2 + \omega^2\mu\varepsilon = k_x^2 + k_y^2$$

from (1) & (5): from (2) & (4):

$$H_x = \frac{1}{h^2}\left[j\omega\varepsilon \frac{\partial E_z}{\partial y} - \gamma \frac{\partial H_z}{\partial x}\right] \qquad H_y = \frac{-1}{h^2}\left[\gamma \frac{\partial H_z}{\partial y} + j\omega\varepsilon \frac{\partial E_z}{\partial x}\right]$$

$$E_y = \frac{-1}{h^2}\left[\gamma \frac{\partial E_z}{\partial y} - j\omega\mu \frac{\partial H_z}{\partial x}\right] \qquad E_x = \frac{-1}{h^2}\left[\gamma \frac{\partial E_z}{\partial x} + j\omega\mu \frac{\partial H_z}{\partial y}\right]$$

- for TEM waves, E_z & $H_z = 0$, substituting 0 in the previous four equations gives all components equal to zero unless h^2 is also equal to zero → $h^2 = 0$ for TEM waves → $\gamma^2 = -k^2$ → $\gamma = jk = j\omega\sqrt{\mu\varepsilon}$ Which is β. however, waveguides cannot support TEM

+ four different field patterns (modes) exist: (for +z-direction)
  - TEM: $E_z = H_z = 0$
  - TM: $E_z \neq 0$, $H_z = 0$
  - TE: $E_z = 0$, $H_z \neq 0$
  - HE: $E_z \neq 0$, $H_z \neq 0$ (hybrid mode)

12.3: Transverse magnetic modes

31/5/2021

- in TM, the magnetic field's components are transverse to the direction of propagation → $H_z = 0$
- since the tangential components of the E field must be continuous therefore E_{zs} should be zero at the four walls of the waveguide.

→ $E_{zs} = 0$ at $y = 0$ (bottom wall) (1) $E_{zs} = 0$ at $x = 0$ (right wall) (2)

$E_{zs} = 0$ at $y = b$ (top wall) (3) $E_{zs} = 0$ at $x = a$ (left wall) (4)

∵ $E_{zs}(x,y,z) = [A_1 \cos(k_x x) + A_2 \sin(k_x x)] \cdot [A_3 \cos(k_y y) + A_4 \sin(k_y y)] e^{-\gamma z}$

- for (1) & (2) to hold, A_1 & A_3 must equal to zero

→ $E_{zs}(x,y,z) = E_0 \sin(k_x x) \cdot \sin(k_y y) e^{-\gamma z}$, s.t. $E_0 = A_2 A_4$

∵ $\sin(k_x a)$ and $\sin(k_y b)$ must both equal zero.

Then $k_x a = m\pi$, $m = 1,2,3 \ldots$ | $k_y b = n\pi$, $n = 1,2,3,\ldots$ must be true.

$$\therefore E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

- to obtain the other components, substitute $H_z = 0$ and the above expression for E_{zs} into the four equations on the previous page

e.g. $E_{xs} = \frac{-\gamma}{h^2} \cdot \frac{\partial E_z}{\partial x}$

$$\rightarrow E_{xs} = \frac{-\gamma}{h^2} \cdot \left(\frac{m\pi}{a}\right) \cdot E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$\because h^2 = k_x^2 + k_y^2 \rightarrow h^2 = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$

- every pair of integers gives a different field pattern (mode), which is referred to as the TM_{mn} mode (e.g., TM_{31}), where integer m represents the number of half cycles in the x-direction, and integer n the number of half cycles in the y-direction.
- neither m nor n can be zero since if one of them is zero then all field components will be zero, TM_{11} is the lowest order of TM_{mn} modes

$\because \gamma^2 + k^2 = h^2 = k_x^2 + k_y^2 \quad \therefore \gamma = \sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2 - k^2}$

where $k = \omega\sqrt{\mu\epsilon}$. Therefore, γ can vary with m, n, or ω.

* Case 1: cutoff, no propagation takes place at this frequency

if $k^2 = k_x^2 + k_y^2 \rightarrow \gamma = 0 \rightarrow \alpha = \beta = 0 \quad \because \gamma = \alpha + j\beta$

$$\therefore \omega_c = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2}$$

* Case 2: evanescent (no propagation, only attenuation)

$k^2 < k_x^2 + k_y^2 \rightarrow \gamma = \alpha \;,\; \beta = 0$

$\omega^2 \mu\epsilon < \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$

* Case 3: propagation

$k^2 = \omega^2\mu\epsilon > \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$

$\rightarrow \gamma = j\beta \;,\; \alpha = 0$

$$\therefore \beta = \sqrt{k^2 - \left[\frac{m\pi}{a}\right]^2 - \left[\frac{n\pi}{b}\right]^2}$$

- propagation takes place because all field components have the factor $e^{-\gamma z} = e^{-j\beta z}$
- for each mn mode, there is a different cutoff frequency and β

* cutoff frequency : the frequency below which attenuation occurs and above which propagation takes place.

- $f_c = \frac{\omega_c}{2\pi}$

$$\rightarrow f_c = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad , \text{s.t } u = \frac{1}{\sqrt{\mu\varepsilon}}$$

- u : phase velocity of a uniform plane wave in the lossless dielectric medium filling the waveguide.
- cutoff wavelength, $\lambda_c = \frac{u}{f_c} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$
- TM_{11} has the lowest cutoff frequency / longest λ_c
- $\beta_z = \omega\sqrt{\mu\varepsilon} \cdot \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

$$\rightarrow \beta_z = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

- guide wavelength : λ_g

$$\lambda_g = \frac{2\pi}{\beta_z} = \frac{2\pi}{k\sqrt{1-(f_c/f)^2}}$$

$\because \lambda = \frac{2\pi}{k} = \frac{u}{f} = \frac{2\pi}{\beta}$

$$\therefore \lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$$

where λ is the wavelength in an unbounded medium

$\rightarrow \lambda_g > \lambda + \lambda$

$$\therefore \frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

- phase velocity : $u_p = \frac{\omega}{\beta_z}$

$u_p > u$

$$\rightarrow u_p = \frac{u}{\sqrt{1-(f_c/f)^2}} = u \cdot \frac{\lambda_g}{\lambda}$$

- intrinsic wave impedance for the mode:

$$\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta_z}{\omega\varepsilon}$$

$$\therefore \eta_{TM} = \frac{k}{\omega\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$\eta_{TM} < \eta$ s.t. $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

- hence, each mode given a different f_c, β_z, u_p, λ_g, λ_c, η_{TM}

- all the previous equations apply for any waveguide of any shape, except the equation for f_c.

12.4 : Transverse electric modes :

- $E_z = 0$, since electric field is transverse to direction of propagation
- tangential components of E must be continuous at the walls:

$\rightarrow E_{xs} = 0$ at $y = 0$ | $E_{ys} = 0$ at $x = 0$

$E_{xs} = 0$ at $y = b$ | $E_{ys} = 0$ at $x = a$

$\therefore$ from the previous four equations linking E_z & H_z to other components

$\rightarrow \frac{\partial H_{zs}}{\partial y} = 0$ at $y = 0$ | $\frac{\partial H_{zs}}{\partial x} = 0$ at $x = 0$

$\frac{\partial H_{zs}}{\partial y} = 0$ at $y = b$ | $\frac{\partial H_{zs}}{\partial x} = 0$ at $x = a$

$$\therefore \quad H_{zs} = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

- m and n again denote the number of half cycle variations in the x-y cross-section of the guide
- f_c, λ, λ_c, λ_g, β_g and u_p are all the same for TE modes as the TM modes (i.e., the same equations hold)
- TE_{mn} can be TE_{01} or TE_{10}, but m and n cannot both be zero as that will cause all the field components to vanish.
- the lowest TE mode (TE_{01} or TE_{10}) depends on a and b.
- if $a > b$, then TE_{10} is the lowest mode because it will have a lower cutoff frequency than TE_{01}

* dominant mode: the mode with the lowest cutoff frequency / longest λ_c

- the cutoff frequency for TE_{10} : $f_{c10} = \frac{u'}{2a}$, and the cutoff wavelength : $\lambda_{c10} = 2a$

- any wave with a frequency lower than the dominant mode's will not propagate in the waveguide.
- the equation for the intrinsic impedance for the TE mode differs from that for the TM mode

$$\rightarrow \quad \eta_{TE} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\omega\mu}{\beta}$$

$$\therefore \eta_{TE} = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\eta}{\sqrt{1-(f_c/f)^2}}$$

- hence, $\eta_{TE} \cdot \eta_{TM} = \eta^2$

example from notes: WG-16, X-band (8-12 GHz), $a = 2.29$ cm, $b = 1.02$ cm

$$\because (f_c)_{mn} = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}, \quad u = c \because \text{air-filled}$$

for TE_{10} : $(f_c)_{10} = \frac{c}{2a} = 6.55$ GHz

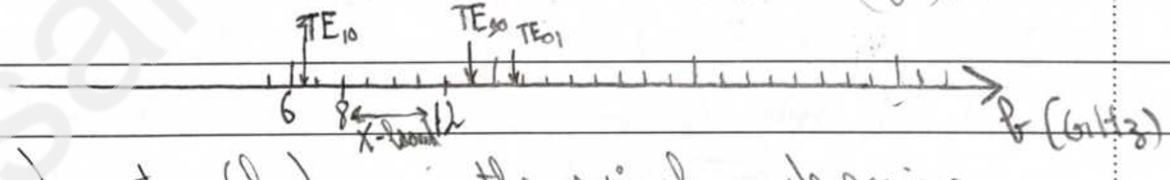
TE_{01} : $(f_c)_{01} = \frac{c}{2b} = 14.7 \times 10^9$ Hz

$TE_{11} \wedge TM_{11}$: $(f_c)_{11} = 16.1$ GHz

TE_{20} : $(f_c)_{20} = 13.1$ GHz

$\therefore (f_c)_{01} > (f_c)_{20} \because a > 2 \cdot b$

if a was smaller than 2b then $(f_c)_{20} > (f_c)_{01}$



$(f_c)_{10}$ to $(f_c)_{20}$ is the single mode region

TE_{20} is the first higher order mode, whereas TE_{10} is the dominant mode example from notes: air-filled rectangular wave guide

5×2 cm $\rightarrow$ $a = 5$ cm & $b = 2$ cm

$f = 15$ GHz (operational frequency)

$E_z = 20 \sin(40\pi x) \sin(50\pi y) e^{-j\beta z}$ V/m

a) what mode is this?

$\because 40\pi x = \frac{m\pi}{a} x \rightarrow m = 2$

& $50\pi y = \frac{n\pi}{b} y \rightarrow n = 1$

$\rightarrow TM_{21}$ $\boxed{\because \sin}$

b) $\beta_z = k\sqrt{1-(\frac{f_c}{f})^2}$ & $(f_c)_{21} = 9.605$ GHz

$\rightarrow \beta_z = \frac{2\pi \cdot 15G}{c} \cdot \sqrt{1-(\frac{f_c}{f})^2}$

$\rightarrow \beta_z = 241.3$ rad/m

c) $\because TM_{21} \rightarrow \eta_{TM} = \eta\sqrt{1-(\frac{f_c}{f})^2} = 289.6\,\Omega$

example from notes: air-filled rectangular, $a = 22.9$ mm

& $b = 10.2$ mm, TE_{11} mode

distance between successive minima = 3cm

$\rightarrow \lambda_g = 2 \cdot 3$cm $= 6$cm

$\because \frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda c^2}$

$\lambda_c = \frac{u}{f_c} = \frac{2}{\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}} \rightarrow \frac{1}{\lambda_c^2} = \frac{(\frac{m}{a})^2 + (\frac{n}{b})^2}{4}$

$\therefore \frac{1}{\lambda_c^2} = 2899.65 \rightarrow \lambda = 0.0178$ m

$\rightarrow f = 16.857$ GHz

example from notes: find a & b for a>b to operate in a single mode between 9 & 14 GHz (air-filled)

dominant mode = TE_{10} $\because$ $a>b$

$(f_c)_{10} = \frac{c}{2a} = 9\ GHz \rightarrow a = 1.6667\ cm$

$(f_c)_{01} = \frac{c}{2b} = 14\ GHz \rightarrow b = 1.07\ cm$

* degenerate modes: modes with the same $f_c, \beta, k_c, \lambda_g, u_p$, but not the same field distributions or intrinsic wave impedance

* group velocity (energy velocity): $u_g = \frac{1}{\partial\beta/\partial\omega}$

$$\rightarrow \quad u_g = u\sqrt{1-\left(\frac{f_c}{f}\right)^2} < u$$

$$\therefore \quad u_p \cdot u_g = u^2$$

* Power Transmission & Attenuation:

$\alpha = \alpha_c + \alpha_d$ (sum of attenuation constants of conductor and dielectric)

only holds for TE_{10}

$$\text{s.t.} \quad \boxed{\alpha_c\Big|_{TE_{10}}} = \frac{2R_s}{b\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[\frac{1}{2}+\frac{b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

$$\text{where} \quad R_s = \frac{1}{\sigma_c\delta_c}$$

$$\wedge \quad \alpha_d = \frac{\sigma_d\eta}{2\sqrt{1-(f_c/f)^2}}$$, holds for any mode.

$$\wedge \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

- $\alpha_c \rightarrow \infty$ as $f \rightarrow f_c$, hence the operational frequency is usually far from the cutoff.

example from notes: loss tangent = 2.55×10^{-4} at 6GHz, $a = 22.86$ mm & $b = 10.16$ mm, $\varepsilon_r = 2.56$, find α_d at 6GHz

$\because \ \alpha_d = \dfrac{\sigma_d \cdot \eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$, $(f_c)_{10} = \dfrac{u}{2a}$, $u = \dfrac{c}{\sqrt{\varepsilon_r}}$ $\rightarrow (f_c)_{10} = 4.101$ GHz

$\because \ \tan(\theta) = \dfrac{\sigma_d}{\omega \varepsilon} \rightarrow \sigma_d = 2.176 \times 10^{-4}$ S/m

$\rightarrow \ \alpha_d = \dfrac{\sigma_d \cdot \frac{120\pi}{\sqrt{\varepsilon_r}}}{1.499} = 0.03512$ Np/m

$\rightarrow \ \alpha_d = 0.305$ dB/m

Subject samples 4 : Date No.

$\boxed{1}$ matched source and infinite length , $a = 0.5$ mm, $b = 1.5$ mm

a) $R = \frac{\sqrt{\pi f \mu_c \sigma_c}}{2\pi \sigma_c}\left[\frac{1}{a} + \frac{1}{b}\right] = 1.1073\ \Omega/m$

$L = \frac{\mu}{2\pi}\ln(\frac{b}{a}) = 0.21972\ \mu H/m$

$\because$ 50 Ω → distortionless → $\frac{R}{L} = \frac{G}{C}$

$\wedge\ Z_0 = \sqrt{\frac{L}{C}} \rightarrow C = 87.88$ pF

or $Z_0 = \sqrt{\frac{R}{G}} \rightarrow G = 0.44292$ mS/m

b) $\because\ C = \frac{2\pi\epsilon}{\ln\frac{b}{a}} \rightarrow \epsilon_r = 1.737824$

c) $\because\ G = \frac{2\pi\sigma_d}{\ln\frac{b}{a}} \rightarrow \sigma_d = 77.4\ \mu S/m$

d) $V(t,z) = V_0^+ e^{-\alpha z}\cos(\omega t - \beta z)$

$\alpha = \sqrt{RG}$ or $\frac{R}{R_0} = 0.022146$ Np/m

$\beta = \omega\sqrt{LC} = 2.760959$

$\rightarrow V(t,z) = V_0^+ e^{-0.022146}\cdot\cos(200\pi\times10^{6}t + -2.76z)$

$V_0^+ = V_0 \cdot \frac{1}{2} = 5V$

$\boxed{2}$ distortionless : 72 Ω $\frac{G}{\omega C} = \frac{R}{\omega L}$ = loss tangent

$L = \frac{\mu}{\pi} = 4\times10^{-7}$ H/m

$R = \frac{2\sqrt{\pi f_0 \mu_c \sigma_c}}{w \cdot \sigma_c} = 0.09038\ \Omega/m$

$\rightarrow \tan(\theta) = 1.1987\times10^{-4}$

$\gamma = \alpha + j\beta \ \wedge\ \alpha = \frac{R}{72} = 1.255\times10^{-3}$

$\wedge\ \beta = \omega\sqrt{LC} \rightarrow \beta = \omega\frac{L}{72} = 10.472$ rad/m

$\because \sqrt{\frac{L}{C}} = 72 \rightarrow C = \frac{L}{72^2}$

$= \omega\frac{\sqrt{C}\cdot L}{\sqrt{L}} = \omega\sqrt{LC}$ $\beta = \omega\sqrt{\frac{L^2}{Z_0^2}} = \omega\frac{L}{Z_0}$

lossy $\Rightarrow R = 4.083\ \Omega/m$

$\boxed{3}$ lossless $\rightarrow Z_0 = \sqrt{\frac{L}{C}}$, $L = \frac{\mu}{2\pi} \ln \frac{b}{a} = \boxed{2.67944 \times 10^{-9}\ H/m}$

& $C = \frac{2\pi\epsilon}{\ln\frac{b}{a}} = \boxed{93.44\ pF/m}$

$\rightarrow Z_0 = 53.51\ \Omega$

& $\beta = \omega\sqrt{LC} = 31.4155\ rad/m$ & $\gamma = j\beta$

$\therefore \gamma_0$ exact $= \sqrt{(R+j\omega L)\cdot j\omega C} = \sqrt{986.936} \angle \frac{3.1391638}{2}$

$\rightarrow \gamma_0 = 31.4155 \angle 1.569682\ rad$

$\rightarrow \gamma_0 \simeq 0.03815 + j31.4155$

Z_0 exact $= \sqrt{\frac{R+j\omega L}{j\omega C}} = \sqrt{2863.2798} \angle \frac{-2.428865}{2}$

$\rightarrow Z_0$ exact $= 53.509615 \angle -1.214326 \times 10^{-3}\ rad$

$\rightarrow Z_0$ exact ≈ 53.50958

$\boxed{1}$ lossless 40Ω , $l = 1.25\lambda = 12.5\ cm$, $V_{max} = 4V$

$\rightarrow$ quarter wave section : $Z_{in} = \frac{Z_0^2}{Z_L} = \underline{10\Omega}$

$V_{max} = |V_0^+| \cdot (1+|\Gamma_L|)$, $\Gamma_L = 0.6$

$\rightarrow |V_0^+| = \pm 2.5 \rightarrow V_0^+ = \pm 2.5V$

& $V_0^+ = \frac{1}{2}(V_{in} + Z_0 I_{in})$

$I_{in} = \frac{V_{in}}{Z_{in}} \rightarrow V_0^+ = \frac{V_{in}}{2}\left[\frac{Z_{in}+Z_0}{Z_{in}}\right]$

$\rightarrow 5 \cdot \frac{Z_{in}}{Z_{in}+Z_0} = V_{in} = \frac{Z_{in}}{Z_{in}+Z_g} \cdot V_g$

$$\rightarrow \boxed{5 \cdot \frac{Z_{in}+Z_g}{Z_{in}+Z_0} = V_g}$$

& $Z_{in} = 10 \rightarrow 5 \cdot \frac{10 + Z_g}{10 + 40} = 10$

$\rightarrow \boxed{Z_g = 90\ \Omega}$

$\because \lambda = 0.1\,m = \frac{2\pi}{\beta} \Rightarrow \beta = 20\pi = \omega\sqrt{\mu\epsilon}$

$\rightarrow \sqrt{\mu\epsilon} = 1\times10^{-8} = \sqrt{LC}$

$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu d^2}{\epsilon w^2}} = \frac{d}{w}\cdot\sqrt{\frac{\mu}{\epsilon}}$

$\because$ non-magnetic: $\epsilon_r = 9 \rightarrow \frac{d}{w} = \frac{1}{\pi} \rightarrow \frac{w}{d} = \pi$

a) $\because \Gamma_L > 0 \wedge R_L > R_0 \rightarrow Z_{min} = Z_{in} \rightarrow V_{min}$

$V_{min} = \frac{V_{max}}{SWR} \wedge SWR = \frac{1.6}{0.4} = 4$

$\rightarrow V_{min} = 1V = V_{in}$

$\rightarrow V_g \cdot \frac{Z_{in}}{Z_g + Z_{in}} = 1V \rightarrow Z_g = 90\,\Omega$

$\boxed{2}$ 53.5Ω ∧ matched source ∧ matched load

distortionless

a) $\because Z_0 = \sqrt{\frac{R}{G}} \wedge R = \frac{\sqrt{\pi f \mu_c \sigma_c}}{2\pi\sigma_c}\left[\frac{1}{a} + \frac{1}{b}\right]$

$\rightarrow R = 1.2936\ \Omega/m$

$\rightarrow G = 4.5195\times10^{-4}\ S/m$

$\because G = \frac{2\pi\sigma_d}{\ln\frac{b}{a}} \rightarrow \sigma_d = 9.65\times10^{-5}\ S/m$

b) $\beta \cdot l$: $\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{LC} = \omega\frac{L}{Z_0}$

$\because L = \frac{\mu}{2\pi}\ln\frac{b}{a} \rightarrow L = 2.68318\times10^{-9}\ H/m$

$\rightarrow \beta = 3.151995\ rad/m$

$\therefore \beta l = 15.756\ rad$

c) matched load → max power delivery

$\therefore V_{in} = 5V = V_L = V_0^+$

$P = \frac{1}{2}Re\{V(z)\cdot I(z)^*\} = \frac{1}{2}Re\left[V_0^+ e^{-\alpha z - j\beta z}\cdot\frac{V_0^+}{Z_0}\cdot e^{-\alpha z + j\beta z}\right]$

$\rightarrow P = \frac{1}{2}\left[\frac{(V_0^+)^2}{Z_0}\cdot e^{-2\alpha z}\right] \rightarrow P = \frac{5^2}{2\cdot 53.5}\cdot e^{-2\alpha z}$

$\alpha = \frac{R}{Z_0} = 0.0242\ Np/m$ $z = l = 5$

$\rightarrow P = \frac{25}{107}\cdot e^{-0.04836z} \rightarrow P = 0.1833\,W$

[3] lossless lines

at B : $Z_2 = Z_0\left[\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}\right]$ $\beta l = \frac{2\pi}{\cancel{\lambda}}\cdot\frac{3\lambda}{8} = \frac{3}{4}\pi$

→ $Z_2 = Z_0\left[\frac{Z_L - jZ_0}{Z_0 - jZ_L}\right] = 118.042 - 92.563j\ \Omega$

$\wedge$ $Z_3 = 95.26 - 30.492j\ \Omega$

→ $Z_3 // Z_2 = 56.818268 - 26.575j\ \Omega$

∴ $Z_{in} = 126.884 - 332.897j\ \Omega$

→ $V_{in} = 65.80836\angle -0.5443$ rad

→ $|V_0^+| = \left|\frac{1}{2}\left(V_{in} + V_{in}\cdot\frac{Z_0}{Z_{in}}\right)\right| = 50\angle -\overset{0}{\cancel{2.286}}$ rad

∴ $P_{in\ \cancel{total}} = \frac{|V_0^+|^2}{2Z_0}(1-|\Gamma|^2) = 2.16499$ W

at B : $P_2 = \frac{|V_0^+|^2}{2Z_{02}}\left[1 - \left|\frac{Z_{L2}-Z_{02}}{Z_{L2}+Z_{02}}\right|^2\right] =$

∵ $V_0^+ = 50 = \frac{V_L}{2}\left(1+\frac{Z_0}{Z_L}\right)e^{j\beta l}$ → $|V_L| = 17.12149$

$P_{L2} = |I_2|^2\cdot\frac{R_{in2}}{2}$ $\wedge$ $|I_B| = I_A = \frac{V_{in}}{Z_{in1}}$

$\wedge$ $I_2 = I_B\cdot\frac{Z_{in3}}{Z_{in2}+Z_{in3}} =$

∴ $P_{L2} = 0.184731^2\cdot 0.40613^2\cdot\frac{118.042}{2} =$

∵ $P_A = P_B = |I_B|^2\cdot\frac{56.81823}{2} \Rightarrow |I_B|^2 = 0.098963$

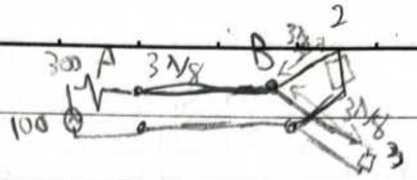
→ $P_{L2} = 0.098963\cdot 0.40613^2\cdot\frac{118.042}{2}$

$= 0.96871$ W

$\wedge$ $P_{L3} = 0.098963\cdot\left|\frac{Z_{in2}}{Z_{in2}+Z_{in3}}\right|^2\cdot\frac{R_{in3}}{2}$

$= 1.3996$ W ✓

$\boxed{3}$



$\because\ l = \frac{3\lambda}{8} \rightarrow \theta_l = \frac{2\pi}{\lambda}\cdot\frac{3\lambda}{8} = \frac{3}{4}\pi$

$\rightarrow \tan(\beta l) = -1$

$\therefore\ Z_{in2} = Z_{02}\cdot\frac{Z_{L2} - jZ_{02}}{Z_{02} - jZ_L} = 118.042 - 92.553j\ \Omega$

$\wedge\ Z_{in3} = Z_{03}\cdot\frac{Z_{L3} - jZ_{03}}{Z_{03} - jZ_{L3}} = 95.2443 - 30.47179j\ \Omega$

Z_{in} at B $= Z_{in2} \parallel Z_{in3}$ define Z_{L1} $= 56.8182$

$\rightarrow Z_{in1} = Z_{01}\cdot\frac{Z_{L1} - jZ_{01}}{Z_{01} - jZ_{L1}} = 126.8837 - 332.8779j$

$\therefore\ V_{in} = V_g\cdot\frac{Z_{in1}}{300 + Z_{in1}} = 65.8084\angle{-0.546}$ rad

$\rightarrow I_{in} = 0.1849305\angle 0.6623$ rad A

$|I_{in}|$ ~~$= |I(z)|\ \forall\ z\ \because$ lossless~~

incorrect ~~before stub~~

$\rightarrow$ current division at stub : $|I_2| = |I_{in}|\cdot\left|\frac{Z_{in3}}{Z_{in2}+Z_{in3}}\right|$

$\wedge\ |I_3| = |I_{in}|\cdot\left|\frac{Z_{in2}}{Z_{in2}+Z_{in3}}\right|$ ✗

$P_{L2} = |I_2|^2\cdot\frac{Z_{L2}}{2} = (0.1849305)^2\cdot(0.40613)^2\cdot\frac{73}{2} =$

current magnitude changes along the T.L

$\because\ V_0^+ = 50V \ \wedge\ V_0^+ = \frac{V_{inB}}{2}\left(\frac{Z_{L1}+Z_{01}}{Z_{L1}}\right)e^{j\beta l}$

$\rightarrow V_{inB} = \frac{2V_0^+}{Z_{L1}+Z_{01}}\cdot Z_{L1}\angle{-\beta l} \Rightarrow |V_{inB}| = 17.12243V$

$\rightarrow |I_{inB}| = 0.28105$ A

$\therefore\ P_{L2} = |I_{inB}|^2\cdot\left|\frac{Z_{in3}}{Z_{in2}+Z_{in3}}\right|\cdot\frac{R_{in2}}{2} = 0.96896W$

$\wedge\ P_{L3} = 1.396$ W

$\boxed{4}$ $Z_L = 5\Omega$, $Z_0 = 50\Omega$ $\rightarrow r = 0.1$

a) open circuited: admittance starts at 0 left $\Gamma = 1$

~~point a : location = 0.2λ , length = 0.3λ~~

~~point b : location = 0.3λ , length = 0.2λ~~

~~shortest stub :~~

point a : location = 0.45λ , length = 0.2λ

point b : location = 0.05λ , length = 0.3λ

shortest distance from load = 0.05λ

λ ∵ $f = 1GHz$ & $\varepsilon_r = 2.3$ $\rightarrow \lambda = \frac{u_p}{f}$ & $u_p = \frac{c}{\sqrt{\varepsilon_r}}$

$\rightarrow \lambda = 0.1978\ m$

$\rightarrow d = 0.989\ cm$ & $l = 5.93\ cm$

shortest stub = 0.2λ = 3.956 cm

distance = 0.05λ = 0.989 cm

b) $Z_0' = \sqrt{50 \cdot 5} = 15.8113\ \Omega$ & $l = \frac{\lambda}{4}$

∵ $Z_0' = \sqrt{\frac{L}{C}} = \frac{\ln\frac{b}{a}}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}$

∵ $Z_0 = \frac{\ln\frac{b}{a}}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0 2.3}} \rightarrow \ln\frac{b}{a} = \frac{\sqrt{230}}{12}$

$\rightarrow \frac{15.8113 \cdot 2\pi \cdot 12}{\sqrt{230}} = \frac{120\pi}{\sqrt{\varepsilon_r'}} \rightarrow \varepsilon_r' = 23$

∴ $\lambda = \frac{c}{\sqrt{23} \cdot 1G} = 6.255\ cm \rightarrow \frac{\lambda}{4} = 1.5638\ cm$

HW$\boxed{5}$: 50Ω lossless , $Z_L = 10 - j50\ \Omega$, 1GHz

$z_L = 0.1 + 1j$

a : distance = $(0.214 - 0.125)\lambda = 0.089\lambda$

length = 0.285λ

b : distance = 0.16λ & length = 0.215λ

a) point a : distance : ~~$(0.2(-0.122)\lambda = 0.078\lambda$~~

length : ~~$0.298\,\lambda$~~

point b : distance : ~~$(0.299-0.122)\lambda = 0.199\lambda$~~

length : ~~0.2λ~~

$\therefore\ \lambda = \frac{u}{f}$, $u = \frac{c}{\sqrt{\epsilon_r}} \rightarrow \lambda = 0.1998\,m$

~~$\rightarrow l_a = 1.543$ cm , $d_a = 5.914$ cm~~

~~$\rightarrow l_b = 3.5$ cm , $d_b = 3.956$ cm~~

point a : distance = $0.078\,\lambda$

length = $0.307\,\lambda$

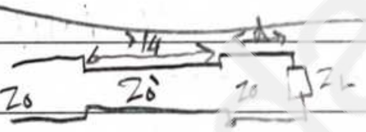
point b : distance = 0.197λ

length = $0.176\,\lambda$

$\rightarrow\ l_a = 1.543$ cm , $d_a = 6.013$ cm

$\rightarrow\ l_b = 3.5$ cm , $d_b = 3.897$ cm

b)



$d = 0.1233\,\lambda \rightarrow Z_{L1} = Z_{min} = 0.1$

$\rightarrow Z_{min} = 5\,\Omega$

$\therefore\ Z_0' = \sqrt{5\cdot 50} = 15.81138\,\Omega$

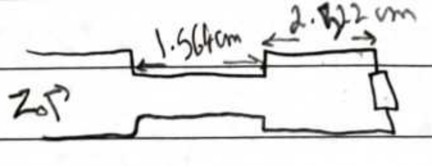
$\therefore\ Z_0 = \sqrt{\frac{L}{C}} = \frac{\ln\frac{b}{a}}{2\pi}\cdot\eta_0\cdot\frac{1}{\sqrt{\epsilon_r}} \rightarrow \ln\frac{b}{a} = 1.2638125$

$\rightarrow Z_0' = \frac{\ln\frac{b}{a}}{2\pi}\,\eta_0\cdot\frac{1}{\sqrt{\epsilon_r'}} \rightarrow \epsilon_r' = 23$

$\therefore\ \lambda' = \frac{c}{\sqrt{23}\cdot f} = 6.2554$ cm

$\therefore\ \frac{\lambda'}{4}$ = length of quarter wave transformer

$= 1.564$ cm



c) from (a) point a gives an inductive stub

~~position : 1.543 cm from load or 11.433 cm~~

~~∴ d = 6.013 cm → -2.88 → jX = 17.897j~~

~~∴ $jX = jLW \to 17.897/w = L = 2.842$ nH~~

series → use impedance

∴ point a: $z = 1 + j2.8$ X

point b: $z = 1 - j2.8 \to x_s = j2.8$

$\to jX = j140 = jWL \to L = 22.3$ nH

d) SWR min $= 5 = \frac{50}{10}$

inductor $\to X = j50 = jWL \to L = 7.959$ nH

Second exam 2021

$\boxed{1}$ ∴ $H_2 = \frac{5}{3\pi} \bar{a}_z e^{-j(16\pi)x}$

perpendicular polarization $\to \theta_t = 90^\circ$

∴ $\theta_i = \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$, $n = c\sqrt{\mu_1\epsilon_1}$

$\to \frac{n_2}{n_1} = \sqrt{\frac{\mu_{r2}\epsilon_{r2}}{\mu_{r1}\epsilon_{r1}}} = \frac{\epsilon_{r2}}{9}$

∴ $\beta = 16\pi = \omega \cdot \sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_{r2}} \to \boxed{\epsilon_{r2} = 4}$

∴ $\theta_c = \sin^{-1}\left(\frac{4}{9}\right) = 0.460554$ rad $= \theta_i$

$n_1 \sin\theta_c = n_2 \to \theta_c = \sin^{-1}\frac{n_2}{n_1}$

∴ $|H_2| = \frac{5}{3\pi} \to |E_2| = \frac{5}{3\pi} \cdot 120\pi = 200$ V/m

∴ $E_{io} = \frac{E_{to}}{\tau_\perp}$, $\tau_\perp = 2$ $\beta_1 = 36\pi$

$\to E_{io} = 100$ V/m

∴ $E_i = 100\, a_y\, e^{-j36\pi(\cos\theta_c \cdot a_z + \sin\theta_c\, a_x)}$

$= 100\, a_y\, e^{-j(50.264 a_x + 101.31 a_z)}$

$\rightarrow -z$ direction

$\boxed{2}$ a) $\because H = j\bar{a}_x e^{jB\textcircled{z}} - 2\bar{a}_y e^{jB\textcircled{z}}$

$\Rightarrow \Delta\phi = \frac{\pi}{2} \wedge H_x \neq H_y \rightarrow$ elliptical

$E = -\eta(\bar{a}_z \times \bar{H})$

$$\begin{array}{ccc} a_x & a_y & a_z \\ 0 & 0 & +\eta \\ j & -2 & 0 \end{array} \quad e^{jBz}$$

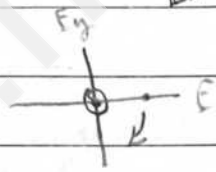
$\rightarrow E = -\left[-2\eta a_x - \eta j a_y\right] e^{jBz}$

$\rightarrow \Delta\phi = \frac{\pi}{2} \wedge E_x \neq E_y \rightarrow$ elliptical

$\therefore E_x = 2\eta \cos(wt + Bz)$

$\wedge E_y = -\eta \sin(wt + Bz)$

B.H EP



$\boxed{3}$ $\because \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \pm 0.2$

$\wedge \theta_{B\perp} = 48.9° \rightarrow \frac{1 - \frac{\mu_1 \varepsilon_2}{\mu_2 \varepsilon_1}}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2} = \sin(48.9)^2$

$\rightarrow 0.5698579 = \frac{1 - \frac{\varepsilon_r}{\mu_r}}{1 - \left(\frac{1}{\mu_r}\right)^2} = \frac{\mu_r^2 - \varepsilon_r \mu_r}{\mu_r^2 - 1}$

$\because \frac{\cancel{120\pi}\left(\sqrt{\frac{\mu_r}{\varepsilon_r}} - 1\right)}{\left(\sqrt{\frac{\mu_r}{\varepsilon_r}} + 1\right)\cancel{120\pi}} \rightarrow \frac{\sqrt{\mu_r} - \sqrt{\varepsilon_r}}{\sqrt{\mu_r} + \sqrt{\varepsilon_r}} = \pm 0.2$

$\therefore \sqrt{\mu_r}(1 \mp 0.2) = (1 \pm 0.2)\sqrt{\varepsilon_r}$

$\rightarrow \mu_r = \left(\frac{1 \pm 0.2}{1 \mp 0.2}\right)^2 \varepsilon_r \rightarrow \frac{\mu_r}{\varepsilon_r} = 2.25$ or $\frac{4}{9}$

$\wedge \varepsilon_r \mu_r - 0.5698577 = \mu_r^2 \left[1 - 0.5678871\right]$

if $\mu_r = \cancel{2.25}\,\frac{4}{9} \varepsilon_r \rightarrow \varepsilon_r^2 \cdot \frac{4}{9} - 0.5698577 = \frac{16}{81}\varepsilon_r^2 \left[1 - 0.5\right.$

$\rightarrow \varepsilon_r^2 \left[\frac{4}{9} - 0.08836\right] = 0.5698871$

$\rightarrow \varepsilon_r = 0.63235 \times \quad \varepsilon_r < 1$ ~~[illegible]~~

if $\mu_r = 2.25 \quad \varepsilon_r^2 \simeq 9.11989$

$\rightarrow \varepsilon_r = 3.02 \wedge \mu_r = 6.794$

$\theta_{B\parallel}$ does not exist

quiz 4 :

matched source , $V_{max} = 300V$, Resistive load

$\therefore |V_0^+| = \frac{1}{2}(V_{in} + I_{in} \cdot Z_0) = \frac{V_{in}}{2}\left(\frac{Z_{in}+Z_0}{Z_{in}}\right)$

& $V_{max} = |V_0^+| \cdot (1+|\Gamma_L|)$

$\rightarrow |V_0^+| = \frac{300}{1+|\Gamma_L|}$

$\therefore |V_0^+| = \frac{V_{in}}{2}\left[\frac{Z_{in}+Z_0}{Z_{in}}\right]$ & $V_{in} = V_g \cdot \frac{Z_{in}}{Z_g+Z_{in}}$

$Z_g = Z_0 \rightarrow |V_0^+| = \frac{V_g}{2}$

$\therefore V_{max} = \frac{V_g}{2}(1+|\Gamma_L|) \rightarrow \frac{300}{250} = 1+|\Gamma_L|$

$\therefore |\Gamma_L| < 0.2 = \left|\frac{R_L - R_0}{R_L + R_0}\right|$

$\rightarrow -0.2 < \Gamma_L < 0.2$

take $-0.2 \rightarrow \frac{R_L - 75}{R_L + 75} \geq -0.2$

$\therefore R_L \geq 50.$

take $\frac{R_L - 75}{R_L + 75} \leq 0.2 \rightarrow R_L \leq 112.5$

$\therefore 50 \leq R_L \leq 112.5$

midterm 1999

①a) lossless : $Z_0 = 60 = \sqrt{\frac{L}{C}} = \frac{d}{w}\sqrt{\frac{\mu}{\varepsilon}} = \frac{d}{w} \cdot \frac{120\pi}{2}$

$\rightarrow d = \frac{w}{\pi} \approx 1mm$

b) $\left.\begin{array}{l} Z_{in1} = \frac{Z_0^2}{Z_L} \\ Z_{in2} = Z_L \end{array}\right\} \overset{Z_{in1} // Z_{in2}}{\longrightarrow} \frac{Z_0^2}{\frac{Z_0^2}{Z_L} + Z_L} = 60\ \Omega$

$\therefore \boxed{SWR = 1}$

$\boxed{2}$ $u_p = 0.8c$, 50Ω lossless 4.4 m not 1.4

$l = 0.7\ m$

$$Z(0.7m) = Z_0 \cdot \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

$\because u_p = 0.8c \rightarrow \beta = \frac{600\pi M}{0.8c} = \frac{5}{2}\pi$

$\rightarrow \beta l = \frac{7}{4}\pi$, $\tan(\beta l) = -1$ ×

$\rightarrow 80 - j60 = 50 \cdot \frac{Z_L - jZ_0}{Z_0 - jZ_L}$ ×

or $\because$

$\rightarrow \beta l = 2.2 \cdot \frac{5}{2}\pi = \frac{11}{2}\pi \rightarrow \tan(\beta l) = \infty$

$\therefore 80 - j60 = \frac{Z_0^2}{Z_L} \rightarrow \boxed{Z_L = 20 + 15j}$

$$\Gamma(z = 2.2m) = \frac{Z(2.2) - Z_0}{Z(2.2) + Z_0} = 0.4685\angle{-0.6747}\ \text{rad}$$

a) $SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2.763$

b) $\Gamma_L = \frac{20 + 15j - 50}{70 + 15j} = 0.4684\angle 2.46692\ \text{rad}$

c) Z_{in} : $\beta l = 11\pi \rightarrow \tan(\beta l) = 0 \rightarrow Z_{in} = Z_L$

$\rightarrow Z_{in} = 20 + j15\ \Omega$

b) as ∵ $\Gamma(z) = \Gamma_L e^{-j2\beta z'}$ → $\Gamma_L = \Gamma(z=2.2) \cdot e^{j2\beta(2.2)}$

→ $\Gamma_L = 0.4685 \angle 2.4669$ rad

$\boxed{2}$ redone smith chart:

$z(2.2) = 1.6 - j1.2$, $\lambda = \frac{0.8c}{300M} = 0.8m$

$|\Gamma| = \frac{12}{25.5} \approx 0.47$, $SWR = 2.78$

a) $\theta_{\Gamma 2.2} = -39.5°$ → $\Gamma(z=2.2) = 0.47 \angle -0.69$ rad

b) take $\lambda = 2.2m$ → 2.75λ to load

→ 0.25λ → $\theta_{\Gamma_L} \approx 141°$ ✓

c) take 55λ towards generator → same position

∴ $z_{in} = z_L = 0.4 + 0.3j$ ⇒ $20 + 15j$

$\boxed{3}$ a) ∵ $\beta_x = |\beta_2| \cdot \sin\theta_t$ ∧ $|\beta_2| = \sqrt{1.05^2 + 2.96^2}$

→ $\theta_t = 0.34088$ rad

∵ $n_1 \sin\theta_i = n_2 \sin\theta_t$, $n_1 = 1$, $n_2 = \sqrt{\varepsilon_{r2}}$

∵ $|\beta_2| = 3.1407165 = \beta = \frac{\omega}{c} \cdot \sqrt{\varepsilon_r}$ → $n_2 = 1.49958$

→ $\boxed{\theta_i = 0.525141 \text{ rad}}$ ≈ 30°

b) ∵ parallel polarization

∧ $E_t = \eta_2 |H_t| \left[\cos\theta_t \bar{a}_x - \sin\theta_t \bar{a}_z\right] \cos$ - -

→ $E_{i0} = \frac{\eta_2 H_{t0}}{\tau_{\parallel}}$ ∧ $\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$

→ $\tau_{\parallel} = 0.572555$ → $E_{i0} = 20.1755$ V/m

∵ $H_{i0} = \frac{E_{i0}}{\eta_0}$

$\beta_1 = \frac{\beta_2}{\sqrt{\varepsilon_{r2}}} = 2.0944$

∴ $H_i = 0.05352 \bar{a}_y \cdot \cos(2\pi 10^8 t - 1.05x - 1.81223z)$ A/m fall 2000 mid :

$\boxed{1}$ a) + z - direction

$∵ E = -\eta(\bar{a}_z \times \bar{H}) = \begin{vmatrix} 0 & 0 & a_z \\ 0 & H & 0 \end{vmatrix}$

$\rightarrow E_{y0} = 0 \wedge E_{x0} = \eta H_0$

∴ linear polarization ∵ one component is zero

b) $∵ \beta = 0.1829 = \frac{2\pi}{\lambda} \rightarrow \lambda = 34.35312 \text{ m}$

$∵ u_p = \frac{\omega}{\beta} \rightarrow u_p = \frac{4\pi \times 10^6}{0.1829} = 68.71 \text{ Mm/s}$

c) $∵ E(z,t) = \eta H_0 \cdot \bar{a}_x$

$∵ \beta = 0.1829 = \cancel{\omega\sqrt{\mu\varepsilon}} \rightarrow \cancel{\varepsilon_r = 19.0656} \rightarrow \cancel{\eta = \frac{120}{\sqrt{4.3664}}}$

$\cancel{\rightarrow E(z,t) = 8.634\, a_x\, e^{-0.0432z} \cos z \; V/m}$

~~c)~~ $∵ \eta = \frac{j\omega\mu}{\gamma} = 84.02697 \angle 0.231944 \text{ rad}$

$\rightarrow E(z,t) \approx 8.403 \cdot e^{-0.0432z} \cdot \cos(\omega t - \beta z + 0.231944)$

d) $∵ \beta^2 - \alpha^2 = \omega^2\mu\varepsilon \rightarrow \varepsilon_r \approx 18$

$\wedge \; 2\alpha\beta = \omega\mu\sigma \rightarrow \sigma \approx 1 \times 10^{-3} \; ℧/m$

e) power density: $\frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta)\, \bar{a}_z$

$\rightarrow 2\alpha z = 1 \rightarrow z = 11.574 \text{ m}$

$\boxed{2}$ a) $∵ \tau = \frac{2\eta_2}{\eta_1 + \eta_2} \wedge \eta_2 = \frac{120\pi}{\sqrt{2}}$

$\rightarrow \tau = \frac{\frac{2}{\sqrt{2}}}{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}}} = 1.101021$

$∴ E_2 = \tau E_0 (1a_y + 2a_z)$

b) $\nabla \cdot \bar{B} = 0$

$\rightarrow \frac{2}{\rho^n} + \frac{-(n-1)}{\rho^n} \rightarrow n - 1 = 2 \rightarrow \boxed{n=3}$

$\boxed{3}$ a) $\beta = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\pi}{5}$

$0.05\pi\sqrt{a^2+3+b^2} = \frac{\pi}{5} \Rightarrow a^2+3+b^2 = 16$

$\boxed{a^2+b^2 = 13}$, $k = a\,\bar{a}_x - \sqrt{3}\,\bar{a}_y + b\,a_z$

$\because k\cdot E = 0 \rightarrow 5a - 5\cdot 3 = 0 \rightarrow a = 3$

$\therefore\ 9 + b^2 = 13 \rightarrow \boxed{b = \pm 2}$

b) $\frac{\gamma^2}{j\omega\mu} \rightarrow \frac{\alpha^2 - \beta^2 + 2j\alpha\beta}{j\omega\mu} = 0.00251 + j0.00145$

$\rightarrow \frac{\alpha^2-\beta^2}{j\omega\mu} = j0.00145 \rightarrow \frac{-\omega^2\mu\varepsilon}{j\omega\mu} = j0.00145$

λ $\frac{2\alpha\beta}{\omega\mu} = 0.00251 \rightarrow \frac{\omega\mu\sigma}{\omega\mu} = 0.00251$

$\therefore\ \underbrace{\omega\varepsilon = 0.00145}_{\varepsilon_r = 261}$ λ $\sigma = 0.00251\ \mho/m$

Or $\because \gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

λ $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

$\rightarrow \frac{\gamma}{\eta} = \sigma + j\omega\varepsilon = 0.00251 + j0.00145$

$\rightarrow \sigma = 0.00251$ λ $\omega\varepsilon = 0.00145$

mid 1999

$\boxed{1}$ a: $\beta = 40\pi = \omega\sqrt{\mu_0\varepsilon_0} \rightarrow \omega = 37.699\ \text{Grad/s}$

$\rightarrow \boxed{f = 6\,\text{GHz}}$

b) $H_i = \frac{1}{\eta_0}(\bar{a}_x \times \bar{E})$

$\begin{matrix} a_x & 0 & 0 \\ 0 & -60 & -j60 \end{matrix}\ e^{-j40\pi x}$

$\vec{H}_i = \frac{1}{\eta_0}(j60\,a_y - 60\,a_z)\,e^{-j40\pi x}$

$\rightarrow H_i = \frac{e^{-j40\pi x}}{2\pi}\left[j\,\bar{a}_y - a_z\right]$ A/m

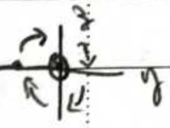
c) $E_{ro} = E_{io}$ $\rightarrow$ $\overline{E}_r = 60\, e^{j40\pi x} (\overline{a}_y + j\overline{a}_z)$

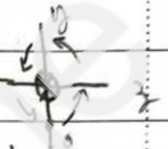
d) $\because$ $E_{iy} = -60 \cos(\omega t - 40\pi x)$

$\wedge$ $E_{iz} = 60 \sin(\omega t - 40\pi x)$

$\therefore$ $|E_{iy}| = |E_{iz}|$ $\wedge$ $\Delta\phi = \frac{3}{2}\pi$

$\rightarrow$ circular polarization, L.H.C.P





$\boxed{2}$ a) $\frac{\sigma}{\omega\epsilon} = 0.1$, $\epsilon_r = 4$ $\because$ loss tangent $<< 1$ $\rightarrow$ good dielectric

$\therefore$ $\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$ $\wedge$ $\beta_r = \omega\sqrt{\mu\epsilon}$

$\therefore$ $\beta = \frac{\omega}{c} \cdot 2 = 40\pi$ rad/m

$\because$ $\sigma = 0.1 \cdot \omega \cdot \epsilon \rightarrow \sigma = \frac{1}{15}$ ℧/m

$\therefore$ $\alpha = \frac{120\pi}{30} \cdot \frac{1}{2} = 2\pi$

$\therefore$ $\overline{E} = E_0 e^{-2\pi x} \overline{a}_z \cos(6\pi \times 10^9 t - 40\pi x)$ V/m

b) $E_0 e^{-2\pi 0} = 2E_0 e^{-2\pi x}$ $\rightarrow$ $-2\pi x = \ln(0.5)$

$\rightarrow x = 0.1032$ m

c) $\eta = \frac{120\pi}{2} = 60\pi$ $\wedge$ $\lambda = \frac{2\pi}{\beta} = \frac{1}{20} = 0.05$ m

$\boxed{3}$ a) $\beta = \omega\sqrt{\mu_0\epsilon_0} = \frac{4}{3}\pi = |\beta| = a\sqrt{2}$ $\rightarrow$ $a = 2.962$

b) $E = -\eta_0 (\overline{a}_k \times \overline{H})$ $\wedge$ $\overline{a}_k = \frac{8.886\overline{a}_x + 8.886\overline{a}_y}{4\pi}$

$$\therefore \overline{a}_k \times \overline{H} = \begin{vmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & \sqrt{2} \end{vmatrix}$$

$\rightarrow 1\overline{a}_x - 1\overline{a}_y + 0\overline{a}_z$

$\rightarrow \overline{E} = [-120\pi\overline{a}_x + 120\pi\overline{a}_y] \sin(\text{——})$

c) $P_{avg} = \frac{E_0^2}{2\eta_0} \xcancel{e^{-2\alpha}} 1 \cdot \overline{a}_k$ $E_0 = 120\pi\sqrt{2}$ mV/m

$\rightarrow P_{avg} = 120\pi\left[\frac{\sqrt{2}}{2} a_x + \frac{\sqrt{2}}{2} a_y\right]$

$= \frac{\sqrt{2}}{2} \cdot \eta_0 [\overline{a}_x + \overline{a}_y]$ μW/m²

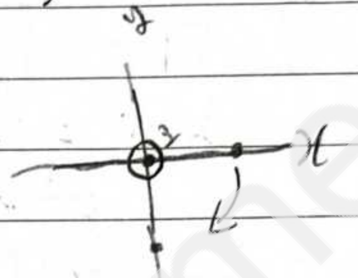
Summer 2000 mid:

$\boxed{1}$ $E(z) = E_0 [\bar{a}_x + \sqrt{2} j \bar{a}_y] e^{j \frac{1}{\sqrt{3}} z}$ $-z$-direction

a,b) $\rightarrow \beta = \frac{1}{\sqrt{3}} = \frac{w}{c} \cdot \sqrt{\varepsilon_r} \rightarrow \varepsilon_r = 3$

$\because \lambda = \frac{2\pi}{\beta} \rightarrow \lambda = 2\sqrt{3}\pi$

c) $E_x = E_0 \cos(wt + \frac{z}{\sqrt{3}})$



$E_y = -E_0 \sin(wt + \frac{z}{\sqrt{3}}) \cdot \sqrt{2}$

$\therefore$ R.H.E.P $\because |E_x| \neq |E_y|$

d) $H(z,t) = \frac{\sqrt{3}}{\eta_0} (\bar{a}_k \times \bar{E})$ $\wedge \bar{a}_k = -\bar{a}_z$

$0 \quad 0 \quad -a_z$ $e^{j \frac{z}{\sqrt{3}}}$

$E_0 \quad \sqrt{2} j E_0 \quad 0$

0

$\rightarrow H(z,t) = \frac{\sqrt{3}}{\eta_0} (j E_0 \bar{a}_x - E_0 \bar{a}_y)$

$\therefore H(z,t) = \frac{\sqrt{3} E_0}{\eta_0} [\sqrt{2} j \bar{a}_x - \bar{a}_y] \cos(10^8 t + \frac{z}{\sqrt{3}})$

$\boxed{2}$ $\because \frac{\sigma}{w\varepsilon} = \frac{4}{200\pi \cdot 81 \frac{10^{-9}}{36\pi}} \gg 1 \rightarrow$ conductor

$\rightarrow \alpha = \sqrt{\pi f \mu \sigma} = 0.03974$ Np/m

$E_0 = 1 V/m \rightarrow E(z=100) = 1 \cdot e^{-0.03974 \cdot 100}$

a) $\rightarrow E(z=100) = 0.0188$ V/m $= 18.8$ mV/m

b) $\frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta) \bar{a}_z$, $z = 0$

$\rightarrow \frac{1}{2|\eta|} \cdot \cos(\theta_\eta)$, $\eta = (1+j)\frac{\alpha}{\sigma}$

$\rightarrow \frac{1}{2 \cdot 0.01403} \cdot \cos(\frac{\pi}{4}) = 25.164$ W/m²

$\boxed{3}$ a) $\beta = 1$ in medium 2, ∵ $\sigma_2 = 0 \rightarrow$ lossless

$\rightarrow 1 = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c}\cdot\sqrt{3\cdot 12} \Rightarrow \omega = 50$ Mrad/s

∴ $f = 7.95775$ MHz

b) ∵ $|\Gamma| = \left|\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right|$ & $\eta_2 = \eta_0 \cdot \frac{1}{2} = 60\pi$ & $\eta_1 = \eta_0$

$\rightarrow \Gamma = \frac{-1}{3}$

$\rightarrow SWR = \frac{4/3}{2/3} = 2$

c) ∵ $E_t = -\eta_2(\bar{a}_n \times \bar{H}_t)$ $\begin{matrix} 0 & 0 & a_z \\ 0 & \frac{1}{2\pi} & 0 \end{matrix}$

$\rightarrow E_t = -60\pi \cdot \frac{-1}{2\pi} = 30\bar{a}_x \cos(\omega t - z)$

& $T = \frac{120\pi}{180\pi} = \frac{2}{3} = 1 + \Gamma$

∴ $E_i = 45\bar{a}_x \cos(\omega t - \frac{1}{6}z)$

$\rightarrow E_r = -15\bar{a}_x \cos(\omega t + \frac{1}{6}z)$

$\bar{H}_r = \frac{1}{\eta}(\bar{a}_n \times \bar{E}_r)$ $\begin{matrix} 0 & 0 & -a_z \\ -15 & 0 & 0 \end{matrix}$

$\rightarrow H_r = \frac{1}{\eta}[15 a_y]$

∴ $\bar{H}_r = 0.03979\bar{a}_y \cos(\omega t + \frac{1}{6}z)$ A/m

second 2000 mid:

$\boxed{1}$ parallel polarization: $E_0(\cos\theta_i \bar{a}_x - \sin\theta_i \bar{a}_z) \rightarrow \frac{-\sqrt{3}}{2} = \sin\theta_i$

a) ∵ $E_x = \frac{-E_0}{2} = -E_0 \cdot \cos(\theta_i) \rightarrow -\cos\theta_i = \frac{-1}{2}$

∴ $\theta_i = \cancel{\frac{2\pi}{3}} = \theta_r = \frac{\pi}{3}$

∵ $n_1 \sin\theta_i = n_2 \sin\theta_t$

$\rightarrow \theta_t = \sin^{-1}\left(\frac{-\sqrt{3}/2}{\sqrt{3}}\right) = \boxed{\frac{\pi}{6}}$

b) ∵ $\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$

$\rightarrow \Gamma_{\parallel} = \frac{60\pi - 60\pi}{60\pi + 60\pi} = 0 \rightarrow \bar{E}_r = 0$

c) $\because \Gamma_{11} = 0 \rightarrow \tau_{11} = \frac{\cos\theta_i}{\cos\theta_t} = \frac{\sqrt{3}}{3}$

$\rightarrow E_t = -E_0 \cdot \tau_{11} = \frac{-E_0\sqrt{3}}{3}$, $\beta_2 = \frac{w}{c}\cdot\sqrt{3} = 108.83$ rad/m

$\therefore \bar{E}_t = \frac{-E_0\sqrt{3}}{3}\cdot\left[\frac{\sqrt{3}}{2}\bar{a}_x - \frac{1}{2}\bar{a}_z\right]\cos\left(wt - 20\pi\sqrt{3}(0.5x + \frac{\sqrt{3}}{2}z)\right)$ V/m

d) $|\Gamma_{11}| = 1$, if $\eta_1\cos\theta_i = 0$ or $\eta_2\cos\theta_t = 0$

$\rightarrow \theta_i = 90$

or $\theta_t = 90 \rightarrow \theta_i = \theta_c \because \varepsilon_2 > \varepsilon_1 \rightarrow \theta_c$ does not exist

$\boxed{2}$ $Z_0 = 50\,\Omega$, distortionless, $\gamma = 0.0666 + j1.885$

a) $\because \beta = w\sqrt{LC} = w\sqrt{\mu\varepsilon} \rightarrow \varepsilon_r \approx 4$

b) $\because 50 = \sqrt{\frac{L}{C}} = \frac{d}{W}\sqrt{\frac{\mu}{\varepsilon}} \rightarrow \frac{d}{W} = 0.26426$

& $\alpha = \sqrt{RG} = \frac{R}{R_0} \rightarrow R = 3.33 = \frac{2\sqrt{\pi f\mu\sigma_c}}{W\sigma_c}$

$\rightarrow \frac{2\sqrt{\pi f\mu}}{3.33\cdot W} = \sqrt{\sigma_c} \rightarrow \sigma_c \approx 640\text{k}\,\mho/\text{m}$

c) $\because \frac{d}{W} = 0.26426 \rightarrow d = 2.6426$ mm

d) $\because \sqrt{RG} = \alpha \rightarrow \frac{\sigma W}{d} = \frac{\alpha^2}{R}$

$\rightarrow \sigma_d = 0.26426 \cdot \frac{\alpha^2}{3.33} = 3.533\times10^{-4}\ \mho/\text{m}$

$\boxed{3}$ $Z_L = 30 + j40\,\Omega$, $Z_0 = 50\,\Omega$ lossless, $l = 0.75\text{ m} = \frac{3}{4}\lambda$

$f = 3\times10^8$ Hz

a) $Z_{in} = Z_0 \cdot \frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$, $\beta l = \frac{2\pi}{\lambda}\cdot\cancel{\lambda} = 2\pi$

$\therefore \tan(\beta l) = 0 \rightarrow Z_{in} = \boxed{Z_L = 30 + j40\,\Omega}$

b) I_{min} at V_{max} at Z_{max}

$z'_{max} = \frac{\theta_{\Gamma_L} + 2n\pi}{2\beta}$, $\Gamma_L = 0.5\angle\frac{\pi}{2}$ ✓

$\rightarrow z'_{max} = \frac{\pi/2 + 0}{2\beta} = \frac{\pi/2}{2\pi} = \underline{0.25\text{ m}}$ ✓

c) $Z_{min} = \frac{Z_0}{SWR}$ & $SWR = \frac{1.5}{0.5} = 3 \rightarrow Z_{min} = 16.667\,\Omega$

$z'_{min} = \frac{\pi/2 + \pi}{4\pi} = 0.375$ m ✓

Subject Date No.

d) $P_{avg} = \frac{|V_0^+|^2}{2Z_0}(1-|\Gamma|^2) = \frac{|V_0^+|^2}{100} \cdot \frac{3}{4}$

$\because V_0^+ = \frac{V_{in}}{2}\left[1+\frac{Z_0}{Z_{in}}\right]$, $V_{in} = V_g \cdot \frac{Z_{in}}{Z_{in}+Z_g}$

$\rightarrow V_0^+ = \frac{V_g}{2}\left[\frac{\cancel{Z_{in}}}{Z_{in}+Z_g}\right] \cdot \left[\frac{Z_{in}+Z_0}{\cancel{Z_{in}}}\right] = \frac{V_g}{2} \cdot [1.3\cancel{99} \angle -0\cancel{.0342}]$

~~$\therefore |V_0^+| = 69.843\ V \rightarrow P_{avg} = 36.585\ W$~~

a) $Z_{in} = Z_0 \cdot \frac{Z_L + j\tan(\beta l) \cdot Z_0}{Z_0 + jZ_L\tan(\beta l)}$, $\beta l = \frac{2\pi}{\lambda} \cdot \frac{3}{4}\lambda = \frac{3}{2}\pi$

$\therefore \tan(\beta l) = \infty \rightarrow Z_{in} = \frac{Z_0^2}{Z_L} = \frac{2500}{Z_L} = \boxed{30-40j\ \Omega}$

d) $V_0^+ = \frac{V_g}{2}\left[\frac{Z_{in}+Z_0}{Z_{in}+Z_g}\right] = 40\sqrt{5}\angle 0.19984\ \text{rad}\ V$

$\rightarrow |V_0^+| = 40\sqrt{5}$

$\therefore P_{avg} = \frac{1600 \cdot 5}{100} \cdot \frac{3}{4} = \boxed{60\ W}$

3) redo with smith chart

$z_L = 0.6 + 0.8j$ | $z_{in} = 0.6 - 0.8j \Rightarrow Z_L = 30-40j$

$\rightarrow |\Gamma_L| = 0.51$ | $z_{max} = 0.124\lambda = 0.124\ m$

c) $z_{min} = (0.124 + 0.25)\lambda = 0.374\lambda = 0.374\ m$

$z_{min} \approx 0.335 \rightarrow Z_{min} \approx 16.74\ \Omega$

Summer 2000, mid:

1) 50 Ω distortionless , $R = 0.5\ \Omega/m$, matched load

a) $\because \frac{\sigma_d}{\omega\epsilon_d} = 0.0018 = \frac{G}{\omega C} = \frac{R}{L\omega}$

~~$\rightarrow L = 11.04\ mH/m$~~

~~$\because \frac{R}{L} = \frac{G}{C} \rightarrow G = \frac{72}{5}\pi \cdot C \Rightarrow G = 0.0911\ \mho/m$~~

~~$Z_0 = \sqrt{\frac{L}{C}} \rightarrow C = 2.1024\ nF/m$~~

~~b) $\because \gamma = \sqrt{RG} + j\omega\sqrt{LC}$~~

~~$\rightarrow \gamma = 0.21807 + j\,128.14$~~

a) $\because \frac{\sigma_d}{\omega\epsilon_d} = 0.0018 = \frac{G}{\omega C} = \frac{R}{\omega L}$

$\rightarrow L = 0.01104\ H/m$

NOTEBOOK

$\because \sqrt{\frac{L}{C}} = Z_0$ & $\sqrt{C} = \frac{\sqrt{2}}{50} \rightarrow C = 4.42\ uF/m$

$\because G = \omega C \cdot 0.0018 \rightarrow G \approx 2 \times 10^{-4}\ \mho/m$

b) $\because \alpha = \sqrt{RG} \wedge \beta_z = \omega\sqrt{LC}$

$\rightarrow \gamma = 0.01 + j5.554$

c) $V(z) = V_0^+ e^{-\gamma z} + \cancel{V_0^- e^{\gamma z}} \rightarrow 0$ $\because$ matched load

$\because V_0^+ = \frac{V_{in}}{2}\left(\frac{Z_{in}+Z_0}{Z_{in}}\right) \rightarrow 2 \wedge V_{in} = \frac{V_g \cdot Z_{in}}{Z_{in}+Z_g}$

$\therefore V_0^+ = \frac{10\sqrt{10}}{6} \cos(8000\pi t - 0.3217)$

~~Redo~~ $\therefore V(z) = \frac{10\sqrt{10}}{6} e^{-0.01z} \cdot \cos(\omega t - 0.3217 - 5.554z)\ \overline{a_z}$

$\boxed{2}$ lossless, air-filled, $Z_0 = 100\ \Omega$, short-circuited

$\because Z(z') = jZ_0 \tan(\beta z') \rightarrow$ at input: $Z_{in} = jZ_0 \tan(\beta l)$

at 137.5 MHz, $Z_{in} = Z_{max} \rightarrow V_{max}$

$Z_{max} = Z_0 \cdot S = \frac{V_{max}}{I_{min}} \wedge S = \frac{V_{max}}{V_{min}} = \infty$

a) $\because |\Gamma| = 1 \wedge V_{max} = |V_0^+| \cdot (1 + |\Gamma|)$

$\rightarrow V_{max} = 2|V_0^+| \rightarrow |V_0^+| = 5V$

$\wedge I_{max} = \frac{V_{max}}{Z_0} = \frac{10}{100} = 0.1A$

b) $\because V_0^+ = \frac{V_{in}}{2}\left(\frac{Z_{in}+Z_0}{Z_{in}}\right) \wedge V_{in} = \frac{Z_{in}}{Z_g + Z_{in}} \cdot V_g$

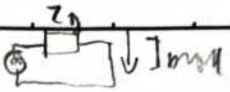
$\because Z_{in} = \infty$ at max $\rightarrow 5 = \frac{V_g}{2} \rightarrow V_g = 10V = V_i$

c) at $I_{max} = 0.2A \rightarrow |V_0^+| = 20V$

~~$20 = \frac{1}{2}(V_{in} + 0.2 \cdot Z_0) \rightarrow 40 - 20 = V_{in}$~~

$\therefore 20 = V_g \cdot \frac{Z_{in}}{Z_g + Z_{in}}$

(1) $\because I_{max} \rightarrow Z_{in} = 0 \rightarrow$



$\rightarrow \frac{V_g}{Z_g} = I_{max} \rightarrow Z_g = \frac{10}{0.2} = 50\,\Omega$

$\boxed{4}$ $\because Z(z') = j Z_0 \tan(\beta z') \rightarrow Z_{in} = j Z_0 \tan(\beta l)$

~~at $f = 165\,MHz \rightarrow Z_{in} = 0 \rightarrow \tan(\beta l) = 0$~~

~~$\therefore \beta l = +n\pi \rightarrow l = \frac{n}{2}\lambda_1$~~

~~at $f = 157.5$, $\tan(\beta l) = \infty \rightarrow \beta l = (2n+1)\frac{\pi}{2} = (2n+1)\frac{\lambda_2}{4}$~~

~~$\because$ air-filled $\rightarrow \lambda = \frac{c}{f}$~~

~~$\therefore l = \frac{n}{2}\frac{c}{165M} = -\frac{(2n+1)}{4}\cdot\frac{c}{157.5M}$~~

~~$\rightarrow 2n = -(2n+1)\cdot\frac{165}{157.5}$~~

~~$\rightarrow 2n\left(1 - \frac{165}{157.5}\right) = \frac{165}{157.5} \rightarrow n = 11$~~

at $f = 157.5\,MHz \rightarrow Z_{in} = \infty$

$\therefore \beta l = (2n+1)\cdot\frac{\pi}{2} \rightarrow l = (2n+1)\frac{\lambda}{4}$

$\wedge \; \lambda = \frac{c}{f} \rightarrow l = (2n+1)\frac{c}{157.5M}$

at $f = 165M \rightarrow Z_{in} = 0 \rightarrow \tan \beta l = 0$

$\rightarrow \beta l = \pm n\pi \rightarrow l = \pm\frac{n}{2}\lambda = \pm n\frac{c}{165\cdot 2}$

$\therefore \frac{\pm n}{165\cdot 2} = \frac{2n+1}{157.5} \rightarrow \frac{157.5}{165\cdot 2} = \frac{2n+1}{\pm n}$

(8) $\beta_2 l - \beta_1 l = \frac{\pi}{2} \quad \wedge \; \beta = \frac{\omega}{u_p} = \frac{2\pi f}{c}$

$\therefore l\left(\frac{2\pi f_2}{c} - \frac{2\pi f_1}{c}\right) = \frac{\pi}{2}$

$\rightarrow \frac{2\pi l}{c}(f_2 - f_1) = \frac{\pi}{2}$

$\rightarrow \frac{4l}{c}(165 - 157.5)M = 1$

$\rightarrow l = \frac{40}{4} = 10m$

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$\boxed{2}$ a) ∵ $V_{i\,max} \rightarrow Z_{in\,max} = \infty$

$\wedge \quad V_{max} = |V_0^+|\cdot(1+|\Gamma|) \quad \wedge |\Gamma| = 1$

$\rightarrow |V_0^+| = \frac{10}{2} = 5V$

∵ $I_{max} = \frac{2|V_0^+|}{Z_0} \rightarrow I_{max} = \frac{10}{100} = 0.1A$

b) ∵ $(\cancel{|V_0^+|} = \frac{1}{2}(V_{in} + \cancel{I_{in}\cdot Z_0})$ $\wedge$ $V_{in} = V_g\cdot\frac{Z_{in}}{Z_g+Z_{in}}$

$V_{in\,max} = V_g\cdot\frac{Z_{in}}{Z_{in}+Z_g} = V_g\cdot\frac{\infty}{\infty}$

$\boxed{\rightarrow V_g = 10V}$

c) at I_{max}, $Z_{in} = 0 \rightarrow I = \frac{V_g}{Z_g} \Rightarrow \frac{10}{0.2} = Z_g = 50\Omega$

$\boxed{4}$ ∵ $Z_{in} = jZ_0\tan(\beta l)$ $\quad \beta = \frac{2\pi}{\lambda}$

at $f = 157.5$ MHz, $Z_{in} = \infty \rightarrow \beta l = n_{odd}\frac{\pi}{2}$

∴ $l = n_{odd}\cdot\frac{\lambda}{4} = \boxed{n_{odd}\cdot\frac{c}{4\cdot157.5}}$ $\quad n_2$

at $f = 165$ MHz, $Z_{in} = 0 \rightarrow \beta l = n_{integer}\pi$

$\rightarrow l = n_1\frac{\lambda}{2} = n\frac{c}{165\cdot2}$

$l \Rightarrow \frac{n_1}{165\cdot2} = \frac{n_2}{4\cdot157.5} \rightarrow \frac{n_2}{n_1} = \frac{21}{11}$

∴ $n_1 = 11 \wedge n_2 = 21$

$\rightarrow l = 11\cdot\frac{c}{165\cdot2M} = 10\,m$

d) ∵ at 1, $Z_{in} = 0$ $\wedge$ at 2, $Z_{in} = \infty$

$\rightarrow \Delta\phi = \frac{\pi}{2}$

∴ $\beta_2 l - \beta_1 l = \frac{\pi}{2} \rightarrow \frac{l\cdot2\pi}{c}(f_2-f_1) = \frac{\pi}{2}$

$\rightarrow l = 10\,m$

first 2003 :

1) ∵ $\beta = 1$, non-magnetic ∧ $\eta = 40\pi\ \Omega$

$\rightarrow \eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} \rightarrow \varepsilon_r = 9$

∴ $\beta = \omega\sqrt{\mu\varepsilon_0\varepsilon_r} = \frac{\omega}{c} \cdot 3$

$\rightarrow \frac{c}{\omega} = 3 \rightarrow \omega = 0.1\ G\ Rad/s$

3) $\eta = \frac{30\pi}{0.25} \rightarrow \eta = 120\pi$ ∴ $\mu_r = \varepsilon_r$

∵ $\alpha = 0 \rightarrow \beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} \cdot \sqrt{\mu_r\varepsilon_r}$

$\rightarrow 2 = 1 \cdot \varepsilon_r \rightarrow \varepsilon_r = \mu_r = 2$

2) $\eta = \cancel{\frac{\sqrt{2}}{50}\angle\frac{\pi}{4}} \quad \frac{50}{\sqrt{2}}\angle\frac{\pi}{4} = \frac{j\omega\mu}{\gamma}$

$\rightarrow \gamma = \frac{j\omega\mu}{\eta}$

∵ $\tan(\theta_\eta) = \frac{\alpha}{\beta} \rightarrow \alpha = \beta$

$\rightarrow \tan\theta$ = loss tangent $= \tan(\frac{\pi}{2}) = \infty$

→ good conductor

∴ $\eta = (1+j)\frac{\alpha}{\sigma} \rightarrow \alpha = 50\ Np/m$

$\rightarrow \beta = 50\ Rad/m$

4) $n = 1.6$

Quarter wave transformer : $\eta_2 = \sqrt{\eta_1\eta_3}$

$n = 1.6 \rightarrow \varepsilon_{r3} = 1.6^2$ ∴ $\eta_3 = \frac{\eta_0}{1.6}$

$\rightarrow \eta_2 = \frac{\eta_0}{\sqrt{1.6}} \approx 298.03965 \rightarrow n_{film} = \sqrt{1.6}$

∵ $\lambda = \frac{u_2}{f}$ ∧ $\cancel{t} = (2n+1)\frac{\lambda_2}{4}$

$u_2 = \frac{c}{\sqrt{1.6}} \rightarrow t = 1.09 \times 10^{-7}\ m \approx 108.9\ nm$

∵ $\lambda_1 = 550n = \frac{c}{f} \rightarrow f = 5.4545 \times 10^{14}$

5) lossless non-magnetic, $SWR = 2 = \frac{\eta_1}{\eta_2}$ or $\frac{\eta_2}{\eta_1}$

$SWR = 2 = \frac{\eta_2}{\eta_1} \rightarrow \epsilon_{r2} > \epsilon_{r1}$ starts at min

$\therefore z_{max} = \frac{(2n+1)}{4} \lambda_1$ & $\frac{\lambda_1}{4} = 0.25m$

$\rightarrow \lambda_1 = 1m = \frac{v}{f}$ & $f = 10^8$ Hz

$\because v = \frac{c}{\sqrt{\epsilon_r}}$ & $\epsilon_{r1} = 9$ $\therefore \boxed{\epsilon_{r2} = 36}$

$(2\sqrt{\epsilon_{r1}})^2$

6) $\tau = \frac{2\eta_2}{\eta_2 + 120\pi}$ & $\eta_2 = 120\pi \cdot \frac{1}{2}$

$\rightarrow \tau = \frac{2}{3}$

$\therefore E_{io} = 45$ V/m

$\because \beta_2 = 1 = \frac{\omega}{c} \cdot \sqrt{36} \rightarrow \omega = 5 \times 10^7$ rad/s

$\rightarrow \beta_1 = \frac{\omega}{c} = \frac{1}{6}$

$\therefore E_r = -15 \bar{a}_y \cos(5 \times 10^7 t + \frac{1}{6} z)$

Part third:

1) matched source & matched load, $\frac{b}{a} = 6$, $b = 1.5mm$

a) $\because \lambda = \frac{c}{f \cdot \sqrt{\epsilon_r}} \rightarrow \lambda = 40000$ m

exact: $C = \frac{2\pi\epsilon}{\ln\frac{b}{a}} = 69.764$ pF/m

$G = \cancel{\frac{2\pi\sigma}{\ln\frac{b}{a}}} = 0$ $R = \frac{\sqrt{\pi f \mu_c \sigma_c}}{2\pi\sigma_c}\left[\frac{1}{a} + \frac{1}{b}\right]$

$Z_0 = \frac{R + j\omega L}{\gamma}$ & $\gamma = \alpha + j\beta$

$\rightarrow R = 0.0137\ \Omega/m$

& $L = \frac{\mu}{2\pi} \ln(6) = 3.58352 \times 10^{-7}$ H/m

$\therefore Z_0 = \sqrt{\frac{R + j\omega L}{j\omega C}} = \sqrt{8090.622} \angle \frac{-0.882934}{2}$

$\rightarrow Z_0 = 89.95 \angle -0.4415$ rad

∵ matched source & load → $V_0^- = 0$

$\therefore V(z) = V_0^+ e^{-\gamma z}$

$V_0^+ = \frac{V_g}{2} = 50$ & $\gamma = Z_0 \cdot j\omega C = 8.4234\times10^{-5} + 1.7824\times10^{-4} j$

$\rightarrow V(z) = 50\, e^{-8.4239\times10^{-5} z} \cdot \cos(\omega t - 1.7824\times10^{-4} z)$

& $I(z) = 0.55586\, e^{-\alpha z} \cdot \cos(\omega t - \beta z + 0.6415)$

$\boxed{2}$ analytically:

a) $\Gamma_L = 0.41523 \angle 1.654$ rad

$\beta = \frac{\omega}{\cancel{\sqrt{LC}}} \cdot 2 = 4\pi \rightarrow z'_{max} = \frac{1.654 + 0\pi}{8\pi} = \boxed{\cancel{0.2634m}}$ 0.06581 m

& $z'_{min} = 0.76324$ m

$Z_{max} = SWR \cdot Z_0$ & $SWR = 2.42015$

$\rightarrow Z_{max} = 121.511$

Smith chart: $z_L = \frac{2}{3} + \frac{2}{3} j$

$(0.25 - 0.1195)\lambda$ & $\lambda = 0.5$ m

$\rightarrow z'_{max} = 0.06525$ m

$Z_{max} \approx 2.4 \approx 120\ \Omega$

b) $+0.75j = 56.25j$ capacitor

$\rightarrow \frac{1}{j\omega C} = \frac{1}{56.25j} \rightarrow \omega C = 56.25$

$\rightarrow C = 29.8$ nF

$-j50 = \frac{1}{j\omega C} \rightarrow C =$

second 2003

[1] $\beta l = 180^\circ = \pi$ rad → $\beta = \frac{\pi}{50mm} = 20\pi = \frac{\omega}{c}\cdot\sqrt{\epsilon_r}$

→ $\sqrt{\epsilon_r} = 1.5$ → $\epsilon_r = 2.25$

∵ lossless → $80 = \sqrt{\frac{L}{C}} = \ln(\frac{b}{a}) \cdot \frac{1}{2\pi} \cdot \sqrt{\frac{\mu}{\epsilon}}$

→ $80 = \frac{\ln b/a}{2\pi} \cdot 120\pi \cdot \frac{2}{3}$ → $\ln\frac{b}{a} = 2$

→ $\frac{b}{a} = 7.38906$ → $b = 3.6945$ mm

[2] analytically: $Z_{in} = Z_0 \frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$

→ $R + 42j = 90$

solve for βl

or smith chart: $z_L = 2$

$\frac{42}{90} = 0.6$, $\lambda = \frac{c}{f} = 6$ m

$l = (0.3744 - 0.25)\lambda = 0.7464$ m

or if $+42j$ → $(0.25 + 0.125)\lambda = 2.25$ m

[3] $V_0^+ = 100$, $V_0^- = 60$, ~~$-z$-direction~~

distortionless → $\frac{R}{L} = \frac{G}{C}$

∵ $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$, $\Gamma(z') = \frac{V_0^-}{V_0^+} e^{2\gamma z}$

→ $\Gamma(z') = 0.6\, e^{-2\gamma z}$

at load, $z = l$ → ~~$\Gamma(z') =$~~ → $\Gamma_L = 0.6$

∴ $Z_0 = 75\,\Omega = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$

∵ $\alpha = 0.0003 = \sqrt{RG}$

→ $G = \frac{R}{75^2}$ → $\frac{R}{75} = 0.0003$

→ $R = 0.0225\ \Omega/m$

, $G = 4\ \mu\mho/m$

→ $-z$-direction

$V = 100 e^{0.0003l} \cdot e^{-0.0003z} e^{j10^{-3}l} \cdot e^{-j10^{-3}z} + 60 e^{0.0003z} \cdot e^{-0.0003l}$

→ $V_0^+ = 100 e^{0.0003l}$, $V_0^- = 60 e^{-0.0003l}$

∵ $\Gamma(l) = \frac{V_0^-}{V_0^+} e^{2\gamma l}$ → $\frac{60}{100} \cdot \cancel{e^{-2\alpha l}} \cdot \cancel{e^{2\alpha l}}$

suppress $e^{j\beta_z z}$

→ $\Gamma_L = \frac{6}{10} = 0.6$

$\boxed{4}$ $14m$, $\beta = \frac{\pi/2}{0.95} \rightarrow \lambda = \frac{2\pi}{\beta} = \frac{4\pi \cdot 0.95}{\pi} = 3.8m$

$\rightarrow l = 5\lambda \rightarrow Z_{in} = Z_L \rightarrow z_{in} = 3 + j4 \ \Omega$

$\boxed{5}$ $\Gamma_L = \frac{50}{150} = \frac{1}{3}$

$\because u_p = 2\times10^8 \rightarrow \lambda = \frac{u_p}{f} = 2m \rightarrow l = 4.95\lambda$

$\rightarrow Z_{in} = \frac{Z_0^2}{Z_L} = \frac{2500}{100} = 25 \ \Omega$

$\therefore V_{in} = 2V$

$\because V_0^+ = \frac{V_{in}}{2}\left(\frac{Z_{in}+Z_0}{Z_{in}}\right) \rightarrow V_0^+ = 3V$

$\wedge$ ~~$V_0^- = 1V$~~

$\because$ Resistive $\wedge$ $R_L > R_0 \rightarrow V_{load} = V_{max}$

$\rightarrow V_{load} = |V_0^+| \cdot (1 + |\Gamma|) \wedge |\Gamma| = \frac{1}{3}$

$\rightarrow V_{load} = 4V$

Or $\because SWR = \frac{V_{max}}{V_{min}} = \frac{R_L}{R_0} \rightarrow V_{max} = 2V_{min}$

$\wedge V_{in} = V_{min} = 2V \rightarrow V_{max} = 4V$

$\boxed{6}$ $z_L = 1.2 + 1.2j$

z_{in} at stub $= 0.7 - 0.6j$

y_{in} at stub $= 0.8 + 1j$

$\rightarrow$ stub length $= 0.125\lambda$

$\rightarrow y_{in} = 0.8 \rightarrow z_{in} = \frac{1}{0.8} = \frac{5}{4}$

$\because z_{in} = \frac{Z_{in}}{Z_0} = SWR = 1.25$

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MOHAMMAD SANAD ALTAHER 130806

أنا محمد سند هيثم الطاهر أشهد أنني قرأت وفهمت وطبقت تعليمات
هذا الامتحان ، ولم أتلق أي مساعدة من أي شخص في حل هذا
الامتحان . أنا لست بغشاش ولاكذاب

Q1) matched load

a) $\therefore R = \frac{1}{\pi a \delta \sigma_c} = \frac{\sqrt{\pi f \mu_c \sigma_c}}{\pi \cdot a \cdot \sigma_c} = \boxed{8.3045 \text{ m}\Omega/\text{m}}$

$C = \frac{\pi \varepsilon}{\ln d/a} = \boxed{12.064 \text{ pF/m}}$

$G = \frac{\pi \sigma_d}{\cosh^{-1}(\frac{d}{2a})} \approx \frac{\pi \sigma_d}{\ln \frac{d}{a}} = \boxed{68.22 \ \mu\mho/\text{m}}$

$L = \frac{\mu}{\pi} \cdot \ln(\frac{d}{a}) = \boxed{1.842 \ \mu\text{H/m}}$

$\therefore \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{7.9168 \mu} \angle \frac{1.5103}{2}$

$\rightarrow \gamma = 2.8136 \times 10^{-3} \angle 0.75515 \text{ Rad}$
$= \boxed{(2.0488 + 1.9284j) \times 10^{-3} \ /\text{m}}$

$\therefore Z_0 = \frac{\gamma}{G + j\omega C} = \boxed{30.34 + 27.931j \ \Omega}$

b) $\therefore P_{load} = \frac{1}{2} \text{Re}\{V_L \cdot I_L^*\} \rightarrow P_{load} = \frac{1}{2} \text{Re}\{\frac{|V_L|^2}{Z_0^*}\}$

$\therefore V_L = V_{in} \cdot e^{-\alpha l}$

$\& \ V_{in} = 200 \cdot \frac{Z_{in}}{Z_g + Z_{in}} \ \& \ Z_{in} = Z_0$

$\rightarrow V_{in} = 88.95 + 38.61j$

$\rightarrow |V_L| = 196.9681 \cdot |e^{-2.048 \times 10^{-3} \times 100}|$

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Q1 continued

h) → $|V_L| \approx 79\ V$ → $\boxed{P_{load} = 55.67\ W}$

$\therefore P_{in} = \frac{1}{2} Re \left\{ \frac{|V_{in}|^2}{Z^{*}} \right\} = 83.8735\ W$

→ $\boxed{P_{lost} = 28.2035\ W}$

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Q2) TE_{10} is dominant for both

a) Waveguide a : $a = 2cm$ & $b = 1.5 cm$

$$(f_c)_{10a} = \frac{u}{2} \cdot \frac{1}{a} = \frac{c}{2a} = 7.5 GHz$$

TE_{01} is the higher-order dominant mode for WG a

$$\rightarrow (f_c)_{01a} = \frac{u}{2} \frac{1}{b} = 10 GHz$$

Waveguide b: $a = 1.8 cm$, $b = 0.75 cm$

$(TE)_{20}$ is second dominant since $a > 2b$

$$\rightarrow (f_c)_{10b} = \frac{u}{2} \frac{1}{a} = \frac{c}{4} \cdot \frac{1}{a} = 4.167 GHz$$

$$(f_c)_{20b} = 8.333 GHz$$

∴ $f_{operation}$ for a single mode in both

$$\boxed{7.5G \leq f_{op} \leq 8.33 G}$$

b) $f = 8 GHz$ $\rightarrow (u_p)_{a10} = \frac{c}{\sqrt{1 - (\frac{f_c}{f})^2}} = \boxed{8.62 \times 10^8 m/s}$

& $(u_p)_{b10} = \frac{c}{2 \cdot \sqrt{1 - (\frac{4.167}{8})^2}} = \boxed{1.757 \times 10^8 m/s}$

c) max electric field = 3MV/m

∵ TE → $E_z = 0$ & TE_{10} → $E_x = 0$

∴ $|E_y|_{max} = 3\frac{MV}{m}$

∵ $|E_y| = \left|\frac{\omega\mu}{h^2}\right| \cdot \left(\frac{m\pi}{a}\right) \cdot H_0$, $h^2 = k_x^2 = \frac{m\pi}{a}$

$$\rightarrow |E_y|_{max} = \frac{a \cdot \omega \cdot \mu}{m\pi} \cdot H_0 = 3 M$$

$$\rightarrow H_{0,max} = 7160.388 A/m$$

∵ P_{avg} for TE_{10} = $\boxed{\frac{\omega^2 \cdot \mu^2 \cdot a^3 \cdot b}{4\pi^2 \eta_{TE_{10}}} \cdot H_0^2}$ $0.0112 H_0^2$

$\eta_{TE_{10}} = \frac{\eta_0}{\sqrt{1 - (\frac{f_c}{f})^2}} = 1083.354 \Omega$ → $\boxed{P_{max} = 573.963 kW}$

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Q3)

a) ∵ $k_x = 0 \rightarrow m = 0$

& $k_y = 100\pi = \frac{n\pi}{b} \rightarrow \boxed{n = 2}$

∴ $\boxed{TE_{02}}$

b) ∵ $\beta_z = 277.06 = k \cdot \sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$(f_c)_{02} = \frac{c}{2\sqrt{\epsilon_r \mu_r}}\left(\frac{2}{b}\right)$, non-magnetic

$\rightarrow f_c = \frac{c}{2\sqrt{\epsilon_r}} \cdot 100$

or $\beta_z = \sqrt{k^2 - k_x^2 - k_y^2}$ & $k_x = 0$

$\rightarrow 277.06 = \sqrt{w^2 \cdot \mu\epsilon - (100\pi)^2}$

$\rightarrow w^2 \mu\epsilon = 175458.3 \rightarrow \boxed{\epsilon_r \approx 4}$

c) $\boxed{E_z = 0}$, $\boxed{E_y = 0}$, $\boxed{H_x = 0}$

∵ $|E_x| = 100 = \frac{\omega\mu}{h^2} \cdot 100\pi \cdot H_0$, $h^2 = k_y^2$

→ H_0 is negative since j shifts the $\cos(wt-\beta z)$

$j\cos(wt-\beta z) = -\sin(wt-\beta z)$

∴ $-100 = \frac{\omega\mu}{100\pi} \cdot H_0 \rightarrow \boxed{H_0 = -0.3979\ A/m}$

∴ $H_y = \frac{\gamma}{k_y} \cdot H_0 \cdot \sin(k_y y) \cdot \cos(wt = \beta z)$

& $\gamma = j\beta$

$\rightarrow \boxed{H_y = 0.351 \cdot \sin(100\pi y) \cdot \sin(wt-\beta z)\ A/m}$

& $H_z = H_0 \cdot \cos(k_y y) \cdot \cos(wt-\beta z)$

$\rightarrow \boxed{H_z = -0.3979 \cos(100\pi y) \cdot \cos(wt - 277.06z)\ A/m}$

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(Q4) series stub → use impedance, shorted, lossless

$b/a = 7.7$

Z_0 stub $= 50\,\Omega$, air-filled

Z_0 line $= 100\,\Omega = \sqrt{\frac{L}{C}} = \frac{\ln \frac{b}{a}}{2\pi} \cdot \sqrt{\frac{\mu}{\epsilon}}$

$\rightarrow \boxed{\epsilon_r \approx 1.5}$ $\therefore \lambda_l = \frac{c}{\sqrt{1.5} \cdot f} = 8.165$ cm

$\therefore z_L = 0.5 + 0.5j$

minimum distance from load to stub → point $1+1j$

$d = (0.162 - 0.088)\lambda = \boxed{0.6042 \text{ cm}}$

stub: $-1j|_{Z_0=100}$ $\lambda_s = \frac{c}{f} = 10$ cm

$\rightarrow$ stub $= -2j|_{Z_0=50}$

$l = 0.324\,\lambda_s = \boxed{3.24 \text{ cm}}$

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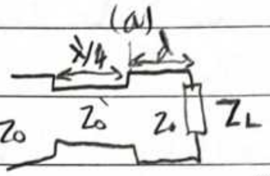
Q5) a) $Z_L = 50 + j50 \ \Omega$, $Z_{0l} = 50 \Omega$



$\lambda_l = \frac{c}{f} = 10$ cm

if $(Z_{in})_a = Z_{max}$

SWR ≈ 2.6 → $Z_{in} = 130 \Omega$

$d = (0.25 - 0.1625)\lambda_l = \boxed{0.875 \text{ cm}}$

if $(Z_{in})_a = Z_{min}$ → $Z_{in} = 19 \Omega$ & $\boxed{d = 3.375 \text{ cm}}$

∴ $(Z_0')_{min} = \sqrt{19 \cdot 50} = 30.822 \Omega$

& $(Z_0')_{max} = \sqrt{130 \cdot 50} = 80.623 \Omega$

∵ transformer has same dimensions & non-magnetic

→ $\frac{Z_0'}{Z_0} = \frac{1}{\sqrt{\varepsilon_r}} \Rightarrow (\varepsilon_r')_{min} = 2.6316$ → $\lambda' = 6.164$ cm

& $(\varepsilon_r')_{max} = X$ $\boxed{(\varepsilon_r')_{max} < 1}$ X

∴ $\boxed{d = 3.375 \text{ cm}}$ & $\frac{\lambda'}{4} = \boxed{l = 1.541 \text{ cm}}$

b) $f = 6$ GHz → $(\theta_e)_a = \frac{2\pi}{\lambda_l} \cdot l$ & $\lambda_l = 5$ cm → $l = \frac{27}{40}\lambda_l$

$Z_{in\,a}|_{6G} = (1.1 - 1.8j) \cdot 50 = 55 - 90j$

$Z_{in} =$