

Four modes of operation of transistor:

- 1 - Saturation: transistor functions as a short circuit between collector and emitter. $\beta I_B > I_C$
- 2 - Cut-off: transistor acts like an open circuit with no current flowing from collector to emitter.
- 3 - Forward active: the current from collector to emitter is proportional to the base current
- 4 - Reverse active: current flow of the forward active mode is reversed.

* "RTL": resistor-transistor logic * "DTL": diode-transistor logic
 * "TTL": transistor-transistor logic * "ECL": emitter coupled logic

- designers should minimize the undefined input region while maximizing the noise margins as the output will be seen as either low (0) or high (1).

* RTL logic:

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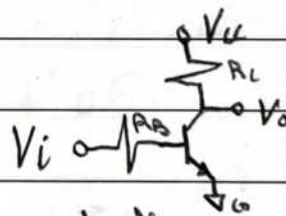
assume the transistor is operating in one of the four modes until otherwise.

- forward-active: $V_{BE(on)} = 0.7V$
- cut-off: $V_{BE} < 0.7V$
- saturation: $V_{BE(sat)} = 0.8V$ \wedge $V_{CE(sat)} = 0.2V$

- Simple RTL inverter:

if $R_B = 10k$, $R_C = 1k$, $\beta = 50$, $V_{CC} = 5V$

$$\circ \circ V_o = V_{CC} - I_C R_C$$



$$\rightarrow I_C = 0 \rightarrow V_o = V_{CC} \quad \wedge \quad I_C = 0 \text{ in cut-off}$$

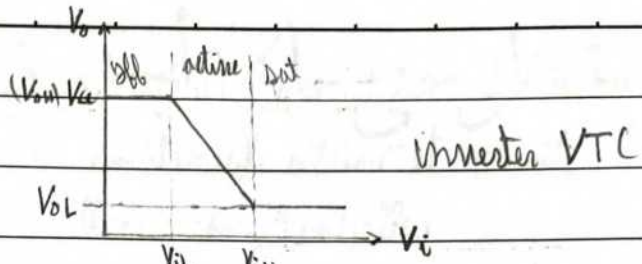
$$\therefore V_i < 0.7V \rightarrow V_o = V_{CC} \rightarrow V_i = \text{low} \rightarrow V_o = \text{high}$$

$$\circ \circ V_o = 0.2V \text{ in saturation} \rightarrow I_C = \frac{V_{CC} - 0.2V}{R_C} = 4.8 \text{ mA}$$

$$\circ \circ I_C = \beta I_B \rightarrow I_B = 96 \mu A \quad \wedge \quad -V_i + I_B R_B + V_{BE(sat)} = 0$$

$$\rightarrow V_i = 0.8 + 96 \mu \cdot 10k = 1.76V \therefore V_i = \text{high} \rightarrow V_o = \text{low}$$

- "VTC": Voltage transfer characteristics
- V_{iL} : minimum input voltage that will be recognized as low input logic level



- V_{iH} : the minimum input voltage that will be recognized as high input logic level
- V_{OH} : the output voltage recognized as logic high (corresponding to V_{iL} in inverter)
- V_{OL} : the output voltage recognized as logic low (corresponding to V_{iH} in inverter)

* noise margins: safety margins preventing erroneous outputs from noisy inputs

- If the output of one inverter is used as the input to the next:

$$\rightarrow NM_L = V_{iL} - V_{OL} = 0.7 - 0.2 = 0.5V$$

$$\rightarrow NM_H = V_{OH} - V_{iH} = 5 - 1.96 = 3.04V$$

Where NM represents the noise margin and the subscript represents the logic level

* fan-out: number of gate inputs driven by output of a single logic gate

- the maximum fan-out an output measures its load driving capability
- the driving device must be able to supply (or sink) the sum of currents needed (or provided) by all the connected inputs while maintaining the output voltage specifications (to supply or sink depends on whether the output is a logic high or low level).

- for an RTL inverter, the fan-out for a low logic level is ∞ .
- for an RTL inverter, the fan-out for a high logic level is $\neq \infty$

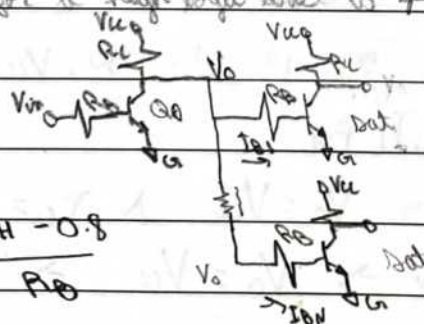
Ex 1: If $V_i = \text{low} \rightarrow Q_0 \text{ off}$
 $\rightarrow V_o = \text{high}$

$$V_o > V_{iH} = 1.96V$$

$$I_{CO} = N \cdot I_{B1} \rightarrow N \cdot I_B = \frac{V_{iH} - 0.8}{R_B}$$

$$I_{CO} = \frac{V_{CC} - V_{iH}}{R_C}$$

$$\therefore \frac{V_{CC} - V_{iH}}{R_C} = N \cdot \frac{V_{iH} - 0.8}{R_B} \rightarrow N_H = \frac{R_B}{R_C} \cdot \frac{V_{CC} - V_{iH}}{V_{iH} - 0.8} = 33.79 \approx 33$$



$$R_B = 10k\Omega$$

$$R_C = 1k\Omega$$

$$V_{BE(sat)} = 0.8V$$

- $\beta I_B > I_C$ in saturation $\rightarrow \sigma \cdot \beta \cdot I_B = I_C$ where σ : saturation parameter.

- in Ex 1, assume transistors are at end of saturation (EOS) $\rightarrow \sigma = 1$

$$\therefore I_{B(\text{sat})} = \frac{I_{C(\text{sat})}}{\beta} = \frac{V_{CC} - V_{CE(\text{sat})}}{\beta \cdot R_C} \quad \beta = 90$$

$$\rightarrow I_{B(\text{sat})} = 9.6 \times 10^{-5} \text{ A} \quad \text{and } I_{C0} = N \cdot I_{B(\text{sat})} = \frac{5 - V_{IH}}{R_C}$$

$$\therefore N = \frac{3.24 \times 10^3}{9.6 \times 10^{-5}} = 33.75 \approx 33$$

f_{an-in} : maximum number of inputs that a gate can handle and maintain proper functionality.

- for an RTL NAND gate, the f_{an-in} is limited by the output voltage.

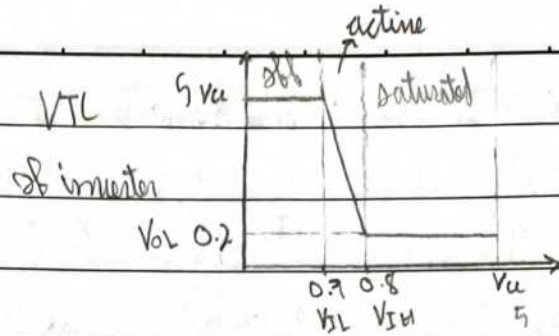
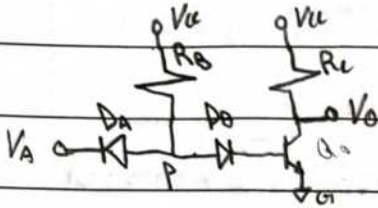
For low logic needs to be lower than V_{IL} (0.7)

$$\text{Thus if } V_{CE(\text{sat})} = 0.2, \text{ max } f_{an-in} = \frac{0.7}{0.2} = 3.5 \approx 3 \text{ gates}$$

- RTL was advantageous because transistors were the most expensive component (before IC technology), hence RTL was the cheapest type of logic.

- RTL dissipates high amounts of current, which causes a lot of losses to heat, hence RTL is not efficient.

* Basic DTL inverter



assuming DA is always on:

when $V_A = 0V$, $V_P = 0.7V$, $V_{BE(on)} = 0.7V \rightarrow Q_0$ is off

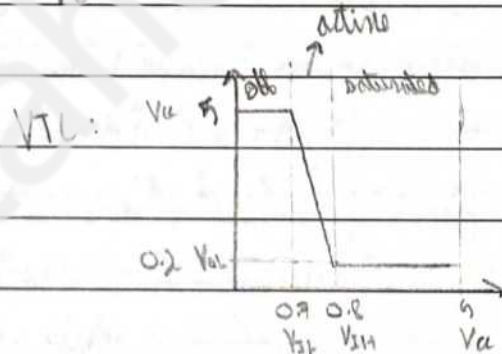
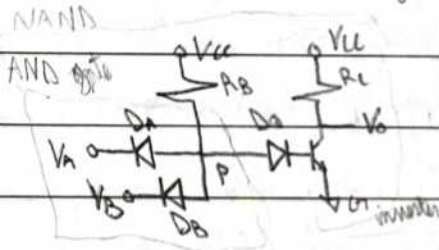
when $V_A = 0.7V$, $V_P = 1.4V \rightarrow Q_0$ is on (in active)

when $V_A = 0.8V$, $V_P = 1.5V \rightarrow V_{BE} = 0.8V \rightarrow Q_0$ is in saturation.

\therefore off for $0 \leq V_A < 0.7$, active for $0.7 \leq V_A < 0.8$

1 saturated for $0.8 < V_A \leq V_{CC}$

* Basic DTL NAND gate:



if V_A is grounded and DA is assumed on:

$V_B = 0.5V$, $V_P = 0.7V$ (and) $\rightarrow D_B$ is off and $V_O = H$

$V_B = 1V$, $V_P = 0.7V$ (and) $\rightarrow V_O = V_{CC}$ if $V_A = 0$ always

if V_B is grounded and D_B is assumed on:

$V_A = 0.7V$, $V_P = 0.9V$, D_A is off, $V_O = H = V_{CC}$

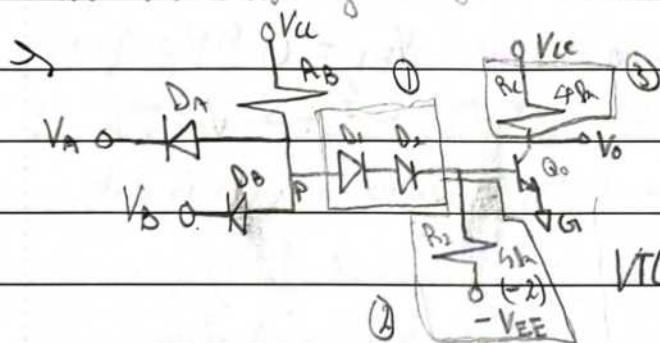
if V_B is fixed at $0.7V$, $V_P = V_A + 0.7V$ if D_A is on

$\rightarrow V_B = V_A = 0.7V \rightarrow V_P = 1.4V \rightarrow Q$ is in active

if $V_B = V_A = 0.8V \rightarrow V_P = 1.5V \rightarrow Q$ is in saturation

if $V_B \wedge V_A > 0.8$, $V_O = 0.2V = L$

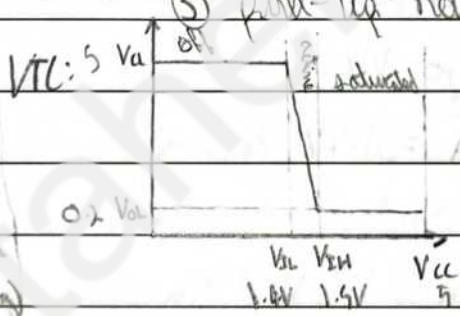
- level-shifting diodes are used to increase the noise margin by shifting V_{IL} .
- pull-up networks provide low resistance paths to V_{CC} when the transistor is off and high resistance paths when transistor is saturated.
- pull-down networks provide a path for collected charges when the transistor goes from saturation mode to cut-off.



① level-shifting diodes

② pull-down network

③ pull-up network



- $-V_{EE}$ is the pull down network (can be 0V but it is given a magnitude to speed up the discharge)

(Reminder): if the transistor is in saturation mode, charges accumulate in the base terminal (can be represented by a capacitor connecting the base and emitter). hence, to turn off a transistor, the charges accumulated at the base must be removed.

- the fan-out for the high output of a DTL NAND gate is objectively ∞ since no current flows through the input diodes. $N_H \rightarrow \infty$

$\therefore Q_1$ must remain saturated

$\rightarrow I_{C(sat)} \leq \beta I_B$

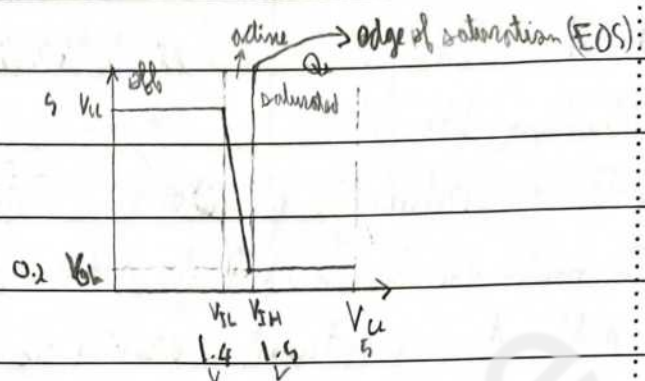
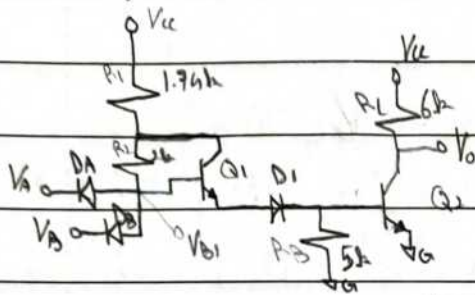
$\wedge I_{C(sat)} = N \cdot I_{DA} + \frac{5}{R_1}$

$\rightarrow N \cdot I_{DA} = \beta I_B \wedge I_B = 0.84 \text{ mA} \rightarrow 0.84 \text{ mA} - 1.25 \text{ mA}$

$\wedge I_{DA} = 1.75 \text{ mA} \rightarrow N = \frac{1.75 \text{ mA}}{1.75 \text{ mA}} = 23.28 \approx 23$

- smaller σ indicates transistor is more saturated

* modified DTL NAND:



if V_B is fixed at $5V (= V_{CC})$:

- if $V_A = 0.2V$ (low) and D_A is on $\rightarrow V_{B1} = 0.9V \rightarrow Q_1$ is on:

but D_1 is off $\rightarrow Q_1$ is off and Q_2 is off $\rightarrow V_o = V_{CC}$

- if $V_A = 5V \rightarrow Q_1$ is on and D_1 is active and Q_2 is on
 $\rightarrow V_o = V_{CC}$

- when $V_A = 1.4V$ and D_A is on $\rightarrow Q_1$ is forward active
 and Q_2 is forward active

- when $V_A = 1.5V$ and D_A is on $\rightarrow Q_1$ is on and Q_2 is on
 $\rightarrow V_o = V_{CC}$

$$\therefore V_{TL} = -D_{A(on)} + V_{BE1(on)} + D_{1(on)} + V_{BE2(on)} = 1.4V$$

$$\wedge V_{TH} = -D_{A(on)} + V_{BE1(on)} + D_{1(on)} + V_{BE2(on)} = 1.5V$$

if $V_A = 0.2V \rightarrow Q_1$ and Q_2 are off, assuming D_A is on

$$\rightarrow V_{B1} = 0.9V \therefore I_A = \frac{V_{CC} - 0.9}{3.7k\Omega} = 1.093 \text{ mA (all other currents = 0)}$$

if $V_A = V_B = V_{CC}$ and D_1 is on, Q_1 is forward active and $\beta = 100$

$$\rightarrow V_{B1} = V_{BE1(on)} + V_{B1(on)} + V_{BE2(on)} = 2.2V$$

$$\therefore I_{A1} = I_{B1} + I_{C1} = I_{E1} \rightarrow I_{B1} = \frac{I_{C1}}{\beta + 1}$$

$$\rightarrow -5 + I_{A1} \cdot 1.7k\Omega + I_{E1} \cdot \frac{2k}{\beta + 1} + 2.2 = 0 \rightarrow I_{A1} = \frac{2.8}{1.7k\Omega + \frac{2k}{101}}$$

$$\rightarrow I_{A1} = 1.582 \text{ mA}$$

$$\therefore V_{BE2} = 0.8V \wedge I_{E1} = I_{B2} + I_{C2}$$

$$\rightarrow I_{B2} = 1.582 \text{ mA} - \frac{0.8}{5k} = 1.422 \text{ mA}$$

$$\wedge I_{C2} = \frac{5 - 0.2}{5k} = 0.96 \text{ mA}$$

- for a high output from a modified DTL NAND, an ∞ number of gates can be driven

- for a low output, the transistor Q_1 must remain saturated

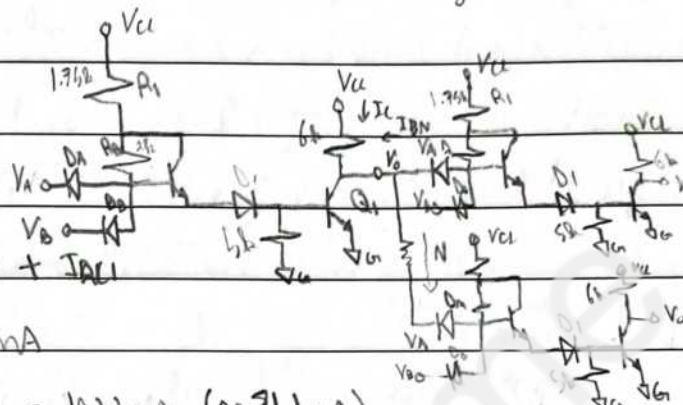
$$\rightarrow \beta I_{B1} \geq I_{C1}, I_{C1} = N \cdot I_{BN} + I_{C1}$$

$$\therefore I_{BN} = \frac{V_{CC} - 0.9}{3.7k\Omega} = 1.093mA$$

$$\beta I_{B1} = 100 \cdot 1.422mA = 142.2mA \quad (or \ 91.1mA)$$

$$\therefore N \cdot I_{BN} + \frac{5-0.2}{6k} = \beta I_{B1} \rightarrow (B=100) \quad N = 129$$

$$(B=50) \quad N = 64$$

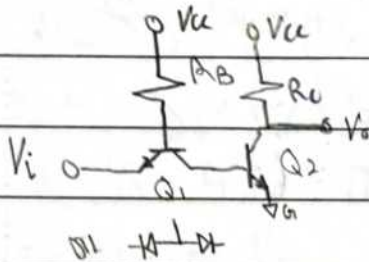


- the average power dissipation is given by:

$$P_{(avg)} = V_{CC} \cdot \frac{I_{CC}(0H) + I_{CC}(0L)}{2}$$

- in designing ICs, compactness and logic density are most important. Since resistors and capacitors are generally the largest components, reducing their count will free up space for extra transistors and save costs.
- TTL logic still requires resistors, whereas MOS gates can be made of transistors only. hence why they are more ubiquitous today.

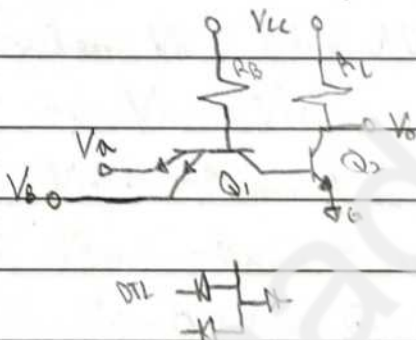
+ Basic TTL inverter:



When V_i is high, Q_1 is off, pushing Q_2 into saturation $\rightarrow V_o = L$ (0.2V)

When V_i is low, the accumulated charges at the base terminal of Q_2 will have a path to the emitter of Q_1 , hence turning Q_2 off.

+ Basic TTL NAND gate:



Q_1 is a multi-emitter BJT

R_C : Composite (lowest gain) (normal emitter)

+ for a basic TTL inverter:

If $V_i = 0.2$, assuming Q_1 is in sat:

$\rightarrow V_{B1} = 0.2 + 0.8 = 1V$ $\because V_{BE(sat)} = 0.8V$

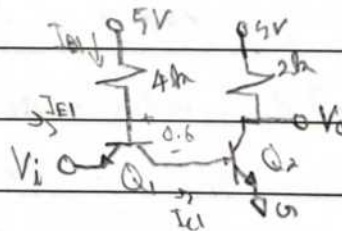
$\rightarrow I_{B1} = 1mA = I_{E1}$

$\wedge I_{B2} = 0 \rightarrow Q_2$ is off $\rightarrow V_o = V_{cc} = 5$

If $V_i = 5V$, assuming Q_2 is in sat $\wedge Q_1$ is in reverse active

$\because V_{BC(sat)} = 0.6V$ $\wedge V_{BE(sat)} = 0.8V \rightarrow V_{B1} = 1.4V$

$\because I_{C1} = I_{B2} = (\beta_{R1} + 1) \cdot I_{B1}$ where β_{R1} is the reverse beta = 0.1



$$\hookrightarrow I_{B1} = \frac{5 - 1.4V}{4k} = 0.9mA \rightarrow I_{C1} = (1 + 0.1) \cdot 0.9mA = 0.99mA$$

$$\hookrightarrow I_{E1} = \beta_{11} \cdot I_{B1} = 0.09mA$$

$$\infty I_{C2} = \frac{V_{CC} - V_{CE(sat)}}{2k} \rightarrow I_{C2} = \frac{5 - 0.2}{2k} = 2.4mA$$

∞ for saturation, $I_C < \beta I_B$ $\wedge \beta_2 = 50$

$$\hookrightarrow I_{C1} = 2.4mA < 50 \cdot 0.99mA = \beta_2 I_{B2} \therefore Q_2 \text{ is saturated.}$$

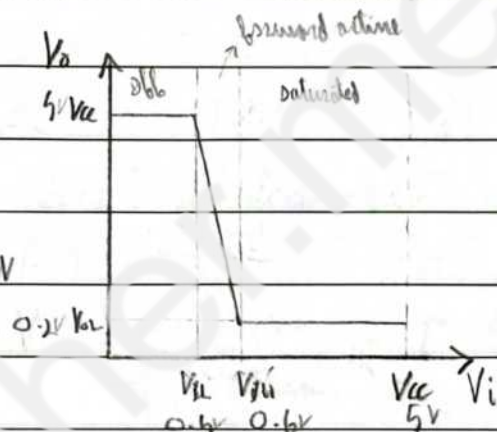
+ VTC for basic TTL inverter: Q_2 :

$$\text{if } V_i = 0.2V, \text{ off}$$

Q_2 turns on when $V_{BE2} = 0.9V$

$$\rightarrow V_{B1} = 0.6 + 0.9 = 1.3V \rightarrow V_i = 0.6V$$

$$\infty V_{CE1(on)} = 0.6V$$



- The voltage for $V_{CE(on)}$ is less than $V_{BE(on)}$ because transistors are not symmetrical and the emitter is more heavily doped compared to the collector.

- immediately after switching an input of a basic TTL inverter from high (5V) to low (0.2V):

$$Q_1 \text{ turns to forward active} \rightarrow V_{B1} = V_i + V_{BE(on)} = 0.9V$$

$$\text{momentarily, } Q_2 \text{ will remain saturated} \rightarrow V_{B2} = 0.8V = V_{BE(sat)}$$

(due to propagation delay)

Q_1 immediately changes to forward active since $V_{B2} = 0.8V$

$$\wedge V_i = 0.2V \rightarrow V_{CE1} = 0.6V > V_{CE1(sat)} = 0.2V$$

therefore, a voltage larger than the saturation voltage keeps the transistor unsaturated.

$$\infty I_{B1} = \frac{5 - 0.9}{4k} = 1.025mA$$

$$\rightarrow \text{initial } I_{C1} (= I_{B2}) = \beta I_{B1} = 51.25mA$$

fan-out for TTL inverter:

- low input case:

∞ Q₁ is sat (stage 1)

Q₂ should be off

→ Q₁ of next stage should be reverse active

→ Q₂ should be sat

→ $V_{BE22} = 0.8V$ $I_{B22} = I_{E12} + I_{B12}$ $I_{E12} = \beta_1 I_{B12}$

for Q₂ sat, Q₁ reverse active → $V_{A12} = 0.6 + 0.8 = 1.4V$

→ $I_{B12} = \frac{5 - 1.4}{4k} = 0.9mA$ → $I_{E12} = \beta \cdot 0.9mA$

∞ $I_{C21} = \frac{5 - V_o}{2k} = N \cdot \beta_{21} \cdot (0.9)$, $\beta_{21} = 0.1$ & $V_o = H.S. V_o \geq 4.2V$
 ↳ Reverse data

∴ $I_{C21} = \frac{5 - 4.2}{2k} = N \cdot 0.1 \cdot 0.9mA = 4.444 \Rightarrow N_{max} = 4$

- high-input case: $V_i = 5$

→ Q₁₁ is reverse active, Q₂₁ is sat, Q₁₂ is sat, Q₂₂ is off

→ $V_{B21} = 0.8V$ → $V_{B11} = 0.8 + V_{BE}(reverse) = 1.4V$

∞ for Q₂₁ to remain in sat: $\beta \cdot I_{B21} \geq I_{C21}$

$I_{C21} = I_{A21} + N \cdot I_{E12}$ $I_{E12} = I_{B12} = \frac{5 - (0.2 + 0.8)}{4k}$

↳ $I_{A21} = \frac{5 - 0.2}{2k} = 2.4mA$

↳ $I_{B21} = I_{C11} = I_{B11} + I_{E11}$, $I_{E11} = \frac{5 - 1.4}{4k} = 0.9mA$

↳ $I_{E11} = \beta_{11} \cdot 0.9mA \approx 0.9mA$ → $I_{C11} = 0.99mA$

∴ $(0.99mA) \cdot \beta \geq 2.4mA + N \cdot (1mA)$

→ $N = \frac{0.99mA \cdot \beta - 2.4mA}{1mA}$ $\beta = 50 \Rightarrow N = 47$

* Standard TTL NAND gate:

- Q_4 is included as a consequence to including Q_2 .

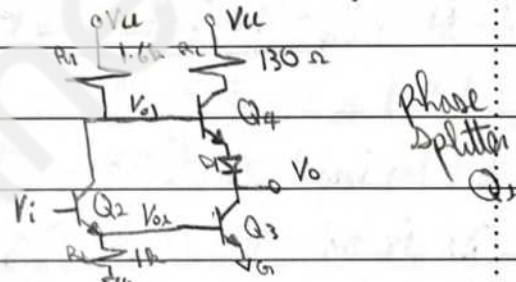
- Since the pull-up network (including Q_4) allows V_o to go from high to low quickly.

- The pull-down network allows the output to go from low to high quickly.

- Transistor Q_2 is included to control Q_4 . When output is low, Q_2 is saturated by the input current from Q_4 , which is also saturated causing the base voltage of Q_4 to be $0.2V$ and cutting it off.

- Q_2 is called a phase splitter:

a phase splitter: allows the input condition to be produced in opposite phase; allowing Q_3 to be on while Q_4 is off.



if $V_i = \text{low}$, $V_{o1} = V_{cc}$ & $V_{o2} = 0 \rightarrow Q_3 = \text{off}, Q_4 = \text{on}$

if $V_i = \text{high}$, $V_{o1} = 0.9V$ & $V_{o2} = V_{BE3} \rightarrow Q_3 = \text{on}, Q_4 = \text{off}$

diode D_1 is included to keep transistor Q_4 off even when $V_{o1} = 0.9V$

hence, $0.9V$ at base terminal of Q_4 can be considered low

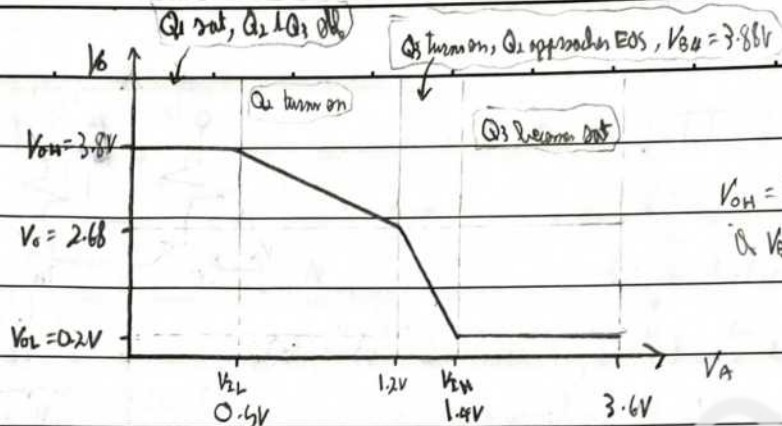
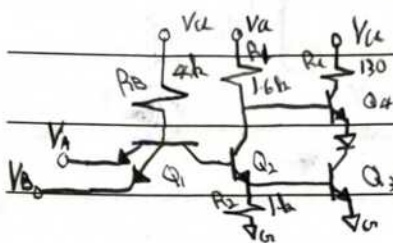
- output transistor pair, Q_3 & Q_4 , along with diode, D_1 , are referred to as the totem pole output

- totem pole output configuration can actively source or sink current and is useful for capacitive loads.

- diode, D_1 , increases the effective turn-on voltage of Q_4 , allowing Q_4 to be turned off before Q_3 is fully on, preventing large surge currents from flowing in the output stage during transitions from high to low or vice versa

- the diode causes the output high voltage level to be reduced by the voltage drop across the diode

VTC of standard TTL NAND gate:



- When V_A & V_B are low, Q_1 is saturated, the base current of Q_1 goes to the emitters, Q_2 gets no base current and is off causing Q_3 to be off as well.
 V_O is high (there is an added voltage drop due to V_{BE}), (Q_1 in sat)
 - If either input (or both) are increased, then some of the base current is directed to the collector of Q_1 , base of Q_2 and causes it to turn on (active mode) and send a current to the base of Q_3 , turning it on in turn.
 - If either input (or both) are high, Q_1 is in reverse active mode, Q_2 is saturated, causing Q_3 to saturate as well. Hence Q_1 will be off and V_O will be low (0.2V)
 $NM_H = 3.8 - 1.4 = 2.4V$
- noise margins in the above circuit are unequal: $NM_L = 0.5 - 0.2 = 0.3V$

$$① V_{CC} = 5, \beta = 96.6, R_C = 2k$$

$$\frac{2 - 0.8}{R_{AB}} = \frac{5 - 0.2}{2k \cdot 96.6} = 483 \text{ } \textcircled{1}$$

$$② \sigma \cdot \beta \cdot I_B = I_{cont} \quad I_{cont} = \frac{5 - 0.2}{2k} = 1.2 \text{ mA}$$

$$I_B = \frac{-0.8}{5k} + \frac{5 - 2.2}{2088} = 1.1861 \text{ mA}$$

$$\Rightarrow \sigma = 0.0159$$

$$③ \sigma = 0.9 \quad \rightarrow \quad 0.9 \cdot 50 \cdot I_B = \frac{5 - 0.2}{1k}$$

$$\rightarrow I_B = \frac{4.8 \text{ mA}}{0.9 \cdot 50} = 0.106667 \text{ mA}$$

$$\wedge V_i = 0.8 + I_B \cdot R_{AB} = 2.199$$

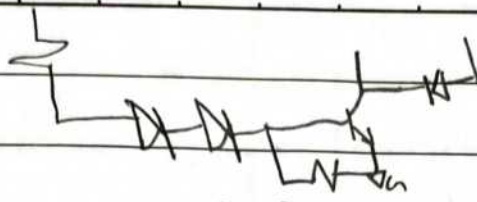
$$④ \frac{5 - V_o}{R_C} = \beta \cdot \frac{V_o - 0.8}{12.8k}$$

$$\rightarrow 5 - V_o = \frac{6 \cdot 12.8k}{12.8k} \cdot (V_o - 0.8)$$

$$\Rightarrow 5 + \frac{6 \cdot 0.8}{12.8} = V_o \left(\frac{6}{12.8} + 1 \right)$$

$$\rightarrow V_o = 3.699 \text{ V}$$

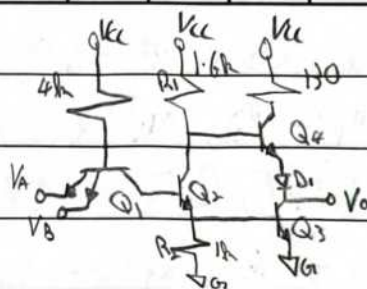
④



$$I_c = \frac{5 - 0.2}{4\Omega} + \frac{5 - 0.7}{2.42\Omega} = 1.2 +$$

* high output:

When either (or both) inputs is low, the output will be high



* EOC: edge of cutoff

$$0 < V_A \text{ or } V_B < 0.5$$

assume Q1 is sat, $V_A = 0.2V \rightarrow V_{B1} = V_{BE(sat)} + 0.2V = 1V$

$$\rightarrow I_{B1} = \frac{5-1}{4k} = 1mA \text{ goes to A or B (emitter)}$$

$$\circ \circ Q1 \text{ is sat} \rightarrow V_{CE} = 0.2V \rightarrow V_{B2} = 0.4V$$

$\circ \circ V_{B2} < V_{BE(on)} \rightarrow Q2$ is off, $D1, Q4$ is at EOC

$\circ \circ Q2$ is off $\rightarrow I_{B3} = 0 \rightarrow Q3$ is off

$\circ \circ V_O$ is unconnected, no current flows through D_2 or D_1

$$\rightarrow V_O = V_{CC} - V_{BE(Q2)} - D_{1(on)} = 5 - 0.7 - 0.7 = 3.6V \text{ (3.8V)}$$

$I_{EB} = 0$ if V_B is high | $I_{EB} = I_{EA} = \frac{I_B}{2}$ if $V_A = V_B = \text{low}$

* indeterminate: $Q2$ is active when $0.5k(V_A \text{ or } V_B) < 1.2V$

assuming $Q1$ is sat $\rightarrow 1.3V < V_{B1} < 2V \rightarrow 0.7V < V_{B2} < 1.4V$

$$\rightarrow 0V < V_{E2} < 0.7V = V_{B3} \rightarrow V_{B3} \text{ off}$$

$\circ \circ V_O$ is open, $I_{E4} = 0, I_{B4} = 0, D1, Q4$ are at EOC

$$\wedge I_{E2} = I_{A1} = I_{E1}$$

$$\rightarrow I_{E2} = \frac{V_{B2} - V_{BE(on)}}{R_2} \rightarrow 0 < I_{A1} < 0.7mA$$

$$\therefore V_O = V_{CC} - I_{A1} \cdot R_1 - V_{BE(Q2)} - D_{1(on)}$$

$$= 5 - I_{A1} \cdot 1.6k - 0.6 - 0.6 \rightarrow 2.68V < V_O < 3.8V$$

$$\circ \circ V_{B4} = V_{CC} - I_{A1} \cdot R_1 = 3.88V \wedge V_{CE} = V_{B4} - I_{E1} \cdot R_2$$

$$\wedge I_{E1} = I_{C1} = I_{A1} \text{ when } 0.7 < V_{B2} < 1.4 \rightarrow V_{CE} = 3.18V$$

$\therefore Q2$ is not in sat, but is active forward.

* indeterminate: Q_3 is active, Q_2 is active, Q_1 is sat

when $0.7V < V_{E2} < 0.9V$, Q_3 will be active

$$\rightarrow 1.4V < V_{B2} < 1.6V \rightarrow 1.2V < (V_A \text{ or } V_B) < 1.4V$$

$$\circ V_0 = V_{CC} - I_{B1} \cdot R_1 = V_{BE4(\text{sat})} - V_{BE3(\text{sat})}$$

$$\therefore V_0 = 3.6 - I_{B1} \cdot R_1$$

$$\wedge I_{B1} = I_{C2} + I_{B4} \quad \wedge I_{E2} = I_{C2} = I_{B2} + I_{B3}$$

$$\wedge I_{B2} = \frac{V_{BE2} - 0.7}{R_2}$$

* low output: when both V_A & V_B are high $\rightarrow (V_A \& V_B) > 1.4V$

$\rightarrow Q_1$ is in reverse active, $V_{BC(\text{rev})} = 0.6V$

$\rightarrow Q_2$ & Q_3 are in sat $\rightarrow V_{B2} = V_{BE2(\text{sat})} + V_{BE3(\text{sat})} = 1.6V$

$$\rightarrow V_{B1} = 1.6 + V_{BC(\text{rev})} = 2.2V \rightarrow I_{B1} = \frac{5 - 2.2}{47} = 0.7 \text{ mA}$$

$$\rightarrow I_{C1} = (1 + \beta_{\text{reverse}}) \cdot I_{B1}, \text{ assuming } \beta_{\text{reverse}} = 0.1$$

$$\rightarrow I_{C1} = 0.77 \text{ mA} = I_{B2}$$

$$\circ Q_2 \text{ is sat} \rightarrow V_{CE2} = V_{CE2(\text{sat})} = 0.2V$$

$$\wedge V_{E2} = 0.8V \rightarrow V_{C2} = V_{B4} = 1V \therefore Q_4 \text{ off (diode is off)}$$

$$\wedge I_{R1} = \frac{5 - 1}{R_1} = 2.5 \text{ mA} = I_{C2}$$

$$\circ I_{E2} = I_{C2} + I_{B2} = 2.5 + 0.77 = 3.27 \text{ mA}$$

$$\wedge I_{E2} = I_{B3} + I_{B2} \quad \wedge I_{B2} = \frac{V_{E2} - 0}{R_2} = 0.8 \text{ mA}$$

$$\rightarrow I_{B3} = 2.47 \text{ mA} \quad \wedge V_0 = 0.2V \text{ (low)}$$

$$\circ \beta_{TB} > I_C \text{ for sat, } I_{B3} = 0 \rightarrow \text{not saturated}$$

- If D_1 is not used, when Q_1 is rev. active and Q_2 & Q_3 are sat

$$\circ V_{B4} = 1V \rightarrow Q_4 \text{ will be sat} \rightarrow V_{CE4} = V_{CE4(\text{sat})} + V_{CE3(\text{sat})}$$

$$\rightarrow I_{C4} = I_{AC} = \frac{V_{CC} - 0.4}{130} = 35.38 \text{ mA} \rightarrow P = 163 \text{ mW}$$

hence, a lot of power is being dissipated for no reason and the large current can burn the circuit

* fan-out for low output logic:

$$\beta I_B > I_C \rightarrow \sigma \beta_3 I_{B3} = I_{C3}$$

$$\text{for } \sigma = 1 \quad \beta_3 = 10, \quad I_{B3} = 2.47 \text{ mA (calculated previously)}$$

$$\rightarrow 24.7 \text{ mA} = I_{C3} \cdot N$$

$\wedge I_{C3} = I_{AN} \text{ (or } I_{EN})$, assuming other input in next stage is high

$$I_{AN} = I_{R_{BN}} = \frac{V_{CC} - V_{BIN}}{R_{BN}}, \quad V_{BIN} = V_0 + V_{BE1} \text{ (next)} = 1 \text{ V}$$

$$\rightarrow I_{AN} = 1 \text{ mA}$$

$$\therefore 24.7 \text{ mA} = N \cdot 1 \text{ mA} \rightarrow \boxed{N = 24}$$

* fan-out for high output logic:

$$\text{If } V_0(\text{min}) = 3.3 \text{ V} \quad \wedge \beta_4 = 100$$

$$\rightarrow I_{E4} = (1 + \beta) \cdot I_{B4} = N \cdot I_{EN} \quad \wedge \quad I_{EN} = I_{AN} - I_{BN}$$

$$\wedge \quad I_{BN} = \frac{5 - V_{BIN}}{4 \text{ k}\Omega} \quad \wedge \quad V_{BIN} = V_{B1}(\text{out}) + V_{BE2}(\text{out})$$

$$\rightarrow V_{BIN} = 2.2 \text{ V} \rightarrow I_{BN} = \frac{2.8}{4 \text{ k}\Omega} = 0.7 \text{ mA}$$

$$\wedge \quad I_{EN} = I_{BN} \cdot \beta_{\text{prev}} = 0.1 \cdot 0.7 \text{ mA} = 0.07 \text{ mA}$$

$$\therefore 101 \cdot I_{EN} = N \cdot 0.07$$

$$\wedge \quad I_{B4}(\text{max}): \quad V_{B4} = 3.3 + 0.7 + 0.7 = 4.7 \text{ V}$$

$$\rightarrow I_{B4}(\text{max}) = \frac{5 - 4.7}{1.6 \text{ k}\Omega} = 0.1875 \text{ mA}$$

$$\therefore (101) \cdot (0.1875 \text{ mA}) = N \cdot 0.07$$

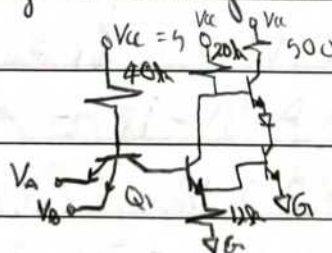
$$\rightarrow N = 270$$

* low-power TTL: achieved by increasing the resistances in the

$$P_H = \frac{b-1}{a+b} \cdot 5 = 0.5 \text{ mW}$$

$$P_L = 0.07 \cdot 5 + 0.2 \cdot 5 = 1.35 \text{ mW}$$

$$\rightarrow P_{avg} = \frac{1.35 + 0.5}{2} = 0.925 \text{ mW}$$



example 17.8:

assuming logic 0 = 0.1V $\wedge V_A = V_B = 0.1V$

if $D_A \wedge D_B$ active $\rightarrow V_1 = 0.8V$

$\therefore D_1 \wedge D_2$ off $\wedge I_1 = 1.05mA$, Q_0 is off $\wedge V_0 = V_{CC}$

if $V_A = V_B = \text{logic } 1 = 5V$ $\rightarrow D_A \wedge D_B$ off, $D_1 \wedge D_2$ on $\wedge Q_0$ sat

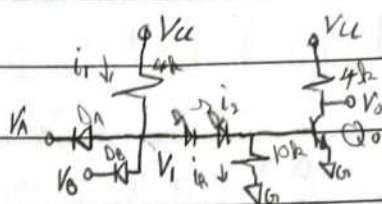
$\rightarrow V_1 = V_{BE(sat)} + V_{D1(on)} + V_{D2(on)} = 2.2V$

$\wedge i_2 = i_1 = \frac{5-2.2}{4k} = 0.7mA$

confirm sat: $\beta I_B > I_C$, $I_C = \frac{5-0.1}{4k} = 1.225mA$

$\wedge I_B + I_A = I_2 = I_1$, $I_A = \frac{0.8}{1k} \rightarrow I_B = 0.62mA$

$\wedge \beta = 25 \rightarrow \beta I_B = 15.5mA > 1.225mA$



Ex 17.8:

$$P_H = V_{CC} \cdot i_{IH} + V_{CC} \cdot I_{CH} = 9.624mW$$

$$P_L = V_{CC} \cdot i_{IL} = 5.25mW$$

if $V_A = V_B = 0V \rightarrow V_1 = 0.9V \rightarrow i_1 = 1.075mA$

$$\therefore P_L = 5.375mW$$

example 17.9:

if $V_A = V_B = 0.1V$ $\wedge Q_1$ is sat

$\rightarrow V_{B1} = 0.9V \rightarrow i_1 = 1.025mA$

$\rightarrow Q_2$ is off $\wedge Q_0$ is off $\rightarrow V_0 = V_{CC}$

if $V_A = V_B = 5V$, Q_1 is reverse active $\wedge Q_2$ is on $\wedge Q_0$ is sat

$\rightarrow V_{B2} = V_{BE1(sat)} + V_{BE2(sat)} = 1.5V \rightarrow V_{B1} = 1.5 + V_{CE(sat)}$

$\therefore V_{B1} = 2.3V \rightarrow i_1 = \frac{2.9}{4k} = 0.675mA$

$\rightarrow I_{B2} = (1 + \beta_{rev}) \cdot i_1 = 0.9425mA$

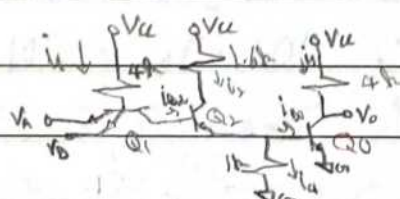
$\because Q_2$ is sat $\wedge V_{B0} = 0.8V \rightarrow V_{C2} = 1V \rightarrow i_2 = \frac{4}{1k}$

$\therefore I_{E2} = i_2 + I_{B2} = 3.2425mA = i_{E2} + I_{B0} \wedge i_{E2} = \frac{0.8}{1k}$

$\rightarrow I_{B0} = 2.4425mA$

confirm sat: $\beta \cdot I_{B0} > I_{C0}$

$$25 \cdot 2.4425 > \frac{4.8}{4k}$$



Ex 17.9:

$$a) \infty V_x = V_y = 0.1V, Q_1 \text{ is sat, } i_1 = \frac{5-0.1}{12k} = 0.34167 \text{ mA}$$

$$Q_2, Q_3 \text{ are cutoff} \rightarrow i_2 = i_3 = i_{E2} = 0 \quad \wedge \quad V_o = V_{cc}$$

$$b) V_x = V_y = 5V, Q_1 \text{ is reverse active, } Q_2, Q_3 \text{ are sat}$$

$$\rightarrow V_{B1} = 2.3V \rightarrow i_1 = \frac{2.7}{12k} = 0.225 \text{ mA} \rightarrow i_{B2} = 0.2475 \text{ mA}$$

$$\infty Q_2 \text{ is sat} \rightarrow V_{C2} = 0.8 + 0.1 \rightarrow i_2 = \frac{4.1}{4k} = 1.025 \text{ mA}$$

$$\therefore i_{E2} = i_{B2} + i_2 = 1.2725 \text{ mA} = i_{B3} + i_4 \quad \wedge \quad i_4 = \frac{0.8}{2k}$$

$$\rightarrow i_{B3} = 0.8725 \text{ mA} \quad \wedge \quad i_3 = \frac{4.9}{6k} = 0.8167 \text{ mA}$$

$$\text{confirm sat: } \beta I_B > I_C \rightarrow 25 \cdot 0.8725 > 0.8167$$

example 17.10:

$$\infty \beta I_B > I_C \rightarrow 25 \cdot 2.4425 = N \cdot i_1 + 1.025$$

$$\wedge i_1 = 1.025 \text{ mA} \rightarrow N = 58$$

Ex 17.10:

$$\text{When } V_x = V_y = 3.6V \rightarrow Q_1 \text{ is rev active, } Q_2, Q_3 \text{ are sat}$$

$$\rightarrow V_{B1} = 2.3V \rightarrow i_{B1} = 0.225 \text{ mA} \rightarrow i_{C1} = 0.27 \text{ mA}$$

$$\infty Q_2 \text{ is sat} \rightarrow V_{C2} = 0.9V \rightarrow Q_3 \text{ is cutoff} \quad \wedge \quad i_{C2} = 1.025 \text{ mA}$$

$$\rightarrow i_{E2} = 1.025 + 0.27 = 1.295 \text{ mA} = i_{B3} + i_{B4}$$

$$\rightarrow i_{B3} = 0.895 \quad \rightarrow V_o = 0.1$$

$$\text{to keep } Q_3 \text{ sat: } 25 \cdot 0.895 = N \cdot \frac{5-0.1}{12k} \rightarrow N = 65$$

$$\text{to keep } I_C = 12 \text{ mA} \rightarrow 12 \text{ mA} = N \cdot \frac{5-0.1}{12k} \rightarrow N = 35$$

TYU 17.5:

$$a) i_1 = \frac{4.2}{15k} = 0.28 \text{ mA}, i_2 = 0 = i_A = i_{RC} = (Q_1)$$

$$c) i_1 = i_2, Q_3 \text{ is sat} \rightarrow V_B = 0.8 \rightarrow V_1 = 2.2V \quad \wedge \quad i_E = 0.18667 \text{ mA}$$

$$i_B = i_2 - i_A = 1.33 \times 10^{-4} \text{ A} \quad \wedge \quad i_C = 0.05333 \text{ mA}$$

$$i_{RC} = \frac{5-0.1}{6k} = 0.817 \text{ mA} \quad \wedge \quad V_o = 0.1V$$

TYU 17.6:

for low output: $i_B = 0.1333 \text{ mA}$ \wedge $i_L = 0.8167 \text{ mA}$

$\therefore \beta \cdot i_B > i_L$ for sat

$\rightarrow \beta \cdot i_B = i_L + N \cdot \frac{V_{CE}}{150} \rightarrow N = 11$ ($N=9$ in book)

b) $I_{C,max} = 12 \text{ mA} = i_{B,C} + N \cdot \frac{V_{CE}}{150} \rightarrow N = 39$ ($N=9$ in book)

TYU 17.7:

low output: $i_B = 0.8167 \text{ mA}$ \wedge $i_{B0} = 0.895$

$\rightarrow 25 \cdot 0.895 = 0.8167 + N \cdot 0.342 \rightarrow N = 63$

17.19:

a) i) if $V_i = 0.1 \text{ V} \rightarrow D_1$ is active, Q_0 is cut off $\rightarrow i_1 = 0.141667 \text{ mA}$

$i_2 = i_3 = 0$ \wedge $V_1 = 0.8 \text{ V}$ \wedge $V_2 = V_{CC} = 2.5 \text{ V}$

ii) if $V_i = 2.5 \text{ V} \rightarrow D_1$ is off, Q_0 is sat $\rightarrow V_1 = 1.5 \text{ V}$

$\wedge i_2 = i_1 = \frac{1}{120} = 0.0833 \text{ mA}$, $V_0 = 0.4 \text{ V} \rightarrow i_3 = 0.2 \text{ mA}$

$\therefore \beta \cdot i_2 > i_3 \rightarrow 25 \cdot 0.0833 > 0.2 \rightarrow \text{sat}$

b) at Q_0 EOL $\rightarrow V_1 = 1.4 \text{ V} \rightarrow V_2 = 0.9 \text{ V}$

at Q_0 EOS $\rightarrow V_1 = 1.5 \rightarrow V_2 = 0.8 \text{ V}$

17.20:

a) if $V_i = 0 \text{ V} \rightarrow V_1 = 0.9 \text{ V}$, $i_1 = 0.4333 \text{ mA}$, $i_B = 0 = i_C$

$\wedge V_0 = 3.3 \text{ V}$

b) if $V_i = 3.3 \text{ V} \rightarrow V_1 = 1.5 \text{ V} \rightarrow i_1 = 0.3 \text{ mA}$

$i_B = i_1 - \frac{0.8}{200} = 0.26 \text{ mA}$, $V_0 = 0.1 \text{ V} \wedge i_C = 0.8 \text{ mA}$

confirm sat: $0.26 \cdot \beta = 6.5 \text{ mA} > 0.8 \text{ mA} \checkmark$

17.21:

i) if $V_{i1} = V_{i2} = 0.1 \rightarrow V = 0.8 \text{ V} \rightarrow i_1 = 0.525 \text{ mA}$

$i_3 = i_2 = 0 \text{ A}$ \wedge $V_0 = 5 \text{ V}$

$$\text{ii) } V_x = V_y = 6V \rightarrow V' = 2.2V, i_1 = 0.36 \text{ mA} = i_4 + \frac{0.8}{15k} \rightarrow i_4 = 0.2967 \text{ mA}$$

$$\wedge i_3 = \frac{4.9}{2.4k} = 2.042 \text{ mA}$$

17.22:

$$\text{i) } i_1 = 0.3125 \text{ mA} \wedge V' = 0.8V, i_4 = i_3 = 0 \wedge V_0 = 9.3V$$

$$\text{ii) } i_1 = 0.1395 \text{ mA} \wedge V' = 2.2V, i_4 = i_1 - \frac{0.8}{15k} = 84.169 \mu\text{A}$$

$$\wedge i_3 = 1.33 \text{ mA}$$

17.23:

$$\text{A) } Q_1 \wedge Q_0 \text{ sat} \rightarrow V_1 = 2.3V \rightarrow i_1 = 0.696 \text{ mA} = i_{B1}$$

$$\wedge i_2 = \frac{3.4}{2k} = 1.7 \text{ mA} \therefore i_4 = 2.396 \text{ mA}$$

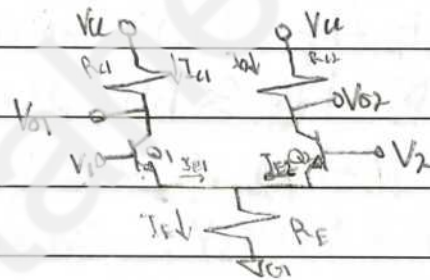
$$i_{B2} = i_4 - i_5 = 2.296 \text{ mA} \wedge i_3 = \frac{4.9}{4k} = 1.225 \text{ mA}$$

$$\text{B) } \circ \circ \text{ B} \cdot i_{B2} = 1.225 + N \cdot \frac{4.2}{4k}$$

$$\rightarrow N = 42$$

- main ECL advantage is the very short propagation delay.
- + disadvantages include:
 - high power consumption
 - poor speed-power product
 - low level of integration
 - not compatible with TTL and CMOS
- ECL logic and interface technology is still used in very high-speed communications gear such as fiber optic transceiver interfaces and synchronous transfer mode networks (ATM)

* basic ECL current switch:



if $V_1 < V_2$

Q1 is off, V_{01} is high

Q2 is forward active, V_{02} is low

if $V_1 > V_2$

Q2 is off, V_{02} is high

Q1 is on, V_{01} is low

hence, V_{01} is the inverting output while V_{02} is the non-inverting if V_2 is fixed.

assuming Q1 and Q2 are forward active:

$$\rightarrow V_{01} = V_{CC} - I_{C1} \cdot R_{C1} \quad \wedge \quad V_{02} = V_{CC} - I_{C2} \cdot R_{C2}$$

$$\wedge I_E = I_{E1} + I_{E2}, \quad \therefore \alpha = \frac{\beta}{\beta + 1}, \quad \alpha_1 = \alpha_2 = \alpha$$

$$\rightarrow I_E = (I_{C1}/\alpha_1) + (I_{C2}/\alpha_2) \quad \rightarrow \quad \alpha I_E = I_{C1} + I_{C2}$$

recall the $I_C - V_{BE}$ Relation: $I_C = I_S \cdot (e^{V_{BE}/V_T} - 1) \approx I_S \cdot e^{V_{BE}/V_T}$

where I_S : [reverse] saturation current, V_T : thermal voltage: 26 mV

26 mV

$$\rightarrow I_{C1} = I_{S1} e^{V_{BE1}/V_T} \quad \wedge \quad I_{C2} = I_{S2} \cdot e^{V_{BE2}/V_T}$$

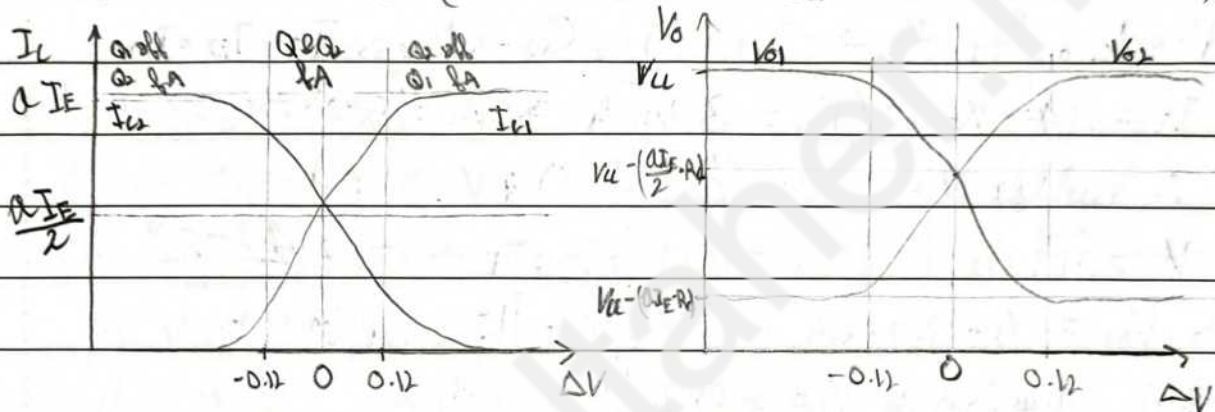
$$\therefore V_{BE1} - V_{BE2} = V_1 - V_2 = \Delta V$$

$$\wedge \quad \frac{I_{C1}}{I_{C2}} = \frac{I_{S1}}{I_{S2}} \cdot e^{(V_{BE1} - V_{BE2})/V_T} \quad \text{if } I_{S1} = I_{S2}$$

$$\rightarrow \frac{I_{C1}}{I_{C2}} = e^{(\Delta V)/V_T}$$

$$\therefore \alpha I_E = I_{C2} (1 + e^{\Delta V/V_T}) \quad \wedge \quad \alpha I_E = I_{C1} (1 + e^{-\Delta V/V_T})$$

$$\rightarrow I_{C1} = (\alpha I_E) / (1 + e^{-\Delta V/V_T}) \quad \wedge \quad I_{C2} = (\alpha I_E) / (1 + e^{\Delta V/V_T})$$



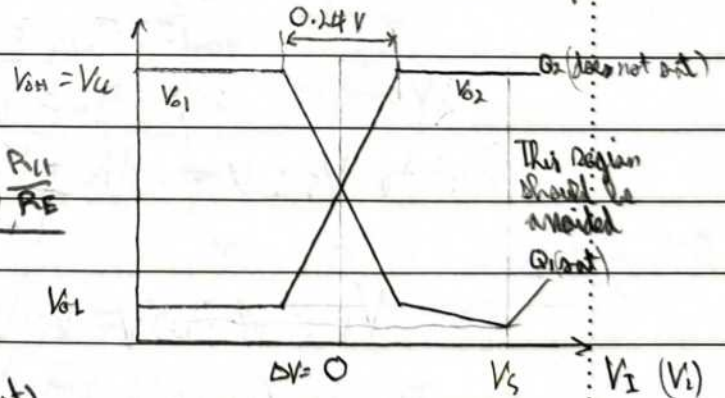
$$\lim_{\Delta V \rightarrow -\infty} (I_{C2}) = \alpha I_{E2}, \quad \text{if } \Delta V = 0.12, \quad e^{\frac{-\Delta V}{V_T}} = 1$$

- Hence, I_E is constant. as $I_{E1} \downarrow, I_{E2} \uparrow$ equally
- If the circuit shown below is to be operated as a logic element, then ΔV should always be smaller than $-0.12V$ or larger than $0.12V$ as the region where $-0.12 < \Delta V < 0.12$ has both Q_1 and Q_2 on and outputs will be indeterminate. (circuit will operate as a diff amp)

- assuming V_2 is fixed and V_1 is V_i

V_i : the voltage at which Q_2 saturates

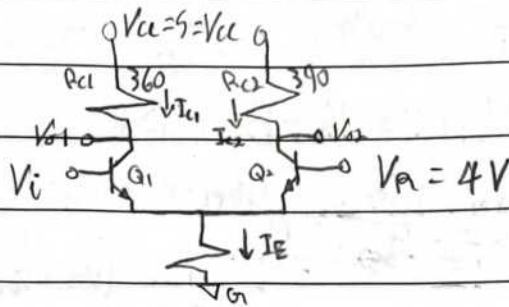
$$V_i = V_1 = \frac{V_{CC} + V_{CE(sat)} + (V_{BE(sat)} + V_{EE}) \frac{R_{C1}}{R_E}}{1 + \frac{R_{C1}}{R_E}}$$



$$V_{CE(sat)} = V_{BE(sat)} - V_{CE(sat)}$$

* Basic ECL inverter/buffer:

- Remember the 0.24V regions in which both transistors are active must be avoided.



→ $V_{IL} < 3.88$, $\text{max } V_{IL} = 3.88V$ $V_R - 0.12V$

→ $V_{IH} > 4.12$, $\text{min } V_{IH} = 4.12V$ $V_R + 0.12V$

- at $V_i = L$; Q_1 off → $I_{C1} = 0$, Q_2 active → $I_E = I_{C2} = I_{E2}$

$I_E = (4 - V_{BE(ON)}) / R_E = 3.3 \text{ mA} \rightarrow V_{O2} = 5 - 3.3 \cdot 390 = 3.913 = \square$

∴ buffer V_{O2} & $V_{EE} \approx 0.4V \rightarrow$ not sat

- at $V_i = H$; Q_2 off → $I_{C2} = 0$, $I_{C1} = I_{E1} = I_E \rightarrow I_E = \frac{4.2 - 0.9}{R_E} = 3.5 \text{ mA}$

→ $V_{O1} = 5 - 360 \cdot 3.5 = 3.94V = \square$, V_{O1} inverter

check active/sat: $V_{CE} = 0.24V \rightarrow$ active

- if $V_i = 5V = V_{CC}$:

assume Q_1 is on: $V_E = V_i - V_{BE(ON)} = 5 - 0.9 = 4.3V$

$I_E = I_{C1} = \frac{4.3}{R_E} = 4.3 \text{ mA}$, $V_{O1} = 5 - 0.36 \cdot 4.3 = 3.452V$

check saturation: $V_{CE} = V_{O1} - V_E = -0.848V$ sat. must redo calc.

recalculate for sat: $V_E = V_i - V_{BE(sat)} = 4.2V \rightarrow I_E = 4.2 \text{ mA}$

∴ $V_{O1} = V_E + V_{CE(sat)} = 4.4V$

→ $I_{C1} = \frac{5 - 4.4}{360} = 1.667 \text{ mA}$

∴ $I_B = 2.533 \text{ mA}$

calculate V_s : $V_i = \frac{V_{CC} + V_{BE(ON)} + (V_{BE(sat)} + V_{EE}) \frac{R_{C1}}{R_E}}{1 + \frac{R_{C1}}{R_E}}$

→ $V_i = 4.329V = V_s$

∴ max $V_{IH} = 4.329V$

+ For $V_A = -1V$, $V_{EE} = -5.2V$, $V_{CC} = 0V$

- if $V_i = L$, $V_i < -1.12V$:

Q_1 off, Q_2 on, $I_{C1} = 0$, $I_{C2} = I_E$

$$I_E = \frac{-1 - 0.9 - (-5.2)}{1k} = 3.5 \text{ mA}$$

$$V_{O2} = 0 - 3.5 \cdot 390 = -1.365V = [L]$$

check active: $V_{CE} = -1.365 + 1.7 = 0.335V \rightarrow$ active

- if $V_i = H$, $V_i > -0.88V$: $V_i = -0.8V$

Q_1 is on, Q_2 is off, $V_E = -1.5V \rightarrow I_E = I_{C1} = \frac{3.9}{1k} = 3.9 \text{ mA}$

$$\rightarrow V_{O1} = -1.332V = [L], \text{ check active: } V_{CE} = -1.332 - (-1.5) = 0.168V$$

$V_{CE} < 0.1$ not

- if $V_i = 0V = [H]$:

Q_2 off, Q_1 active $\rightarrow V_E = -0.9V \rightarrow I_E = I_{C1} = \frac{4.5}{1k} = 4.5 \text{ mA}$

$$\rightarrow V_{O1} = -1.62V = [A]$$

check if assumption that Q_1 is active is correct:

$$V_{CE} = -1.62 - (-0.9) = -0.72V$$

hence assumption is false and Q_1 is not

$$\rightarrow V_E = 0 - V_{BE}(\text{sat}) = [-0.8V], I_E = \frac{4.4}{1k} = 4.4 \text{ mA}$$

$$\rightarrow V_{O1} = 0.2 - 0.8 = -0.6V \rightarrow I_{C1} = \frac{0 + 0.6}{360} = 1.667 \text{ mA}$$

$$\therefore I_{B1} = 2.733 \text{ mA}$$

* Basic OR/NOR ECL gate:

- if V_1 or V_2 (or both) are

high then their respective

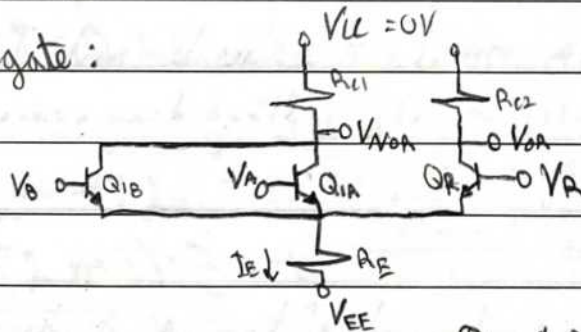
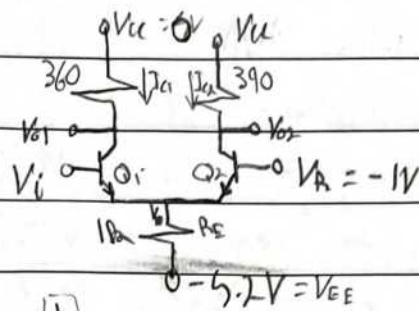
transistors will be forward

active while Q_A will be off

$$\rightarrow V_{NOR} = L \quad V_{OR} = H = V_{CC} = 0V$$

- if both V_1 & V_2 are low, Q_{1A} & Q_{1B} will be off, Q_A will be forward active

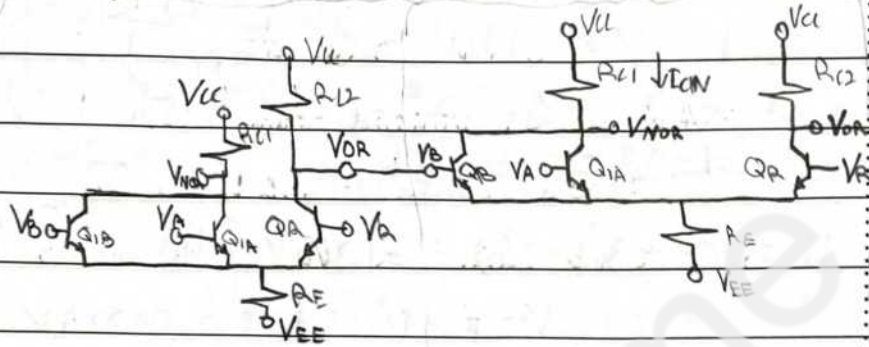
$$V_{OR} = [L], \quad V_{NOR} = [H] = 0V = V_{CC}$$



Q_{1A} & Q_{1B} are in parallel

* fan-out of ECL OR/NOR gate: $V_{CC} = 0V$

- for an ECL OR/NOR gate supplying the input of another OR/NOR gate from its NOR



Terminal:

driver

load

1) if $V_B = V_A = L \rightarrow V_{OR} = L$ next stage $V_B = L \rightarrow Q_{1B}$ off
hence an infinite number of gates can be driven

$$N_L \rightarrow \infty$$

2) if either V_A or V_B of the driver circuit is high (or both), Q_{1A} will be off and $V_{OR} = V_{CC} - I_{C2} \cdot R_{C2}$

- if V_A of the second stage is low $\rightarrow Q_{1A}$ is off, I_{C2} passes completely through Q_{1B} , which gives the output of the emitter of the load $Q_{2B} = I_{C2} + I_{C1}$, where I_{C2} is the total current I_{C2} of the driver divided by the N loads.

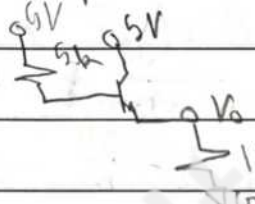
- if V_A is high, the current I_{C1} will be divided evenly among Q_{1B} and Q_{1A} , giving $I_{E2} = I_{C2} + \frac{I_{C1}N}{2}$

Q_{1B} must not saturate and I_{C2} must not get large enough to cause the voltage drop across R_{C2} to reduce V_{OR} from high to low.

to be continued after quiz practice and quiz

P2.2: $V_i = 0.1V \rightarrow V_{B1} = 0.9 \rightarrow I_B = 0.82 \text{ mA}$
 $V_{B2} = V_{C1} = 0.3V \rightarrow Q_2 \text{ off}, Q_3 \text{ off} \rightarrow I_{C3} = 0 = I_{C2}$
 $V_i = 4V \rightarrow Q_1 \text{ reverse active}, Q_2 \text{ sat} \rightarrow V_{B2} = 0.8V$
 $\rightarrow V_{B1} = 0.8 + V_{CE(sat)} = 1.4V \rightarrow I_1 = 0.92 \text{ mA}$
 $Q_3 \text{ sat} \rightarrow \text{well} \rightarrow V_{B3} = 0.8V \rightarrow V_{B2} = 1.6V$
 $\rightarrow V_{B1} = 2.2V \rightarrow I_1 = 0.56 \text{ mA}$
 $Q_2 \text{ sat} \rightarrow V_{C2} = 0.2 + 0.8 = 1V \rightarrow I_2 = 1.6 \text{ mA}$
 $V_0 = 0.2V \rightarrow I_3 = 2.4 \text{ mA}$
 total power supply current = 4.56 mA

Q2:



$-91 + 5k \cdot I_B + 0.7 + I_E \cdot 1k = 0$
 $I_E = (1 + \beta) \cdot I_B$ and $\beta = 60 \rightarrow I_B(5k + 1.61 \cdot 1k) = 4.3$
 $\rightarrow I_B = 76.78 \mu A$
 $\rightarrow V_0 = 76.78 \cdot 91 \cdot 1k = 3.9161 \text{ not sat}$
 $= 3.92V$

Q3: $Q_1 \text{ Rev active}, Q_2 \text{ sat}, Q_3 \text{ sat} \rightarrow V_{B3} = 0.8V, V_{B2} = 1.6V$
 $V_{B1} = 2.2V$
 $Q_3 \text{ sat} \rightarrow V_0 = 0.2V$

3 gates connected $Q_{IN} \text{ sat} \rightarrow V_{B1N} = 0.9V \times 1V$
 $\rightarrow I_{E1N} = \frac{4.1}{5k} = 0.82 \text{ mA} \times 0.8 \text{ mA}$
 $\rightarrow I_{C1} = (0.82 \text{ mA}) \cdot 3 + \frac{5-0.2}{2k} = 4.86 \text{ mA}$
 $(0.8 \text{ mA}) \cdot 3 + \frac{1}{1} = 4.8 \text{ mA}$

Q4: $V_0 = 0.2V \times$

$$I_{C1} = \frac{5-0.2}{2k} + 2 \left(\frac{5-(0.2+0.8)}{4k} \right) \times \times$$

Q_1 sat ∞ input low $\rightarrow Q_2$ off $\rightarrow V_0 = V_{CC}$

V_0 input to two gates

Q_1 in reverse active $\rightarrow Q_2$ in sat $\rightarrow V_{BE1} = 1.4V$

$$\rightarrow I_{B1N} = 0.9 \text{ mA} \quad \beta_{RC1} = 0.2$$

$$\rightarrow I_{E1N} = 0.18 \text{ mA}$$

$$\rightarrow I_{C1N} = I_{C1} = 0.18 \cdot 2 = 0.36$$

$$\therefore V_0 = 5 - 0.36 \cdot 2k = \boxed{4.28V}$$

* H. \rightarrow L input

$$I_{E1} = \frac{5-0.9}{4k} + I_{C1} = \frac{5-0.9}{4k} + 91 \cdot \frac{5-0.9}{4k} = 53.33 \text{ mA}$$

$$I_{C1} = (\beta_F + 1) I_B \quad \wedge \quad I_B = \frac{5-0.9}{4k}$$

$$I_{B1} = \frac{V_{CC} - V_{BE1}}{R_1} \quad \wedge \quad V_{BE1} = 0.2 + 0.9 + 0.8 = 1.9$$

$$\rightarrow I_{B1} = 3.3 / 16k = 206.25 \mu\text{A}$$

$$I_{C1} = \frac{V_{CC} - V_C}{130} \quad V_C = 0.2 + 0.9 + 0.2 = 1.1$$

$$\rightarrow I_{C1} = \frac{3.9}{130} = 0.30 \text{ mA}$$

$$\rightarrow I_{E1} \text{ charging} = 30 + 206.25 = 32.0625$$

Q5: Q_1 forward $\rightarrow V_{BE1} = 0.9 \quad I_{B1} = \frac{4.1}{4k} = 1.025 \text{ mA}$

$$\rightarrow I_{E1} = (\beta + 1) \cdot I_{B1} = 57.4 \text{ mA}$$

$$\rightarrow I_{C1} = I_{B2} = \beta \cdot I_{B1} = 56.375 \text{ mA output base}$$

P3: $V_A = 4V \rightarrow Q_1$ in rev. active, Q_3 & Q_2 sat

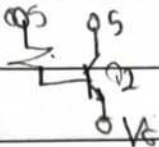
$$\rightarrow V_{BE2} = 1.6V \rightarrow V_{BE1} = 2.2V \rightarrow I_{E1} = 0.56 \text{ mA}$$

$$\rightarrow I_{C1} = 0.56 \cdot (1 + 0.1) = 0.616 \text{ mA}$$

$$\infty Q_2 \text{ sat} \rightarrow I_{C2} = \frac{5-1}{2k} = 1.6 \text{ mA}$$

$$\rightarrow I_{E2} = 2.216 \text{ mA}$$

$$I_{B3} = I_{E2} - 0.8 / 1k = 1.416$$

P4:  Q_2 sat \rightarrow I no current since $R_2 = \infty$
 $V_{BE(sat)} = 0.7V \rightarrow V_o = 5 - 0.6 = 4.4V$

P5: Q_2 cut off $\rightarrow Q_1$ in active $\rightarrow I_{E1} = \frac{3.6}{1k} \rightarrow I_{E1} = 0.18mA$
 $\therefore I_{C1} = 0.18mA \quad V_o = 4.64V$

Quiz 2:

1) Q_2 sat $\rightarrow V_o = 0.2V$

$$\beta I_B \sigma = I_C$$

$$I_B = \left(\frac{5 - 1.4}{1k} \right) (1 + 0.11) = 0.999mA$$

$$I_C = \frac{4.8}{2k} + 8.1mA = 10.4$$

$$10.4 = 0.999 \cdot 42 \cdot \sigma \rightarrow \sigma = 0.24986$$

2) $V_{B1} = 2.2 \rightarrow I_{B1} = 0.56mA$

$$V_o = 0.2mA$$

$$0.8mA \cdot 5 = 4mA$$

$$= 6.4$$

3) Q_2 & Q_3 sat $\rightarrow V_o = 0.2V$

$$12 \cdot 1 = 12mA$$

$$\frac{5 - 1.71}{1.6k} =$$

4) $5 \cdot 1 = 5mW$ Q_1 P

$$\frac{5 - 1.4}{1k} =$$

$$= 0.9 \times 3$$

$$0.18 = P_{D(sat)}$$

$$I_{E1} = I_{B1} \cdot \beta_{D(sat)} = 0.162mA$$

If we calculate

I_{B1}

$$\therefore I_{C2} = 0.162 \cdot 3 = 0.486mA$$

power consumed

$$\therefore Q_2 P = 5 \cdot 0.486 = 2.43mW$$

for a single circuit

$$\rightarrow P_{total} = 7.43mW$$

* modified OR/NOR ECL gate: MEELI

Q3 and Q4 are always forward active

If Q3 is forward active

→ Q3 & Q4 off → $V_A = V_B = L$ $V_{NOR} = H$

If Q4 is off, Q3 or Q4 or both

are forward active $V_{OR} = H$

1) If V_A and V_B are low:

$$0 + 270 \cdot I_{B4} + 0.7 + I_{E4} \cdot 2k = -5.2 = 0 \quad \wedge \quad I_{E4} = (\beta + 1)I_{B4}$$

$$\rightarrow I_{B4} [270 + (\beta + 1) \cdot 2k] = 5.2 - 0.7$$

$$\text{If } \beta = 100 \rightarrow I_{B4} = 22.25 \mu A \rightarrow V_{NOR} \approx 0.7V = H$$

$$\wedge I_{B3} - I_{B4} = I_E = \frac{-1.4 - -5.2}{1.2k} = 2.82258$$

$$\wedge 0 + I_{B3} \cdot 300 + 0.7 + I_{E4} \cdot 2k - 5.2 = 0$$

$$\rightarrow (2.82258 + I_{B3}) \cdot 300 + 0.7 - 5.2 + (\beta + 1)I_{B4} \cdot 2k = 0$$

$$\rightarrow I_{B3} (300 + (\beta + 1)2k) = 3.653226, \beta = 100$$

$$\rightarrow I_{B3} = 18.098 \mu A \rightarrow V_{OR} = -1.5922V = L$$

2) If V_A and/or V_B are high:

$$V_A = V_B = H = -0.7V \rightarrow V_E = -1.4 \rightarrow I_E = I_{E3} + I_{E4} = \frac{-1.4 - -5.2}{1.2k} = 3.0645 \mu A$$

$$\rightarrow V_{NOR} = -3.0645 \mu A \cdot 270 - 0.7 = -1.527V = L = V_{NOR}$$

$$\because Q3 \text{ off } \wedge V_{E3} \text{ negligible} \rightarrow V_{OR} = -0.7V = H$$

$$\because V_A = -1 \rightarrow V_{IH} = -0.88 \wedge V_{IL} = -1.12$$

$$\wedge \text{NOR: } V_{OH} = -0.7V \wedge V_{OL} = -1.528V$$

$$\text{OR: } V_{OH} = -0.7V \wedge V_{OL} = -1.59V$$

$$\therefore \text{noise margins: OR: } NM_H = 0.18V \wedge NM_L = 0.43$$

$$\text{NOR: } NM_H = 0.18V \wedge NM_L = 0.408$$

* modified OR/NOR ECL gate: MELLT fan-out

if V_A & V_B are high, OR output will be high and OR off

$$I_{E4} = I_{RE4} + N \cdot I_{load} \quad \text{assume } Q_{1A} = \text{inactive}$$

assuming $V_{OR} = -0.75V$ $\therefore \uparrow I_{E11} = I_{REF} = I_{load} + I_{C1}$

$$\uparrow +0.75 + 0.7 + 1.4k \cdot I_{E1} - 5.2 = 0$$

$$\rightarrow I_{E1} = 3.0242 \text{ mA} = (\beta + 1) \cdot I_B$$

$$\therefore I_B = I_{E1} \wedge \beta = 100 \rightarrow I_B = 29.9426 \mu A$$

$$\therefore I_{E4} = (\beta + 1) I_{B4} = I_{REF} + N I_B$$

$$\uparrow I_{REF} = \frac{-0.75 - (-5.2)}{2k} = 2.25 \text{ mA} \quad \uparrow I_{D1} \cdot 300 + 0.7 - 0.75 = 0 \rightarrow I_{D1} = 0.1667 \text{ mA}$$

$$\therefore (\beta + 1) \cdot 0.1667 \text{ mA} = 2.25 \text{ mA} + N \cdot 29.943 \mu A$$

$$\rightarrow N = \frac{14.608}{0.0299} = 488 \quad \text{fan-out for high output logic}$$

* modified 2:

$$V_A = 0.7 + 0.9 + I_{A2} \cdot 2.3k - 5.2$$

$$\uparrow V_B = 0.7 + I_{E5} \cdot 2k \quad I_{E5} = \beta I_{B5} \quad \text{assume } I_{B5} = 0 \text{ negligible}$$

$$\rightarrow 0.9 + I_{A2} \cdot 2.3k - 5.2 = (\beta + 1) I_{B5} = (\beta + 1) (I_{A1} - I_{A2})$$

$$\uparrow I_{A1} \cdot 300 + 0.7 + 0.9 + I_{A2} \cdot 2.3k - 5.2 = 0$$

$$\rightarrow I_{A1} = \frac{5.2 - 1.4 - I_{A2} \cdot 2.3k}{300}$$

$$\rightarrow -4.4 + I_{A2} \cdot 2.3k = 101 \cdot \left(\frac{3.8}{300} - I_{A2} \left(\frac{2.3k}{2.3k} + 1 \right) \right)$$

$$\rightarrow I_{A2} \cdot 2.3k = 4.4 + 1.29933 - 895.333 \cdot I_{A2}$$

$$\rightarrow I_{A2} (2300 + 895.333) = 4.4 + 1.299333$$

$$\rightarrow I_{A2} = 1.82 \text{ mA} \rightarrow I_{A1} = -$$

$$\rightarrow 0 + 300 I_{A1} + 0.7 + 0.9 + 2.3k I_{A2} = 5.2, \quad I_{A1} = I_{A2}$$

$$\rightarrow I_{A1} = 1.462 \text{ mA}$$

$$\uparrow V_B = -0.4374 \quad \uparrow V_A = V_B - V_{BE(ON)} = -1.1374 \approx -1.14V$$

Quiz 1 Redone:

$$1) NM_H = 3V \Rightarrow V_{OH} - V_{IH} = 3V$$

$$NM_L = V_{IL} - V_{OL} \quad \because V_{OH} = V_{CC} = 5V$$

$$\rightarrow V_{IH} = 5 - 3 = 2V \quad \therefore -2 + I_B R_B + 0.8 = 0$$

$$\wedge -5 + I_C R_C + 0.2 = 0$$

$$\rightarrow I_C = \frac{4.8}{2k} = 2.4 \text{ mA}$$

$$\text{assume EOS: } \sigma I_B \beta = I_C, \sigma = 1 \rightarrow I_C = \beta I_B$$

$$\rightarrow I_B = 24.8 \mu\text{A} \quad \rightarrow R_B = 48.3$$

$$2) \sigma I_B \beta = I_C \rightarrow \sigma = \frac{I_C}{I_B \beta}, \quad I_C = 0 \text{ if } V_O = H$$

$$\rightarrow I_C = \frac{5 - 0.2}{4k} = 1.2 \text{ mA}$$

$$\wedge I_B = I_{D1} - \frac{0.8}{5k}$$

$$\wedge I_{D1} = I_{A1} \quad \because V_A \wedge V_B \text{ high}$$

$$V_O = 0.8 + 0.7 + 0.7 = 2.2V \rightarrow I_{D1} = 1.306153 \text{ mA}$$

$$\rightarrow I_B = 1.186153 \text{ mA} \rightarrow \sigma = 0.019516$$

$$Q3: \sigma = 0.9 = \frac{I_C}{I_B \beta} \quad \wedge I_C = \frac{5 - 0.2}{1k} = 4.8 \text{ mA}$$

$$\rightarrow I_B = 0.106667 \text{ mA} \quad \wedge I_B = \frac{V_O - 0.8}{1.5k} \rightarrow V_O = 2.19733V$$

$$Q4: I_C = 0 \text{ if } V_A \wedge V_B \text{ high}$$

$$\rightarrow I_C = \frac{V_{CC} - V_O}{4k} + \frac{5 - 0.9}{2.2k} = \frac{4.8}{4k} + \frac{4.1}{2.2k} \quad \text{since other input is } 2$$

$$Q5: Q1 \text{ cutoff, assume gates are sat } \rightarrow$$

$$-V_{CC} + R_C \cdot N I_B + V_O = 0$$

$$\wedge -V_O + I_B \cdot R_B + 0.8 = 0 \rightarrow V_O = I_B \cdot R_B + 0.8$$

$$\rightarrow -V_{CC} + I_B (N \cdot R_C + R_B) + 0.8 = 0$$

$$\rightarrow I_B = \frac{5 - 0.8}{23.4k} = 223.4 \mu\text{A}$$

$$\rightarrow V_O = 3.66V$$

Ques 5/17-18:

P1: Assume EOS, $V_{IH} \rightarrow V_{O1}$

$$V_{O1} = 0.2V \rightarrow I_{A1} = \frac{5-0.2}{1k} = 4.8mA$$

inputs tied together $\rightarrow \frac{4.8mA}{2} = I_C = 2.4mA$

EOS $\rightarrow \sigma \cdot B \cdot I_B = I_C$ & $\sigma = 1 \rightarrow I_B = \frac{2.4}{90} = 0.12mA$

$$\therefore V_{IH} = 0.8 + 0.12mA \cdot 15k = 2.6V$$

2) for high logic:

$$-5 + N I_B \cdot 1k + I_B \cdot 15k + 0.8 = 0$$

$$\therefore V_{O \text{ min } H} = V_{IH} \rightarrow V_O = 5 - 1k \cdot N I_B = 2.6V$$

noise margin $\rightarrow V_{OH} < V_{IH} < V_{OL}$

$$\rightarrow N \cdot I_B = 2.4mA$$

$$\therefore -5 + 2.4 + I_B \cdot 15k + 0.8 = 0 \rightarrow I_B = 0.12$$

$$\rightarrow N = 20$$

P2) 1) $V_A = V_B = 0.1V \rightarrow V_O = 0.8V \rightarrow I_{A1} = \frac{4.2}{4k} = 1.05mA$

$$\rightarrow I_{A2} = 0.625mA \quad \text{and } I_{B2} = 0 \quad \text{EOS}$$

2) $0.8 \cdot B \cdot I_B = I_C$, $V_O = 0.2V \rightarrow I_C = 1.2mA$

$$V_{O1} = 2.3V \rightarrow I_{E1} = \frac{5-2.3}{4k} + \frac{5-1.7}{2k} = 2.325mA$$

$$\rightarrow I_{B2} = 2.325 - \frac{0.8}{1k} = 2.245mA$$

$$\therefore 0.8 \cdot 2.245mA \cdot B = 1.2mA \rightarrow B = 0.6682$$

3) if $V_A = V_B = 5V \rightarrow P_d = 5(0.675 + 1.65 + 1.2)m = 17.625mW$

4) for a low output: $I_C = N \cdot \left(\frac{5-0.9}{4k} \right) + \frac{5-0.2}{4k}$

EOS $I_C = B I_B = 120 \cdot 2.245mA$

$$\rightarrow N = 42$$

P3: 1) Q_1 reverse active, $Q_3 = Q_2 = \text{sat} \rightarrow V_{B2} = 1.6V$

$$\therefore I_{B1} = \frac{5 - 1.6 - 0.6}{6k} = 0.46667 \text{ mA} \rightarrow I_{C1} = I_{E2} = 0.5333 \text{ mA}$$

$$\wedge I_{B3} = I_{E2} - \frac{0.8}{1.5k} \quad \wedge I_{E2} = 0.5333 \text{ mA} + \frac{4}{2k}$$

$$\rightarrow I_{B3} = 1.97999 \text{ mA}$$

2) Q_1 off, $V_{B1} = 0.3V$, Q_2 sat $\rightarrow V_{B1} = 1.1V \rightarrow$

Q_3 off, Q_4 forward active:

$$-5 + I_{B4} \cdot 2k + 0.9 + 0.9 + (1+\beta)I_{B4} \cdot 1k = 0$$

$$\rightarrow I_{B4} = 0.1666 \text{ mA} \rightarrow V_{O} = 3.289 \text{ V}$$

3) $-5 + I_{B4} \cdot 2k + 1.4 + 10I_{B4} \cdot 1k = 0$

$$\rightarrow I_{B4} = \frac{3.6}{12k} = 0.2916667 \text{ mA}$$

$$\rightarrow V_{O} = 2.916667 \text{ V} \rightarrow V_{E4} = 3.616667 \text{ V}$$

$$\wedge V_{C} = 5 - \beta \cdot I_{B} \cdot 80 = 4.79 \text{ V}$$

$$\rightarrow V_{CE} = 1.17$$

a) low-output: $I_{B3} \approx 2 \text{ mA}$

$$\text{EOS} \rightarrow 1 \cdot 2 \text{ mA} = I_{C} \quad \wedge I_{C} = \beta \cdot I_{E1}$$

$$\wedge I_{E1} = \frac{5 - 0.9}{6k} \rightarrow \beta = 26$$

* ECL positive:

$$P1: V_A = 0$$

$$a) V_i = 0 \rightarrow I_E = \frac{3.3}{450} = 7.33 \text{ mA}$$

$$\rightarrow I_{E1} = I_{E2} = \frac{7.33}{2} = 3.6667 \text{ mA}$$

$$\rightarrow I_{C1} = I_{C2}$$

b) $V_i = -0.12$ $\because V_i < V_A \rightarrow Q_1$ off, $V_{B1} = 2V$

$$Q_2 \text{ on} \rightarrow I_{E2} = 7.333 \text{ mA} \quad \wedge I_{C2} = 7.26 \text{ mA}$$

mid practice:

$$2) V_A = \frac{V_{OL} + V_{OH}}{2} \quad Q_0 \text{ fA}$$

$$V_0 = 0 + I_{OH} \cdot 0.3k + 0.7 + I_{EO} \cdot 2k - 5 = 0$$

$$\Rightarrow V_{OH} = -0.7V$$

$$\wedge V_{OL} \rightarrow V_x \text{ or } V_y \text{ on} \rightarrow I_C = \frac{(V_A + 0.12) - 0.7 - -5}{1.24k}$$

$$\Rightarrow V_{OL} = 0.3k \cdot \frac{4.3 + V_A + 0.12}{1.24k} + 0.7 - 5$$

$$\text{or assume } V_x = 0 \Rightarrow I_C = \frac{-0.7 - -5}{1.24k} = 3.468 \text{ mA}$$

$$\Rightarrow V_{OL} = -1.7403 \Rightarrow V_A = -1.22 \text{ V}$$

$$I_E = +2.903 \text{ mA} = I_C \Rightarrow V_{OL} = -1.5709$$

$$\Rightarrow V_A = -1.135V$$

$$4) V_{IH} = 4.79 + 0.12 = 4.89V \quad \wedge V_{OH} = V_{OL} = 4V$$

$$\Rightarrow NM_H = 0.13$$

$$b) V_x = V_y = H, \text{ Q.A. off} \Rightarrow I_E = \frac{5 - 0.85 - 0.7}{1.24k}$$

$$\Rightarrow I_E = 2.78225 \text{ mA} \Rightarrow I_{C1} = I_{C2} = \frac{I_E}{2} = 1.39 \text{ mA}$$

$$12) Q_x \text{ is active, } I_E = \frac{5 - 0.8}{1.3k} = 3.23077 \text{ mA}$$

$$\Rightarrow V_0 = 5 - 3.23077 \cdot 0.3 = 4.031V$$

$$NM_L = V_{IL} - V_{OL}$$

$$NM_H = V_{OH} - V_{IH}$$

$$2) V_{OH} = 0, V_{OL} = -1.7403 \Rightarrow V_A =$$

$$1) Q_3 \text{ off}, Q_1 \text{ off} \quad -5 + I_{B2} \cdot 5k + 0.7 + (1+\beta)I_{B2} \cdot 1k = 0$$

$$\rightarrow I_{B2} = \frac{5 - 0.7}{(1+\beta)1k + 5k} = 91.6667 \mu A$$

$$\rightarrow V_o = 3.94$$

$$2) V_A = V_B = H \rightarrow Q_1 \text{ sat} \wedge Q_2 \text{ sat}$$

$$\rightarrow V_{B2} = 2.3 \rightarrow I_{B1} = \frac{5 - 2.3}{38k} = \frac{2.7}{38k}$$

$$I_{B2} = I_{E1} - \frac{0.8}{2k}$$

$$I_{E1} = \frac{2.7}{38k} + \frac{3.3}{2k} \rightarrow I_{B2} = 1.32$$

$$3) V_{BE(on)} = 0.7$$

Q_x off

$$5 - 0.3 \cdot I_C = V_o$$

$$I_E = I_B + I_C$$

$$I_E = (1+\beta)I_B$$

$$-5 + \beta I_B \cdot 0.3 + 0.7 + I_B(1+\beta) \cdot 1.3 = 0$$

$$\rightarrow I_B = \frac{4.3}{\beta \cdot 0.3 + (1+\beta) \cdot 1.3} = 96.11 \mu A$$

$$V_o = 5 - 15 \cdot I_B \cdot 0.3 = 4.6575$$

A) Q_1 off - Q_3 off:

$$Q_1 \wedge Q_3 \text{ sat} \rightarrow -V_i + I_{B1} \cdot 4k + 0.8 = 0$$

$$\wedge -V_i + I_{B2} \cdot 10k + 0.8 = 0$$

$$-5 + 5k \cdot I_{B2} + 0.7 = V_i$$

$$I_{B3} + I_{B1} = I_{B2}(1+\beta)$$

$$6) V_A = V_B = 4V \rightarrow Q_1 \text{ is } \text{Res. act.}$$

$$Q_2 \text{ act } Q_3 \text{ act}$$

$$V_C = L$$

or

$$7) V_{IH} = 3.6$$

$$V_{IH} \Rightarrow -V_{IH} + I_B \cdot R_B + 0.8 = 0$$

$$V_{IH} = I_B \cdot R_B - 0.8$$

$$\text{EOS} = I_B \cdot R_B = I_C \quad \wedge \quad I_C = \frac{1}{2} \frac{3.6 - 0.2}{4.4k}$$

$$\rightarrow I_B = 79 \mu\text{A} \rightarrow V_{IH} = 0.8489$$

$$8) V_X = V_Y \text{ active}$$

$$I_E = -0.96 - 0.9 + 9 / 1.24k = 2.859 \text{ mA}$$

$$\wedge I_{EY} = \frac{1}{2} I_E = 1.429$$

$$9) Q_1 \text{ Res. active}$$

$$I_C = \frac{V_C - 0.2}{2k} + 7 \cdot 1 \text{ mA} = 1.4 \text{ mA}$$

$$10) V_A = V_B = H$$

$$V_P = 2.2V \rightarrow I_{O1} = 1.34 \text{ mA}$$

$$\rightarrow I_B = 1.18 \text{ mA}$$

$$\sigma = 59.1 \cdot 1.18 = I_C, \quad I_C = \frac{9 - 0.2}{4k} = 1.2 \text{ mA}$$

$$\rightarrow \sigma = 0.01721$$

$$11) \frac{P_H + P_L}{2}, \quad P_H = 0$$

$$P_L = 3.6 \cdot \frac{3.4}{4.49} = 26.78 \text{ mW}$$

12) put de base

$$13) \quad 0.9 \quad \frac{0.8V}{\frac{5-0.4}{5k}} \cdot 53 = 43.46 \text{ out}$$

$$14) \quad \sigma = \frac{I_C}{I_B}$$

$$-4.1 + I_{B1} \cdot R_B + 0.8 + 0.2 = 0 \rightarrow I_{B1} = 0.2541 \text{ mA}$$

$$V_B = 0.4 \rightarrow I_{C1} = \frac{5-0.4}{1k} = 4.6 \text{ mA}$$

$$\rightarrow \sigma = 0.362$$

$$15) \quad V_A \text{ high} \rightarrow V_B = L \quad I_C = \frac{4}{5k} \cdot 6 = 4.8 \text{ mA}$$

$$\wedge I_{B3} = I_{E2} - \frac{0.8}{1k} \quad \wedge I_{E2} = \frac{4}{2.5k} + (1+\beta_{E2}) \cdot 0.96$$

$$\rightarrow I_{E2} = 2.216 \text{ mA} \rightarrow I_{B3} = 1.416 \text{ mA}$$

$$\rightarrow \sigma =$$

$$I_C = 4.8 + \frac{5-0.2}{2k} = 7.2 \text{ mA}$$

$$V_B = L = 0.2 \rightarrow I_C = \frac{5-0.2}{2k} + 6 \cdot \frac{4}{5} = 7.2 \text{ mA}$$

$$\wedge I_B = I_{E2} - 0.8 \text{ mA} \quad I_{E2} = \frac{4}{2.5} + \frac{2.8}{5} \cdot 1.1$$

$$\rightarrow I_{E2} = 2.216 \rightarrow I_{B3} = 1.416 \text{ mA}$$

$$\sigma =$$

$$\frac{4}{5k} \cdot 5 + \frac{5-0.2}{2k} = 6.4 \text{ mA}$$

$$I_{B2} = 1.1 \cdot \frac{5-2.2}{5k} = 0.616$$

$$0.616 + \frac{5-1}{2.5k} = 2.216 \text{ mA} \rightarrow I_{B2} = 1.416$$

$$\rightarrow \sigma = \frac{6.4}{50 \cdot 1.416}$$

* MOSFET: N-channel or P-channel

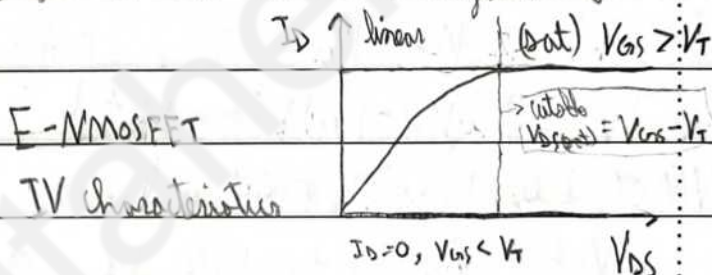
- subcategory: enhancement or depletion (simplified to single letter)

→ NMOS: E-NMOS & D-NMOS | PMOS: E-PMOS & D-PMOS

- depletion type (D-NMOSFET) is normally on, whereas enhancement mode (E-NMOSFET) is normally off

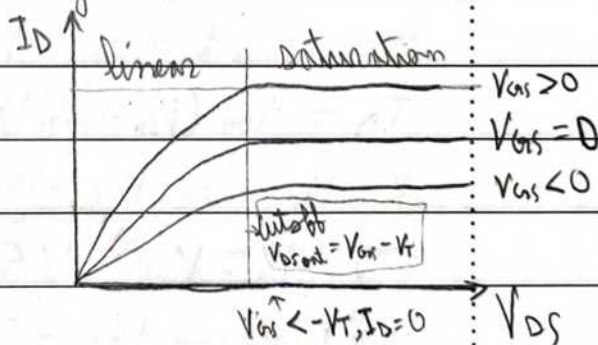
- depletion type has a built in channel whereas enhancement type must create it

- the V_{GS} required to create the channel is called the threshold voltage (V_T) and is positive for E-NMOS and negative for D-NMOS.



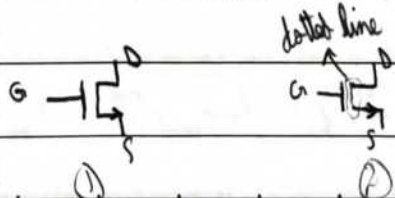
- for D-NMOS, the required V_{GS} to deplete the channel is the threshold voltage (V_T) and is negative

- an oxide separates the gate from the β -substrate (which separates the source and drain). The thickness of this oxide layer is (t_{ox}).

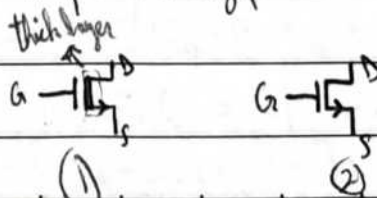


* symbols for enhancement type NMOSFET, and corresponding symbols for depletion type

enhancement type:



depletion type



either ① or ② will be consistently used.

* three modes of operation of the NMOS:

1- cutoff:

$V_{GS} < V_{TN}$, $I_D = 0$ for enhancement : $V_{GS} > 0$

2- saturation:

$V_{GS} > V_{TN}$

V_{TN} : threshold voltage

$V_{DS} > V_{GS} - V_{TN}$

β_n : channel conduction parameter, $\beta_n = \frac{W}{L} \cdot \frac{\mu_n C_{ox}}{2}$

$I_D = \beta_n (V_{GS} - V_{TN})^2$

W & L : channel length and width / C_{ox} : capacitance of oxide per unit area

3- ohmic (linear):

$V_{GS} > V_{TN}$

The minimum parameters of a MOSFET are V_T & β_n

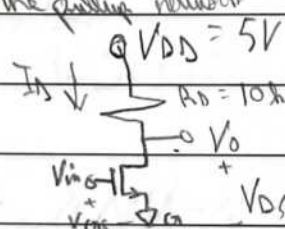
$V_{DS} < V_{GS} - V_{TN}$

$I_D = \beta_n [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$

R_D is the pulling network

* NMOS inverter with resistive load:

$V_{GS} = V_{in}$ and $V_o = V_{DD} - I_D \cdot R_D$



+ if $V_{GS} < V_{TN}$: $I_D = 0$ and $V_o = V_{DD} = 5V$

+ if $V_{in} > V_{TN}$:

$V_{TN} = 1V$, $\beta_n = 100 \mu A/V^2$

- assume sat then check assumption:

$R_D \cdot \beta_n = 1 V^2$

$I_D = \beta_n (V_{in} - V_{TN})^2$

$V_o = V_{DS} = V_{DD} - [\beta_n (V_{in} - V_{TN})^2] \cdot R_D > V_{in} - V_{TN}$ (for sat)

$\rightarrow V_o = V_{DS} = 5 - (V_{in} - 1)^2 > V_{in} - 1$

solving $V_{DD} - \beta_n R_D [V_{GS} - V_{TN}]^2 = V_{GS} - V_{TN}$

for $V_{GS} = V_{in} = V_s$

\rightarrow

$$V_s = V_{TN} - \frac{1}{2\beta_n R_D} + \sqrt{\left(\frac{1}{2\beta_n R_D}\right)^2 + \frac{V_{DD}}{\beta_n R_D}}$$

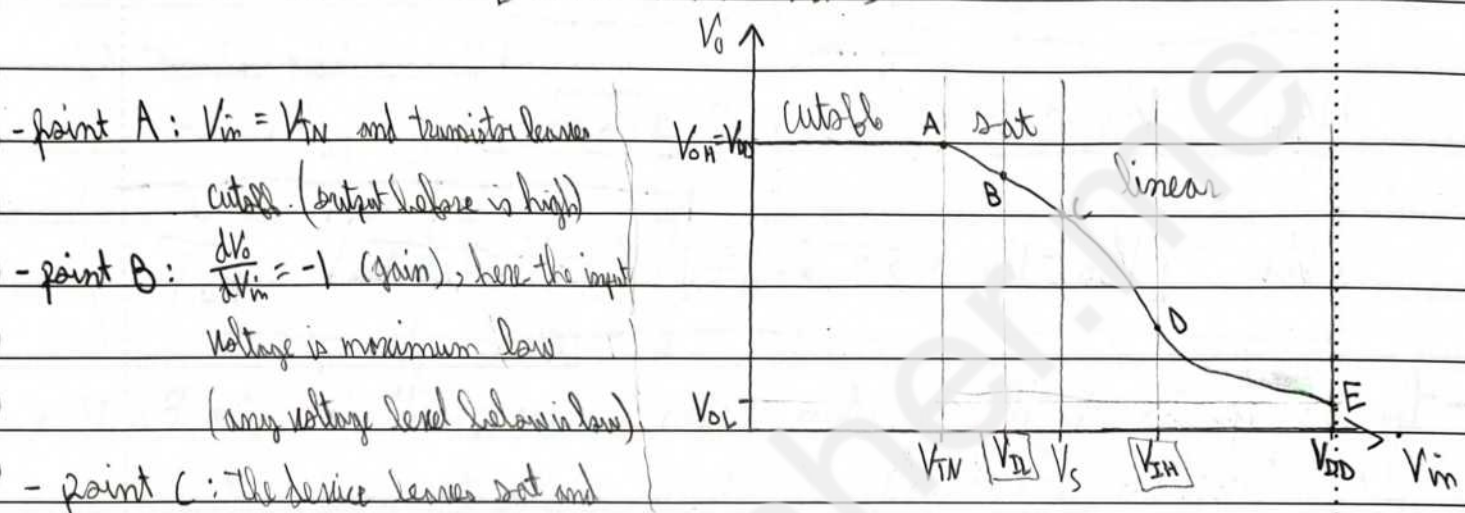
$\rightarrow V_s = 2.79V$

- Assume linear:

$$I_D = \beta_n [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

$$\rightarrow V_o = V_{DD} - R_D \beta_n [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

$$\therefore V_o = 5 - [2(V_{GS} - 1)V_{GS} - V_{GS}^2]$$



- point A: $V_{in} = V_{TN}$ and transistor leaves cutoff (output voltage is high)

- point B: $\frac{dV_o}{dV_{in}} = -1$ (gain), here the input voltage is maximum low (any voltage level below low)

- point C: The device leaves sat and enters linear mode

- point D: $\frac{dV_o}{dV_{in}} = -1$, voltages larger than V_{in} at this point will be high

- point E: the output at this point is low, $V_{input} = V_{OD} = V_{OH}$

+ to calculate input voltage low (V_{IL}):

$$\frac{dV_o}{dV_{in}} = -1 = -2R_D \beta_n (V_{in} - V_T)$$

$$\rightarrow V_{in} = \frac{1}{2R_D \beta_n} + V_{TN}$$

$$V_o = V_{DD} - \frac{1}{4R_D \beta_n}$$

+ to calculate input voltage high (V_{IH}):

$$V_o \text{ at D: } V_o = \sqrt{\frac{V_{DD}}{3A_D \beta_n}}$$

$$\rightarrow V_{IH} = \frac{2}{\sqrt{3}} \sqrt{V_{DD} \cdot R_D \beta_n} + V_{TN} \cdot R_D \beta_n - \frac{1}{2}$$

+ to calculate output voltage low (V_{OL}):

$$V_{OL} = \left[(V_{DD} - V_{TN}) + \frac{1}{2R_D \beta_n} \right] \pm \sqrt{\left[(V_{DD} - V_{TN}) + \frac{1}{2R_D \beta_n} \right]^2 - \frac{V_{DD}}{2R_D \beta_n}}$$

∴ for a resistive load inverter:

$$V_{OH} = V_{DD}$$

$$V_{IL} = V_{TN} + \frac{1}{2 R_{o1} k_n}$$

$$V_{IH} = V_{TN} + 2 \sqrt{\frac{V_{DD}}{3 R_{o1} k_n} - \frac{1}{2 R_{o1} k_n}}$$

$$V_{OL} = \left[V_{DD} - V_{TN} + \frac{1}{2 R_{o1} k_n} \right] \pm \sqrt{\left[V_{DD} - V_{TN} + \frac{1}{2 R_{o1} k_n} \right]^2 - \frac{V_{DD}}{R_{o1} k_n}}$$

- the sharpness of the transition region increases with increasing R_o .
(as $R_o \cdot k_n$ increases)

- the minimum output voltage (the logic zero level) for which input decreases with increasing R_o .
(V_{OL} decreases as $R_o \cdot k_n$ increases)

- hence the inverter approaches ideal as R_o increases.
(limit $R_o \cdot k_n \rightarrow \infty$)

- power dissipation: $P_d = \frac{V_{DD}}{2} [I_D(V_{in} = V_{OL}) + I_D(V_{in} = V_{OH})]$

for inverter with resistive load: $P_d = \left[\frac{V_{DD}}{2} \right] \cdot \left[\frac{V_{DD} - V_{OL}}{R_D} \right]$

* steps to finding:

1- check V_{OS} , if $V_{OS} > V_T$ then transistor on, else off

2- check V_{OS} . if $V_{OS} > V_{OS} - V_T$, then saturated

else if $V_{OS} < V_{OS} - V_T$, then linear active

3- if V_{OS} can't be checked first, assume sat then check if assumption is correct.

design example: $k_n = 150 \mu A/V^2$, $V_{TN} = 0.5V$

1) R_D for $V_o = 0.1V$ when $V_i = 3V$

∴ linear $\rightarrow V_o = V_{DD} - R_D \cdot k_n [2(3-0.5) \cdot 0.1 - 0.1^2]$

$\rightarrow 0.1 = 3 - R_D \cdot 150 \mu [0.49] \rightarrow R_D = \frac{2.9}{7.35 \times 10^{-5}} = 39.5 \Omega$

2) transition point: $V_o = V_s - V_{TN}$ sat

$\rightarrow V_{DD} - 39.5 \Omega \cdot 150 \mu [V_s - 0.5]^2 = V_s - 0.5$

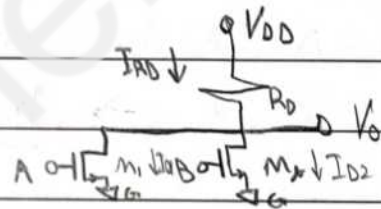
define: $x = V_s - 0.5 \rightarrow 39.5 \Omega \cdot 150 \mu \cdot x^2 + x - V_{DD} = 0$

$\rightarrow x = 0.63217$ or -0.8809

$\rightarrow V_s = 1.13217V$

* NMOS NOA with resistive load:

- transistors do not saturate



A	B	V_o	M_1	M_2	V_{GS1}	$V_o = V_{DD} - I_{D1} R_D$
0	0	V_{DD}	off	off	1	$V_o = V_{DD} - I_{D1} R_D$
0	1	V_{GS1}	off	linear	0	$V_o = V_{GS1} = V_{GS2}$
1	0	V_{GS2}	linear	off	0	
1	1	V_{GS2}	linear	linear	0	

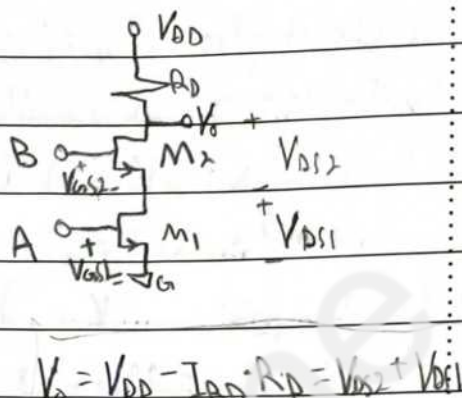
∴ $I_{D1} = k_{n1} V_{GS1} [2(V_{GS1} - V_{TN}) - V_{GS1}]$ & $I_{RD} = \frac{V_{DD} - V_o}{R_D} = I_{D1} + I_{D2}$

for $A=B$, $I_{D1} = I_{D2} \rightarrow I_{RD} = m \cdot I_{D1}$, m : number of transistors (total) (here $m=2$)

$\rightarrow \frac{V_{DD} - V_o}{R_D} = m k_{n1} V_o [2(A - V_{TN}) - V_o]$, $A = \text{Voltage input at A}$

* NMOS NAND with resistive load:

A	B	V_o (logic)	V_o (V)	M_1	M_2
0	0	1	V_{DD}	off	off
0	1	1	V_{DD}	off	linear
1	0	1	V_{DD}	linear	off
1	1	0	$V_{DS1} + V_{DS2}$	linear	linear



$$V_o = V_{DD} - I_{DQ} \cdot R_D = V_{DS2} + V_{DS1}$$

$$I_{D1} = \beta_{n1} V_{GS1} [2(V_{GS1} - V_{T1}) - V_{DS1}] \quad \wedge \quad I_{RD} = \frac{V_{DD} - V_o}{R_D} = I_{D1} = I_{D2}$$

$$I_{D2} = \beta_{n2} V_{GS2} [2(V_{GS2} - V_{T2}) - V_{DS2}]$$

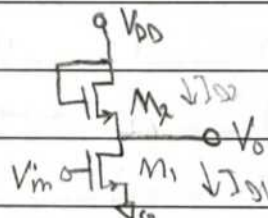
$$V_{GS1} = A \quad \wedge \quad V_{GS2} = B - V_{DS1}$$

$$\rightarrow \frac{V_{DD} - (V_{DS1} + V_{DS2})}{R_D} = \beta_{n1} V_{GS1} [2(A - V_{T1}) - V_{DS1}] = \beta_{n2} V_{GS2} [2(B - V_{DS1} - V_{T2}) - V_{DS2}]$$

* NMOS inverter with resistive load

$$V_{GS2} = V_{GS2} \rightarrow V_{GS2} - V_T < V_{DS2}$$

$\rightarrow M_2$ is always saturated regardless of M_1



* Cases:

1- $V_{in} < V_T \rightarrow M_1$ is off $\wedge I_{D1}$ is zero;

$$I_{D1} = I_{D2} = 0 \rightarrow \beta_{n2} (V_{GS2} - V_{TN2})^2 = 0 \rightarrow V_{GS2} = V_{TN2}$$

$$\therefore V_o = V_{DD} - V_{GS2} = V_{DD} - V_{TN2}$$

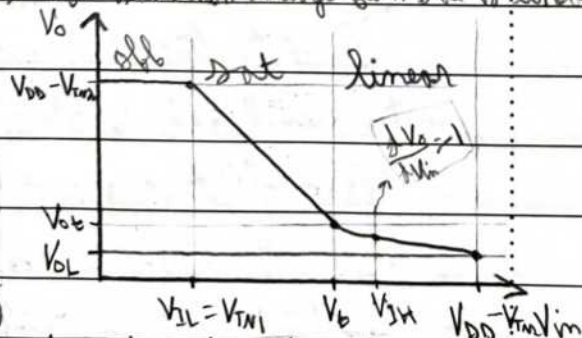
2- $V_{TN1} < V_{in} < V_t \rightarrow M_1$ is sat, V_t : transition voltage from sat to linear

$$I_{D1} = \beta_{n1} (V_{GS1} - V_{TN1})^2 = \beta_{n1} (V_{in} - V_{TN1})^2$$

$$V_{GS2} = V_{DD} - V_{DS1} = V_{DD} - V_o$$

$$I_{D2} = \beta_{n2} [V_{GS2} - V_{TN2}]^2 = \beta_{n2} [V_{DD} - V_o - V_{TN2}]^2$$

$$I_{D1} = I_{D2} \rightarrow \beta_{n1} (V_{in} - V_{TN1})^2 = \beta_{n2} (V_{DD} - V_o - V_{TN2})^2$$



$$\therefore V_o = V_{DD} - V_{TN2} - \sqrt{\frac{\mu_{n1}}{\mu_{n2}}} \cdot (V_{in} - V_{T1}) \rightarrow V_{T1} = V_{TN1}$$

3- $V_{in} = V_t \rightarrow M_1$ saturated ohmic (edge)

$$V_{GS1} = V_{GS1} - V_{TN1} \rightarrow V_{ot} = V_t - V_{T1} \quad \wedge \quad I_{D1} = I_{D2}$$

$$\mu_{n1} (V_{GS1} - V_{TN1})^2 = \mu_{n2} (V_{GS2} - V_{TN2})^2$$

$$\rightarrow \mu_{n1} (V_t - V_{T1})^2 = \mu_{n2} (V_{DD} - V_{ot} - V_{T2})^2 = \mu_{n2} (V_{DD} - V_t + V_{T1} - V_{T2})^2$$

4- $V_{in} > V_t \rightarrow M_1$ is ohmic:

$$I_{D1} = \mu_{n1} V_{DS1} [2(V_{GS1} - V_{T1}) - V_{DS1}] \quad , \quad V_o = V_{DS1} \quad , \quad V_{GS1} = V_{in}$$

$$\wedge \quad V_{GS2} = V_{DD} - V_o$$

$$\infty \quad I_{D1} = I_{D2} \quad \wedge \quad I_{D2} = \mu_{n2} (V_{GS2} - V_{T2})^2$$

$$\rightarrow \mu_{n1} V_{DS1} [2(V_{GS1} - V_{T1}) - V_{DS1}] = \mu_{n2} (V_{GS2} - V_{T2})^2$$

- in this region, V_{TH} can be found by taking the derivative of the above equation: $\frac{dV_o}{dV_{in}} = -1$

- taking the second case and solving for V_t :

$$V_t = \frac{\sqrt{\frac{\mu_{n1}}{\mu_{n2}}} \cdot V_t + V_{DD}}{1 + \sqrt{\frac{\mu_{n1}}{\mu_{n2}}}} \quad , \quad \text{define } \beta_a = \frac{\mu_{n1}}{\mu_{n2}}$$

$$\rightarrow V_t = \frac{\sqrt{\beta_a} \cdot V_t + V_{DD}}{1 + \sqrt{\beta_a}} \quad , \quad \beta_a \text{ should be large}$$

$$\infty \quad \mu_{n1} = \frac{W_1}{L_1} \frac{\mu_n \text{ Cox}}{2} \quad \wedge \quad \mu_{n2} = \frac{W_2}{L_2} \frac{\mu_n \text{ Cox}}{2} \quad , \quad \frac{\mu_n \text{ Cox}}{2} \text{ is usually constant}$$

$$\therefore \beta_a = \left(\frac{W_1/L_1}{W_2/L_2} \right) \quad ; \quad \text{ratio of the aspect ratios of the transistors}$$

$\rightarrow M_1$ should be wide and M_2 narrow

example: $V_t = 1V$, $\mu_{n1} = 10^{-3} \text{ A/V}^2$, $\mu_{n2} = 0.34 \times 10^{-3} \text{ A/V}^2$, $V_{DD} = 5V$

$$1) \quad V_t = \frac{\sqrt{\beta_a} \cdot V_t + V_{DD}}{1 + \sqrt{\beta_a}} \quad \wedge \quad \sqrt{\beta_a} = 1.715 \rightarrow V_t = 2.493V$$

$$V_{ot} = V_t - V_t = 1.493V$$

$$2) \quad \text{for } V_{in} = V_{DD} - V_t = 4V \text{, ohmic } \rightarrow V_o [2(3) - V_o] = \frac{\mu_{n2}}{\mu_{n1}} (5 - V_o - 1)^2$$

$$\rightarrow \frac{\mu_{n1} + \mu_{n2}}{\mu_{n1}} V_o^2 - \frac{6\mu_{n1} + 8\mu_{n2}}{\mu_{n1}} \cdot V_o + \frac{\mu_{n2}}{\mu_{n1}} 16 = 0$$

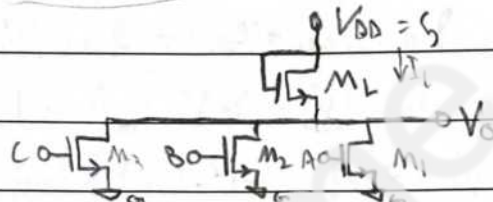
$$\rightarrow V_o = 0.6989V$$

or 5.8085 out of range 40

- note on previous example: maximum output voltage of the circuit is $V_{DD} - V_T$, hence the high input corresponding to V_{OL} is $V_{in} = V_{DD} - V_T$

* NMOS NOR gate:

A	B	C	V_{out}
0	0	0	1
0	0	1	0
0	1	0	0
1	1	1	0



$$I_D = \beta_{n1} V_{GS1} [2(V_{GS1} - V_{T1}) - V_{DS1}]$$

$$I_{D1} = \beta_{n2} (V_{GS2} - V_{T1})^2, I_{D1} = m_1 I_{D1}$$

$$\therefore \beta_{n1} (V_{GS1} - V_T)^2 = m \beta_{n11} V_{GS1} [2(V_{GS1} - V_{T1}) - V_{DS1}]$$

- Worst case when $m=1$ (one transistor), hence $V_{OL} < V_T$

example: $V_T = 1V, \beta_{n1} = 10^{-3} A/V^2, \beta_{n11} = 0.34 \times 10^{-3} A/V^2$

$$\rightarrow \beta_n = \frac{10^{-3}}{0.34 \cdot 10^{-3}} = 2.94$$

1) V_{OL} if $A=B=C=4V$

$$\beta_{n2} (V_{GS1} - V_T)^2 = 3 \cdot \beta_{n1} \cdot V_{OL} [2(4-1) - V_{OL}]$$

$$\rightarrow (5 - V_{OL} - 1)^2 = 3 \cdot 2.94 \cdot V_{OL} [6 - V_{OL}]$$

$$\rightarrow 16 - 8V_{OL} + V_{OL}^2 = 8.82 \cdot [6V_{OL} - V_{OL}^2]$$

$$\rightarrow 9.92 \cdot V_{OL}^2 - 60.92 V_{OL} + 16 = 0 \rightarrow V_{OL} = 0.295V$$

2) $m=1 \rightarrow 16 - 8V_{OL} + V_{OL}^2 = 2.94 [6V_{OL} - V_{OL}^2]$

$$\rightarrow 3.94 V_{OL}^2 - 25.64 V_{OL} + 16 = 0 \rightarrow V_{OL} = 0.699V$$

- hence, one active gate gives the highest output voltage

* NMOS NAND gate:

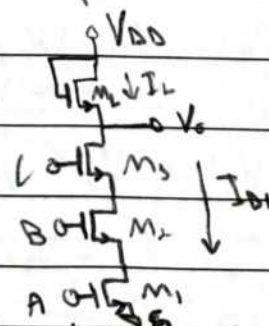
- if any input is low then $V_o = V_{DD} - V_T$

- if all inputs high then output: low logic

all transistors will be ohmic, $V_o = V_{DS1} + V_{DS2} + V_{DS3}$

$$V_{GS1} = A = V_{DD} - V_T, V_{GS2} = V_{DD} - V_T - V_{DS1}$$

$$V_{GS3} = V_{DD} - V_T - V_{DS1} - V_{DS2}$$



example: $k_1 = k_2 = k_3 = k = 2 \times 10^{-3} \text{ A/V}^2$, $V_{T1} = V_{T2} = V_{T3} = 1 \text{ V}$

$$I_D = 1 \text{ mA}, V_{DD} = 5 \text{ V}$$

$$1) V_{GS1} = V_{DS3} + V_{DS2} + V_{GS1}$$

$$\therefore V_{GS1} = 5 - 1 = 4 \text{ V}$$

$$\text{ohmic} \rightarrow I_D = k \cdot V_{GS1} [2(V_{GS1} - V_T) - V_{DS1}] = 1 \text{ mA}$$

$$\rightarrow 0.5 = 6V_{GS1} - V_{GS1}^2 \rightarrow V_{GS1} = 84.52 \text{ mV}$$

$$\therefore V_G = V_{DD} - V_T \rightarrow V_{GS2} = V_G - V_{GS1} = 3.9155 \text{ V}$$

$$I_D = k V_{GS2} [2(V_{GS2} - V_T) - V_{DS2}] \rightarrow -V_{GS2}^2 + 5.831 V_{GS2} - 0.5 = 0$$

$$\rightarrow V_{GS2} = 87.05 \text{ mV}$$

$$\therefore V_G = V_{DD} - V_T \rightarrow V_{GS3} = V_G - V_{GS1} - V_{GS2} = 3.8284 \text{ V}$$

$$\rightarrow -V_{GS3}^2 + 5.65686 V_{GS3} - 0.5 = 0 \rightarrow V_{GS3} = 89.81 \text{ mV}$$

$$\therefore V_{OL} = 89.81 + 87.05 + 84.52 = \boxed{0.2614 \text{ V}}$$

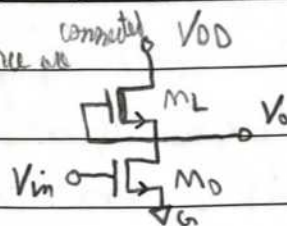
$$2) \therefore \text{load always saturated: } I_D = k_L (V_{GS_L} - V_T)^2$$

$$V_{GS_L} = V_{DD} - V_T - V_{OL} = 3.739 \text{ V} \rightarrow k_L = 71.55 \mu\text{A/V}^2$$

$V_{GS} = 0 \rightarrow M_L$ is always on, since gate and source are

$V_{GS} < V_{GS} - V_{TL} \rightarrow M_L$ is linear

$V_{GS} > V_{GS} - V_{TL} \rightarrow M_L$ is saturated



- since the depletion NMOS has a negative threshold voltage, the device is normally on

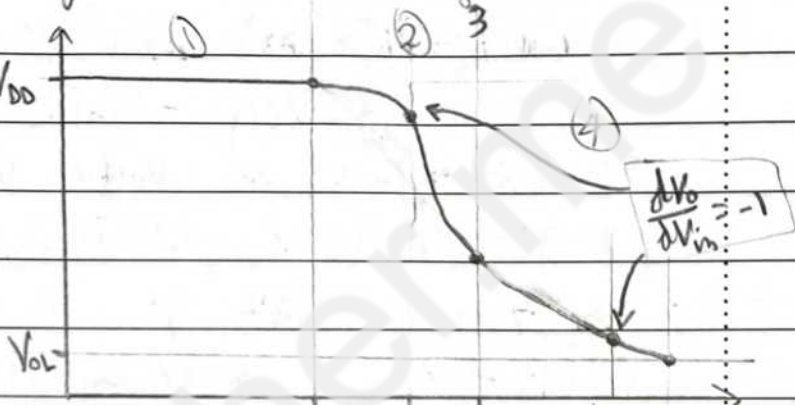
① $V_{in} < V_{TD}$, M_D is off, $I_D = 0$: $V_{OH} = V_{DD}$

- M_L could be sat or ohmic.

assuming sat:

$$I_D = \beta_L (0 - V_T)^2 \neq 0$$

hence not sat \rightarrow linear



$$\therefore I_D = \beta_L V_{GS} [2(V_{GS} - V_T) - V_{GS}] = 0 \quad V_{GS} = -2V_{TL} \text{ or } V_{GS} = -2V_{TL}$$

$\rightarrow V_{GS} = 0$ or $V_{GS} = -2V_{TL}$, which is larger than $V_{GS} - V_{TL}$ hence breaks the condition for sat

$$\therefore V_O = V_{DD}$$

② $V_{in} > V_{TD}$, M_D becomes saturated, M_L remains linear:

$$\beta_1 (V_{GS1} - V_{TD})^2 = \beta_2 V_{GS2} [2(V_{GS2} - V_{TD}) - V_{GS2}]$$

$$\therefore \beta_1 (V_i - V_{TD})^2 = \beta_2 (V_{DD} - V_o) [-2V_{TL} - (V_{DD} - V_o)]$$

then find V_{SL}

③ at $V_{in} = V_{SS}$, M_D will stay in sat, whereas M_L will enter sat:

$$EOS: V_{GS1} = V_{GS1} - V_{TL} = V_{SS} - V_{TL} = V_{SS}$$

$$\beta_1 (V_{GS1} - V_{TD})^2 = \beta_2 (V_{GS2} - V_{TL})^2 \quad \& \quad V_{GS2} = 0 \quad V_{GS1} = V_{SS}$$

$$\therefore V_{SS} = V_{TD} \pm \sqrt{\frac{\beta_2}{\beta_1}} \cdot V_{TL}$$

$$\rightarrow V_{SS}^2 = \frac{\beta_2}{\beta_1} \cdot V_{TL}^2$$

	①	②	③	④
M_L	linear	linear	sat	sat
M_D	cut off	linear	sat	linear

④ $V_{in} > V_{SS}$, M_D is linear, M_L remains saturated

$$\beta_1 V_{GS1} [2(V_{GS1} - V_{TH}) - V_{DS1}] = \beta_2 (V_{GS2} - V_{TH})^2$$

$$\rightarrow \beta_1 V_0 [2(V_i - V_{TH}) - V_0] = \beta_2 (0 - V_{TH})^2$$

then find V_{TH} , \Rightarrow V_{OL} if $V_i = V_{DD}$

- V_{OL} is the output voltage corresponding to an input equal to V_{DD}

- larger $\frac{\beta_1}{\beta_2}$ makes the curve steeper and closer to ideal

example: $V_{TH} = 1V$, $V_{TL} = -1V$, $\beta_1 = 1 \times 10^3 A/V^2$

$\beta_2 = 0.2 \times 10^3 A/V^2$, $V_{DD} = 5V$. find V_{S1} , V_{SS} , and V_{OL}

$$\circ \circ V_{S1} = V_{TH} \pm \sqrt{\frac{\beta_2}{\beta_1}} \cdot V_{TL} \rightarrow V_{S1} = 0.553V \text{ (or } 1.4492)$$

$0.553 \rightarrow V_{GS2} = V_{in} - V_{S1} \rightarrow$ negative $\rightarrow M_1$ cutoff

$$\therefore V_{SS} = 1.4492V$$

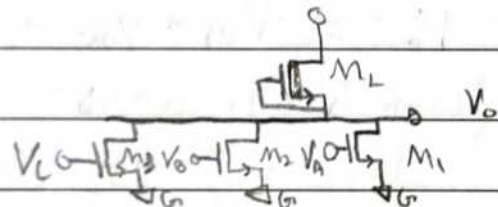
$$\circ \circ V_{SS} = \pm V_{TL} \sqrt{\frac{\beta_2}{\beta_1}} \rightarrow V_{SS} = 0.449V$$

$$\text{for } V_{OL} \text{ (i)} \rightarrow \beta_1 V_0 [2(V_{DD} - V_{TH}) - V_0] = \beta_2 (-V_{TH})^2$$

$$\therefore 2V_0(4) - V_0^2 = 0.2 \rightarrow V_0 = 25mV$$

* N-MOS NOR gate:

A	B	C	V_o
0	0	0	1 V_{DD}
0	0	1	0 V_{S1}
0	1	1	0 V_{S1}
1	1	1	0 V_{S1}



load is sat and others active $\rightarrow I_D = \beta_L (V_{GS2} - V_{TH})^2$

$\wedge I_D = m I_D$, m : number of transistors for input

$M_1 \rightarrow 3$ are linear: $I_D = \beta_D \cdot V_{DS} [2(V_{GS} - V_{TH}) - V_{DS}]$

$$\therefore m I_D = m \beta_D \cdot V_{DS} [2(V_{GS} - V_{TH}) - V_{DS}] = \beta_L (V_{GS2} - V_{TH})^2$$

- worst case is when $m=1$ $\wedge V_{OL} < V_{TH}$

example on NMOS NOR

$$① A=B=C=5V \rightarrow m=3$$

$$\rightarrow 3 \cdot 10^{-3} \cdot V_0 [2(V_{DD} - V_{TH}) - V_0] = 0.34 \times 10^{-3} (-V_{TH})^2$$

$$\rightarrow 8V_0 - V_0^2 = \frac{19}{150} \rightarrow V_0 = 14.14 \text{ mV}$$

$$② \text{ if } A=5 \text{ \& } B=C=0V \rightarrow m=1$$

$$\therefore 8V_0 - V_0^2 = 0.34 \rightarrow V_0 = 42.93 \text{ mV}$$

* NMOS NAND gate:

- any low input gives $I_L = 0$ & $V_0 = V_{DD}$

- if all inputs are high ($A=B=C=V_{DD}$), then all

MOSFETs are in linear mode

$$\rightarrow V_{GS1} = A = V_{DD}$$

$$V_{GS2} = B - V_{DS1} = V_{DD} - V_{DS1}$$

$$V_{GS3} = V_{DD} - V_{DS1} - V_{DS2}$$

$$V_0 = V_{DS1} + V_{DS2} + V_{DS3}$$

example on NMOS NAND:

$$① V_0 = V_{DS1} + V_{DS2} + V_{DS3} \quad \wedge \quad I_D = 1 \text{ mA} = \beta_{n1} (-V_{TH})^2$$

$$= \beta_{n1} \cdot V_{TH} [2(V_{GS2} - V_{TH}) - V_{DS2}]$$

$$\therefore -V_{DS2}^2 + 2(V_{GS2} - 1) \cdot V_{DS2} - 0.5 = 0$$

$$\rightarrow V_{DS1} = 62.99 \text{ mV}, \quad V_{DS2} = 64.02 \text{ mV}, \quad V_{DS3} = 69.09 \text{ mV}$$

$$\therefore V_{OL} = 0.1921 \text{ V}$$

$$② \therefore 1 \text{ mA} = \beta_{n1}$$

* CMOS: complementary metal oxide semiconductors

* CMOS inverter: consists of NMOS and PMOS

① $0 < V_{in} < V_{TN}$, M_n off, M_p ohmic:

$$I_{Dp} V_{SDP} [2(V_{SGP} - |V_{TP}|) - V_{SDP}] = 0 \rightarrow V_{SDP} = 0$$

$$\rightarrow V_o = V_{DD}$$

② $V_{TN} < V_{in} < V_{SS}$, M_n sat, M_p ohmic:

$$\rightarrow I_{Dn} (V_{DSn} - V_{TN})^2 = I_{Dp} V_{SDP} [2(V_{SGP} - |V_{TP}|) - V_{SDP}]$$

$$\therefore I_{Dn} (V_{in} - V_{TN})^2 = I_{Dp} (V_{DD} - V_o) [2(V_{DD} - V_{in} - |V_{TP}|) - (V_{DD} - V_o)]$$

③ $V_{in} = V_{SS}$, M_n sat, M_p sat:

$$\rightarrow I_{Dn} (V_{DSn} - V_{TN})^2 = I_{Dp} (V_{SGP} - |V_{TP}|)^2$$

$$\therefore I_{Dn} (V_i - V_{TN})^2 = I_{Dp} (V_{DD} - V_i - |V_{TP}|)^2$$

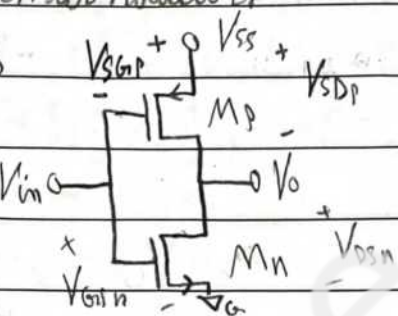
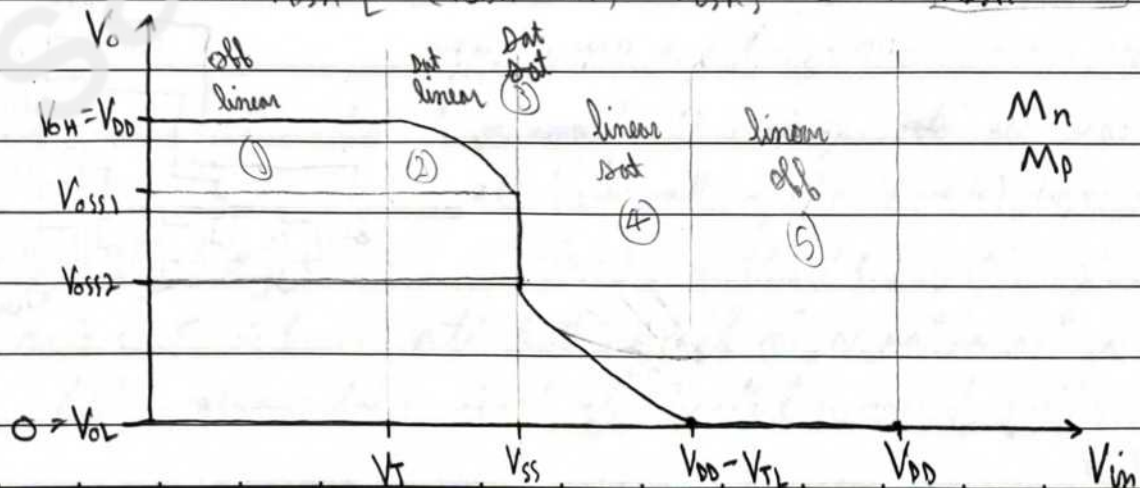
④ $V_{SS} < V_{in} < V_{DD} - V_{TN}$, M_n ohmic and M_p sat:

$$I_{Dn} V_{DSn} [2(V_{DSn} - V_{TN}) - V_{DSn}] = I_{Dp} (V_{SGP} - |V_{TP}|)^2$$

$$\therefore I_{Dn} V_o [2(V_i - V_{TN}) - V_o] = I_{Dp} (V_{DD} - V_i - |V_{TP}|)^2$$

⑤ $V_{DD} - V_T < V_{in} < V_{DD}$, M_n is ohmic, M_p is off:

$$\rightarrow I_{Dn} V_{DSn} [2(V_{DSn} - V_{TN}) - V_{DSn}] = 0 \rightarrow V_{DSn} = 0V = V_o$$



tips for using PMOS:
 - replace V_{GS} with V_{SG}
 and replace V_{DS} with V_{SD} in order to use NMOS equations
 - use: $|V_{TP}|$ for the threshold
 MODE: $V_{SGP} > |V_{TP}| \rightarrow$ on
 $V_{SDP} > V_{SGP} - |V_{TP}| \rightarrow$ sat

* advantages of CMOS logic:

- low power dissipation
- better packaging design
- high noise immunity

- if $I_{on} = I_{off}$, $k_n (V_{GSn} - V_{TN})^2 = k_p (V_{SGP} - |V_{TP}|)^2$

$$\rightarrow V_{SS} = \frac{V_{TN} + \sqrt{\frac{k_p}{k_n}} [V_{DD} - |V_{TP}|]}{1 + \sqrt{\frac{k_p}{k_n}}}$$

if $k_n = k_p$ & $V_{TN} = |V_{TP}|$

$$\rightarrow V_{SS} = \frac{V_{DD}}{2}$$

- at V_{SS} , M_n switches from sat to linear:

$$EOS \rightarrow V_{GSn} = V_{GSn} - V_{TN} = V_{SS} - V_{TN} = V_{SS1}$$

- at V_{SS} , M_p switches from linear to sat:

$$\rightarrow V_{SDP} = V_{SGP} - |V_{TP}| = V_{DD} - V_{SS} - |V_{TP}|$$

$$\& V_{SDP} = V_{DD} - V_o \rightarrow V_{DD} - V_{SS} - |V_{TP}| = V_{DD} - V_o$$

$$\therefore V_o = V_{SS} + |V_{TP}| = V_{SS2}$$

example on inverter:

① if $k_n = k_p$ & $V_{TN} = |V_{TP}| \rightarrow V_{SS} = \frac{V_{DD}}{2} = 2.5V$

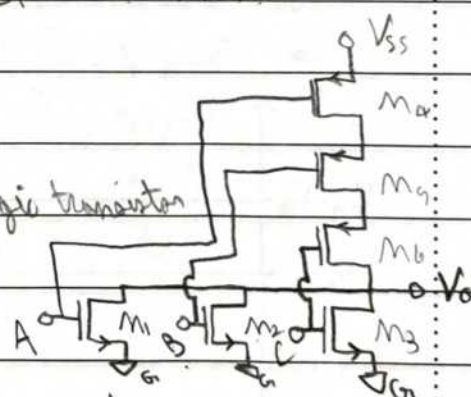
② if $V_{in} = V_{SS} \rightarrow$ both sat

$$I_p = k_p (V_{SGP} - |V_{TP}|)^2 \quad \& \quad V_{SG} = V_{DD} - V_{in}$$

$$\rightarrow I_p = k_p (2.5 - 1)^2 \rightarrow I_p = 4.5 \text{ mA}$$

* NOR CMOS:

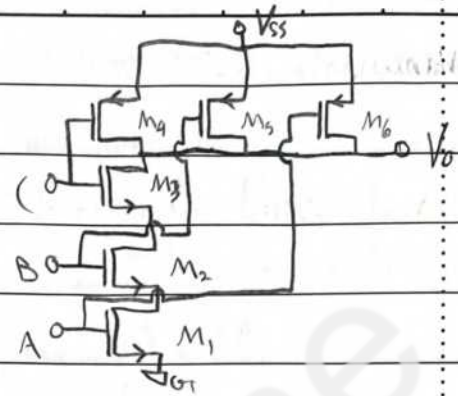
- two transistors are required for every input
- every input transistor requires a complementary logic transistor
- if one input (at least) is high, then one of the logic transistors (at least) will be off



- if a transistor is in linear but its current is zero then the voltage difference between its drain and source will be zero

8 NAND CMOS:

- if one of the input transistors has a low voltage, it will be cutoff, causing the other transistors' currents to equal zero. Hence the output voltage will be high.



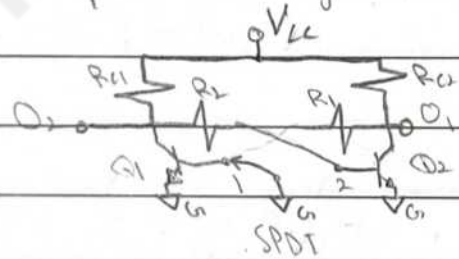
* Multivibrators: sequential logic circuits that operate continuously between two distinct states of high and low

+ Types of multivibrators:

- Astable: free-running multivibrator that has no stable states but switches continuously between two states, producing a train of square-wave pulses at a fixed frequency.
- Monostable: a one-shot multivibrator that has only one stable state and is triggered externally and returns back to its first, stable, state.
- Bistable: a flip-flop that has two stable states, which produces a single pulse (either positive or negative)

* Bistable multivibrator:

- When SPDT is at position [1], Q_1 will be cutoff and output O_1 will go high. Q_2 will instead go to sat and O_2 will be low



- This state will remain unless the switch is flipped (externally)

$$\rightarrow I_{R1} = I_{B2} = \frac{V_{CC} - V_{BE2 \text{ sat}}}{R_{C1} + R_2}$$

$$\therefore V_{O1} = V_{CC} - I_{R1} \cdot R_{C1} \quad \wedge \quad V_{O2} = V_{BE2 \text{ sat}} = 0.1V$$

- If the switch connects to position [2], Q_2 will cutoff O_2 will be high, and so on.
- Linear and reverse active modes are ignored in multivibrators.

given $C = 50 \text{ fF} = 50 \times 10^{-15} \text{ F}$, $V_{OL} = 0.6 \text{ V}$, $V_{OH} = 5 \text{ V}$

$V_T = 1 \text{ V}$ $\mu_{kn} = 100 \text{ uA/V}^2$

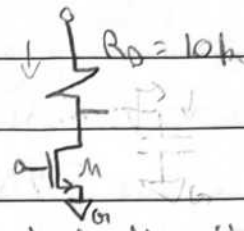
- if the input is low, output should switch to high

$$i = C \frac{dV_o}{dt} = \frac{V_{DD} - V_o}{R_D}$$

- if the input switches from low to high; output should switch back to low: $I_D = I_{RD} + I_{Cs}$, $I_{Cs} = -C \frac{dV_o}{dt}$

if $V_o > 4 \text{ V}$: $M \rightarrow \text{sat} \rightarrow I_D = \mu_{kn} [V_{GS} - V_{TN}]^2 = \frac{V_{DD} - V_o}{R_D} - C \frac{dV_o}{dt}$

if $V_o < 4 \text{ V}$: $M \rightarrow \text{ohmic} \rightarrow I_D = \mu_{kn} R_D [8V_o - V_o^2] = \frac{V_{DD} - V_o}{R_D} - C \frac{dV_o}{dt}$

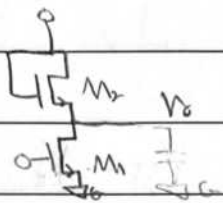


§ NMOS with saturated ENMOS no load:

input going from high to low:

$M \rightarrow \text{sat} \rightarrow I_{D2} = \mu_{kn} (V_{DD} - V_o - V_{TN})^2$

$i_c = C \frac{dV_o}{dt} = \mu_{kn} (4 - V_o)^2$

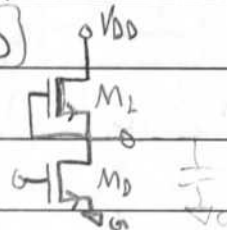


§ NMOS inverter with DNmos no load: $V_{GS1} = 0$

for $V_o < 3$: $V_{DS1} > V_{GS1} - V_{TN} \rightarrow \text{sat}$

$$\therefore I_D = C \frac{dV_o}{dt} = \mu_{kL} (V_{GS1} - V_{TN})^2 = \mu_{kL} (-V_{TN})^2$$

$$\therefore V_o(t) = V_{OL} + \frac{\mu_{kL}}{C} |V_{TN}|^2 t$$



$V_{TL} = -2 \text{ V}$ $V_{SD} = 1 \text{ V}$

$\mu_{kL} = 25 \text{ uA/V}^2$

$V_{OL} = 0.1 \text{ V}$

$V_{OH} = 5 \text{ V}$

$\mu_{kD} = 75 \text{ uA/V}^2$

for high to low input

for $V_o > 3$: $V_{DS1} < -V_{TN} \rightarrow \text{ohmic}$

$$I_D = C \frac{dV_o}{dt} = \mu_{kL} R_D (2(V_{GS1} - V_{TN}) \cdot V_{DS1} - V_{DS1}^2)$$

$$\rightarrow C \frac{dV_o}{dt} = \mu_{kL} R_D (4(V_{DD} - V_o) - (V_{DD} - V_o)^2)$$

- NMOS with DNmos load no input goes from low to high

if $V_o > 4$: M_1 ohmic, M_0 sat $\therefore V_{GS0} > V_{GS0} - V_{TN}$

$$\rightarrow I_D = \mu_{kD} (V_{GS0} - V_{TN})^2 = 1.2 \text{ mA}$$

if $V_o > 3 \rightarrow M_1$: $I_D = \mu_{kL} (4(V_{DD} - V_o) - (V_{DD} - V_o)^2)$

* for $3 < V_0 < 4$, M_0 & M_1 ohmic

$$M_0: I_D = \beta_{n0} (8V_0 - V_0^2) = \beta_{n1} (4(V_{DD} - V_0) - (V_{DD} - V_0)^2)$$

* for $V_0 < 3$, M_1 is sat & M_0 is linear

- hence, an CMOS driver connected to a CMOS load will pass through three stages (M_1 linear & M_0 sat, both linear, and M_1 sat & M_0 linear) while switching from low input to a high input (or high output to low)

* CMOS inverter:

• high to low input:

$$M_p \text{ sat initially: } I_D = \beta_{p0} (V_{DD} - V_i - |V_{TP}|)^2$$

$$\text{for } V_0 < |V_{TP}| \quad = \beta_{p0} (V_{DD} - |V_{TP}|)^2 = C \frac{dV_0}{dt}$$

$$\therefore V_{SS} - V_0 > V_{SS} - V_i - |V_{TP}| \quad (V_{DD} > V_{SS} - |V_{TP}|) \quad \text{condition for } P_{out}$$

- if $V_0 > |V_{TP}| \rightarrow M_p$ goes to ohmic mode

$$\rightarrow C \frac{dV_0}{dt} = \beta_{p0} (2(V_{DD} - |V_{TP}|)(V_{DD} - V_0) - (V_{DD} - V_0)^2)$$

• for low to high input:

- initially M_p will be sat, M_n will be cutoff

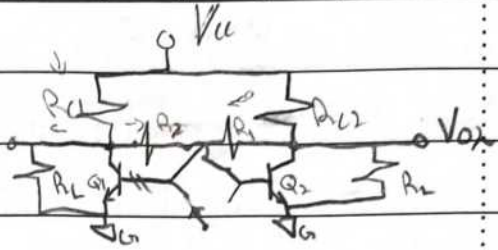
$$\rightarrow C \frac{dV_0}{dt} = -\beta_{n0} (V_{DD} - V_{TN})^2$$

- next the driver will enter linear, whereas M_p will remain off:

$$C \frac{dV_0}{dt} = -\beta_{n0} (2(V_{DD} - V_{TN})V_0 - V_0^2)$$

* loaded bistable MV:

- Q_1 is off since its base is grounded
- Q_2 is saturated



$$I_{C2} = \frac{V_{CE2 sat}}{R_L} = \frac{0.1}{R_L}$$

$$I_{B1} = I_{A1} - I_{B2} \rightarrow \frac{V_{O1} - V_{CE2 sat}}{R_{B2}} = \frac{V_{CC} - V_{O1}}{R_{C1}} - \frac{V_{O1}}{R_L}$$

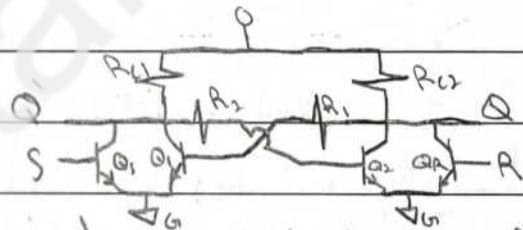
- for Q_2 to be saturated, $I_{C2} \geq \beta_2 \cdot I_{B2}$

$$I_{C2} = \frac{V_{CC} - V_{O2}}{R_{C2}} - \frac{V_{O2}}{R_L}$$

$$\therefore \beta_2 \left[\frac{V_{CC} - V_{O1}}{R_{C1}} - \frac{V_{O1}}{R_L} \right] \leq \left[\frac{V_{CC} - 0.1}{R_{C2}} - \frac{0.1}{R_L} \right]$$

* RS flip-flop using bistable:

- If the S input is low while the R input is high



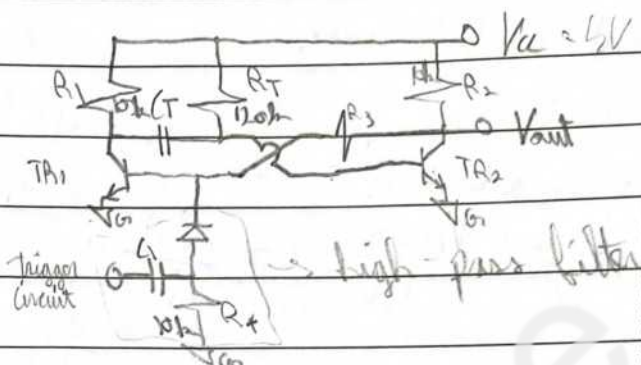
Q_1 will be off, Q_2 will be sat \rightarrow output Q will be low
 \because Q output is low and connected to the base of Q_1 , then the output Q' will be high since Q_1 will be off

R	S	Q		Q_s	Q_r
0	0	$Q(t-1)$	holds previous	off	off
0	1	1	set	sat	off
1	0	0	reset	off	sat
1	1	X	indeterminate		

* monostable multivibrator:

- before triggering, TR_2 will be saturated, giving TR_1 off.

The voltage difference across
 $CT = 5 - 0.8$



$V_{cc} - V_{BE(sat)2}$, $V_{out} = 0.1V$

- the voltage on both terminals of the capacitor must drop by the same amount, hence CT after the trigger:

$0.1 \frac{CT}{T} = 0.8 - (V_{cc} - 0.1)$

since TR_1 is saturated so $V_{BE1} = 0.8V$ & TR_2 goes off

\therefore during trigger: $V_{out} = V_{cc} - I_{B2} \cdot R_2$ & $I_{B2} = \frac{V_{cc} - V_{BE(sat)1}}{R_2 + R_3}$
 $\rightarrow \frac{5 - 0.8}{20k} = 0.24 \mu A \rightarrow V_{out} = 2.6V$

otherwise, $V_{out} = V_{CE(sat)} = 0.1V$

CT charging: $V_{B2}(t) = V_{B2}(\infty) - [V_{B2}(\infty) - V_{B2}(0)]e^{-t/T}$

$V_{B2}(\infty) = V_{cc}$ & $V_{B2}(0) = 0.8 - (V_{cc} - 0.1)$

$\therefore V_{B2}(t) = V_{cc} - [2V_{cc} - 0.9]e^{-t/T}$
 $= 5 - 9.1e^{-t/T}$ & $T = R_T \cdot C_T$

- to find the pulse duration, set $V_{B2}(t) = V_{B2}(T)$ equal to the stable value of $V_{BE(sat)} = 0.8$

$\therefore V_{B2}(T) = 0.8 = 5 - 9.1e^{-T/T} = V_{cc} - [2V_{cc} - 0.9]e^{-t/T}$

$\rightarrow e^{-T/T} = \frac{V_{cc} - 0.8}{2V_{cc} - 0.9} \approx \frac{1}{2}$ assuming $V_{cc} \gg 1$

$\rightarrow T = T \cdot \ln(2) = R_T \cdot C_T \cdot \ln(2)$

- monostable circuits are called "one-shot" as they give a high output pulse which returns to the stable steady-state low output
- the duration between pulses from the triggering circuit should be $5 \cdot R_T \cdot C_T$

NMOS :

$V_{GS} < V_{TN}$
off

$V_{GS} > V_{TN}$

$V_{DS} > V_{GS} - V_{TN}$

$V_{DS} < V_{GS} - V_{TN}$

sat : $I_D = \frac{1}{2} \mu_n C_{ox} (V_{GS} - V_{TN})^2$ ohmic : $I_D = \mu_n C_{ox} [(V_{GS} - V_{TN}) V_{DS} - \frac{1}{2} V_{DS}^2]$



example :

a) $V_{in} = V_{GS} = 3V \rightarrow V_G = 0.1V$

$V_G = V_{DSM}$ if $V_G = 0.1 \rightarrow V_{DS} < V_{GS} - V_{TN}$

\rightarrow ohmic $\therefore I_D = \mu_n C_{ox} [2(V_{GS} - V_{TN}) V_{DS} - V_{DS}^2]$

$I_D = \frac{3 - 0.1}{R_D}$

$\rightarrow \frac{2.9}{R_D} = 150 \mu [2(2.5)0.1 - (0.1)^2]$

$\rightarrow R_D = 39.45 \text{ k}\Omega$

b) substituting in equation

$$V_S = V_{in} - \frac{1}{2R_D \mu_n C_{ox}} \pm \sqrt{\left(\frac{1}{2R_D \mu_n C_{ox}}\right)^2 + \frac{V_{DD}}{R_D \mu_n C_{ox}}}$$

$\rightarrow V_S = 0.9 - 0.084495 \pm 0.9190148$

$\rightarrow V_S = 1.13252$

example : $V_t \rightarrow$ transition point from sat to linear

N_1 sat \wedge load N_2 linear

1) $\frac{\mu_{n1}}{\mu_{n2}} (V_{GS1} - V_{TN1})^2 = \mu_{n2} [2(V_{GS2} - V_{TN2}) V_{DS2} - V_{DS2}^2]$

$\frac{\mu_{n1}}{\mu_{n2}} = \frac{1}{0.34}$ \wedge $V_{GS1} = V_{in} = V_t$ \wedge $V_{T1} = V_{T2} = V_t$

$V_G = V_{GS} = V_t - V_{TN} \rightarrow \frac{1}{0.34} (V_t - 1)^2 = 2(6 - V_t)(6 - V_t) - (6 - V_t)^2$

$\rightarrow \frac{1}{0.34} [V_t^2 - 2V_t + 1] = 2[36 - 11V_t + V_t^2] - (36 - 12V_t + V_t^2)$

$\rightarrow \frac{1 - 0.34}{0.34} V_t^2 - \frac{2 + 3.4}{0.34} V_t - 21.0588 = 0$

$\rightarrow V_t = 2.39965$ or $\rightarrow V_{ot} = 1.39965$

1) load is always sat:

$$V_{OL} = V_E - V_T$$

$$I_{in1} (V_E - V_T)^2 = I_{in2} (V_{DD} - V_O - V_T)^2$$

$$I_{in1} I_{in2} = I_{in2} \rightarrow V_E^2 - 2V_E + 1 = 0.34 (5^2 - 10V_E + V_E^2)$$

$$\rightarrow V_E = 2.49330 \quad \wedge \quad V_{OL} = 1.49330$$

2) $V_{in} = 5 - 1 = 4V$

M_1 is ohmic

$$V_{GS1} = 4$$

$$\rightarrow I_{in1} [V_{DD} - V_O - 1]^2 = I_{in2} [2(V_{GS1} - V_T) V_{GS1} - V_{GS1}^2]$$

$$\rightarrow 0.34 [4 - V_O]^2 = 6V_O - V_O^2 \quad V_O = V_{DS1}$$

$$\rightarrow 0.34 [16 - 8V_O + V_O^2] = 6V_O - V_O^2 \rightarrow V_O = 0.69892$$

example:

$$1) I_{in1} (V_{GS1} - V_T)^2 = m I_{in2} V_{GS1} [2(V_{GS1} - V_T) - V_{GS1}]$$

$$\rightarrow 0.34 (5 - V_O - 1)^2 = 3 \cdot [2V_O(3) - V_O^2]$$

$$\rightarrow 0.34 (4 - V_O)^2 = 18V_O - 3V_O^2$$

$$\rightarrow V_{OL} = 0.274913$$

$$2) 0.34 [16 - 8V_O + V_O^2] = 6V_O - V_O^2$$

$$\rightarrow V_{OL} = 0.6989$$

example: $V_{in} \rightarrow$ all inputs high, $I_O = 1mA$, all ohmic

$\rightarrow M_1$: $\frac{0}{2}$ / input high = high output = $V_{DD} - V_{TL}$

$$1mA = 2 \times 10^{-3} [2(V_{DD} - V_{TL} - V_{GS1} - V_T) V_{GS1} - V_{GS1}^2]$$

$$\rightarrow 0.5 = 2(3) V_{GS1} - V_{GS1}^2$$

$$\rightarrow V_{GS1} = 0.084924$$

$$M_2: 1mA = 2 \times 10^{-3} [2(V_{DD} - V_{TL} - V_{GS1} - V_{GS2} - V_T) V_{GS2} - V_{GS2}^2]$$

$$\rightarrow 0.5 = 5.830952 V_{GS2} - V_{GS2}^2$$

$$\rightarrow V_{GS2} = 0.08904882$$

$$M_3: V_{GS3} = V_{DD} - V_{TL} - V_{GS1} - V_{GS2} = 3.528072918$$

$$\therefore 0.9 = 6.6568544 V_{DS3} - V_{DS3}^2$$

$$\rightarrow V_{DS3} = 0.0848143$$

$$\rightarrow V_{OL} = V_{DS1} + V_{DS2} + V_{DS3} = \underline{0.261387V}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS1} - V_{TL})^2$$

$$\rightarrow \frac{1}{2} \frac{\mu_n}{\mu_n} (4.938613 - 1)^2 \rightarrow \mu_n = 71.545 \text{ MA/V}^2$$

example:

$$V_{SS} = V_{T1} \pm \sqrt{\frac{\mu_n}{\mu_p}} V_{TL}$$

$$= 1 \pm \sqrt{0.2} \cdot (-1) = 0.957786 \text{ or } 1.442214V$$

$$\rightarrow V_{SS} = 1.442214V$$

$$\mu_n V_{SS}^2 = \frac{\mu_n}{\mu_p} V_{TL}^2 \Rightarrow V_{SS} = \pm \sqrt{\frac{\mu_n}{\mu_p}} \cdot V_{TL}$$

$$\Rightarrow V_{SS} = \pm \sqrt{0.2} = 0.44721V$$

to find V_{OL} : M_1 is sat $V_{GS} > V_T$ & $V_{DS} > V_{GS} - V_T$

$$\rightarrow I_{OL} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS1} - V_T)^2 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [1]^2$$

$$\mu_n I_{DD} = \mu_p I_{OL} = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} [2(V_{GS1} - V_{TN})V_O - V_O^2]$$

$$\rightarrow 0.2 = 8V_O - V_O^2 \Rightarrow V_{OL} = 0.0261V$$

example:

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS1} - V_{TL})^2 = m \frac{1}{2} \mu_p C_{ox} \frac{W}{L} [2(V_{GS1} - V_{TN})V_O - V_O^2]$$

$$\rightarrow 0.34 = m [8V_O - V_O^2]$$

$$a) m=3 \rightarrow V_{OL} = 0.014192V$$

$$b) m=1 \rightarrow V_{OL} = 0.04293V$$

example: $I_D = 1 \text{ mA}$

$$M_1: 0.5 = 8V_{DS1} - V_{DS1}^2 \rightarrow V_{DS1} = 0.062116 \text{ V}$$

$$M_2: 0.5 = 2(4 - V_{DS1})V_{DS2} - V_{DS2}^2 = 0.06401 \text{ V}$$

$$M_3: 0.5 = 2(4 - V_{DS1} - V_{DS2})V_{DS3} - V_{DS3}^2 = 0.06509 \text{ V}$$

$$\rightarrow V_{DS} = 0.19211 \text{ V}$$

$$b) \quad I_D = 1 \text{ mA} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \rightarrow \mu_n C_{ox} \frac{W}{L} = 1 \text{ mA/V}^2$$

example

$$a) \quad \mu_n C_{ox} \frac{W}{L} = \mu_p C_{ox} \frac{W}{L} \quad \text{and} \quad V_{TH} = |V_{TP}| \rightarrow V_{SS} = \frac{V_{DD}}{2}$$

$$\rightarrow V_{SS} = 1.5 \text{ V}$$

$$b) \quad \text{both sat} \rightarrow I_D = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (V_{GS} - |V_{TP}|)^2 \\ = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \\ = 2 \times 10^{-3} \cdot (2.5 - 1)^2 = 4.5 \text{ mA}$$

past mid exam practice:

$$1) \quad \mu_p C_{ox} \frac{W}{L} = \frac{\mu_n C_{ox} \frac{W}{L}}{k_{n2}}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (-1.2)^2 =$$

$$V_{SS} = V_{TH} \pm \sqrt{\frac{\mu_n C_{ox} \frac{W}{L}}{\mu_p C_{ox} \frac{W}{L}}} \cdot V_{TL}$$

$$\rightarrow 1.6 = 1 \pm 1.2 \cdot \sqrt{\frac{1}{k_{n2}}} \rightarrow 0.5 = \frac{1}{k_{n2}}$$

$$\rightarrow k_{n2} = 4$$

2) $V_{SS} \rightarrow$ both sat

$$\rightarrow I_{DD} = I_{DL} \rightarrow \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (-V_{TL})^2$$

$$\rightarrow k_{n2} = \frac{(1.2)^2}{(1.6 - 1)^2} \rightarrow k_{n2} = 4$$

$$3) \quad NM_L = V_{IL} - V_{OL}, \quad V_{OL} =$$

$$b) \quad V_{OL} = 0.2 \quad V_{IL} = 1.2 \rightarrow NM_L = 1$$

$$7) NM_H = V_{OH} - V_{IH} \quad V_{OH} = V_D$$

$$V_{IH} = V_{TN} \rightarrow NM_H = 4$$

$$V_{IL} = V_{TN} + \frac{1}{2A_{OD}} \quad \lim_{R_D \rightarrow \infty} = V_{TN}$$

$$8) \infty V_I = 4.2V \rightarrow \text{high}$$

$$V_{SG} = 0.8V (5 - 4.2)$$

for on $V_{SG} > |V_{TP}| \quad \times \rightarrow$ load is off

$$\rightarrow V_{DL} = 0 \rightarrow V_{DSL} = -5$$

$$9) P_{avg} = \frac{P_{IH} + P_{IL}}{2} \quad \Delta P_{IL} = 0$$

$$P_{IH} = \frac{(5 - 0.5) \cdot 1.2}{1.2} = \frac{20.25 \text{ mW}}{2}$$

$$\frac{1}{2} [VI] = \frac{5}{2} \cdot \frac{4.9}{1.2} = 11.25 \text{ mW}$$

$$10) \infty V_{in} = 0.3 \rightarrow n_1, \text{ off}$$

$$V_o = 5 \rightarrow V_{DS1} = V_{DD} - V_o = 0$$

$$11) \infty V_{in} = 1.2 \rightarrow V_{GS1} - V_{TN} = 0.2$$

$$V_o \approx 0.2$$

$$n_1: \quad 5 - V_o \approx 1.2$$

$$\text{if } V_o > 0.2 \rightarrow$$

linear

$$\text{if } V_o < 0.2 \rightarrow 5 - V_o > 1.2$$

or

$$V_o = 4.6 \rightarrow V_{DS1} = 0.4V < V_{GS1} - V_{TL} \quad \times$$

\Rightarrow load is linear

$$V_{DS1} = 4.6 \rightarrow 4.6 > V_{GS1} - V_{TN} \rightarrow \text{not}$$

$$B) I_{D1} \cdot (V_{in} - 0.8)^2 = (V_{DD} - V_o - 1.2)^2$$

$$\rightarrow 1 = 5 - V_{DS1} - 1.2$$

$$V_{DS1} = 2.8V$$

$$C) V_{GS1} = 1.5 \quad \rightarrow \quad V_{GS1} ? 0.5$$

$$V_{SGP} = 3.5$$

$$V_{SDP} > V_{SGP} - |V_{TP}| \quad \text{for sat}$$

$$V_{SDP} = V_{SS} - V_o$$

$$\circ \circ \quad I_{D1} \neq I_{D2} \rightarrow V_{SS} = \frac{V_{TN} + \sqrt{\frac{\mu_{p0}}{\mu_{n0}}} [V_{DD} - |V_{TP}|]}{1 + \sqrt{\frac{\mu_{p0}}{\mu_{n0}}}}$$

$$\rightarrow V_{SS} = \frac{1 + 0.5 [V_{DD} - 1]}{1.5}$$

$$\rightarrow V_{SS} = \frac{2}{3} + \frac{1}{3} V_{SS} - \frac{1}{3}$$

$$\rightarrow \frac{2}{3} V_{SS} = \frac{1}{3} \rightarrow V_{SS} = 0.5V$$

$$\therefore \text{Mn sat}$$

$$V_{SS} = \frac{V_{TN} + \sqrt{\frac{\mu_{p0}}{\mu_{n0}}} [V_{SS} - |V_{TP}|]}{1 + \sqrt{\frac{\mu_{p0}}{\mu_{n0}}}}$$

$$\rightarrow V_{SS} = \frac{1 + 0.5 [V_{SS} - 1]}{1 + 0.5}$$

$$\rightarrow 1.5 V_{SS} - 0.5 V_{SS} = 0.5 \rightarrow V_{SS} = 0.5$$

V_{SS} estimation \neq V_{SS} source

$$\rightarrow 1.5 V_{SS} - 0.5 V_{SS} = 0.5$$

$$\rightarrow V_{SS} = \frac{0.5 + 2.5}{1.5} = 2V$$

$\circ \circ \quad V_{TN} < V_{in} < V_{SS} \rightarrow$ Mn sat & Mp ohmic

$$I_D = 200 \mu [1.5 - 1]^2 = 50 \mu A$$

$$Q3) V_{DSL} = V_{DD} - V_o$$

$$V_{DSL} = V_{DD} - V_o \rightarrow V_{DSL} = V_{DSL}$$

$$\text{sat} \rightarrow V_{DSL} \approx V_{DSL} - 1.2$$

$$V_o = V_{DD} - V_{DSL} = 5 - 1.2 = V_o$$

$$V_{DSL} = 5 - V_o = 1.2 V$$

$$Q4) I_{DD} = I_{DL} \rightarrow (V_{DSO} - V_{TL})^2 = (V_{DSL} - V_{TL})^2$$

$$V_{DSL} = 5 - V_o \quad \wedge \quad V_{DSO} = V_{in} = 2.4$$

$$\rightarrow 2.4 - 0.8 = 3.8 - V_o \rightarrow V_o = 1.2 V$$

Q5) max power dissipation in CMOS occurs when both transistors are sat

$$Q6) V_{DS1} = 4.5 > V_{in} - V_{TN} \rightarrow \text{sat } M_1$$

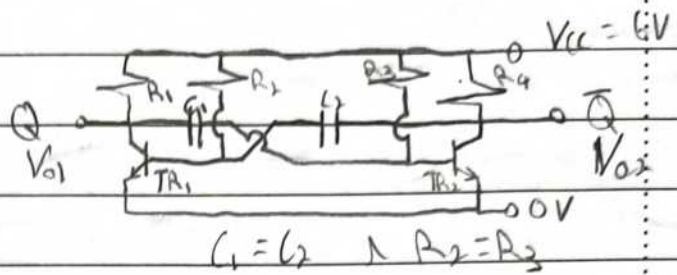
$$\text{M}_2: V_{DS2} = 0.5 < |1 - 1.2| \rightarrow \text{linear}$$

Astable multivibrators:

1- if TR_1 is off, TR_2 is on

$V_{B1} = V_{CC}, V_{B2} = 0.1$

$V_{CC} \frac{C_1}{1} = 0.8V = 0.8 - V_{CC} \frac{C_2}{1} = 0.1$



2- C_2 will start charging through R_2 until $V_{B1} = 0.8$

when TR_1 turns to on and TR_2 turns off $\rightarrow V_{B1} = 0.1V \wedge V_{B2} = V_{CC}$

3- C_1 will then start to charge through R_3 until $V_{B2} = 0.8V$ which is when TR_2 becomes on and TR_1 goes off

4- then 2 repeats and so on

- given $V_{B1}(t) = V_{B1}(\infty) - [V_{B1}(\infty) - V_{B1}(0)] e^{-t/\tau}$

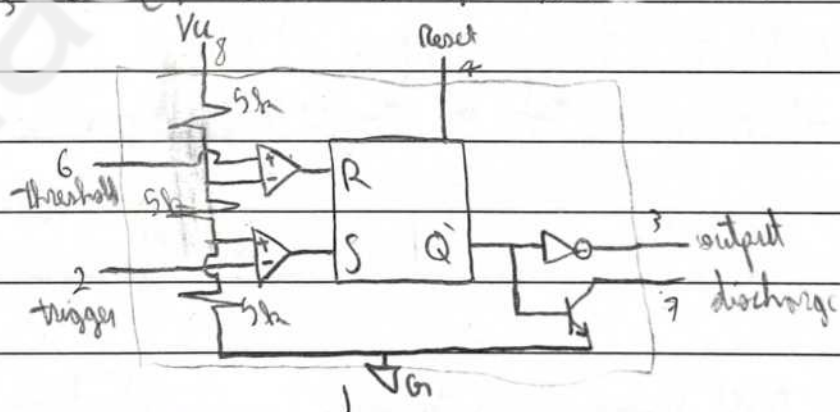
\rightarrow time for V_{B1} to equal $(0.8V) : T_1 = R_2 C_2 \ln(2)$

$\wedge T_2 = R_3 C_1 \ln(2)$ time for V_{B2} to equal $0.8V$

$\therefore T = R_3 C_1 \ln(2) + R_2 C_2 \ln(2)$

if $R_2 = R_3 \wedge C_1 = C_2 \rightarrow T = 2RC \ln(2)$

& 555-timer:



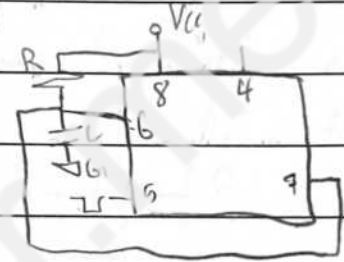
-The open-loop op-amp can be used as a comparator whose output is high if the voltage at its positive terminal is higher than that at its negative terminal else the output will be low (i.e., negative > positive \rightarrow output low)

* mono-stable MV using 555:

1- before trigger: $S=0, R=0, Q=0, Q'=1$, transistor is on. This state is stable. (C is discharged)

2- after trigger: $S=1, R=0, Q=1, Q'=0$, transistor is (negative trigger) off, C starts charging

3- Remove the trigger before V_C reaches $\frac{2}{3}V_{CC}$:
 $S=0, R=0, Q=1, Q'=0$, transistor is off, C will continue charging



4- when V_C exceeds $V_{CC} \cdot \frac{2}{3}$:

$S=0, R=1, Q=0, Q'=1$, transistor is on, C will discharge.

5- when V_C goes below $\frac{1}{3}V_{CC}$:

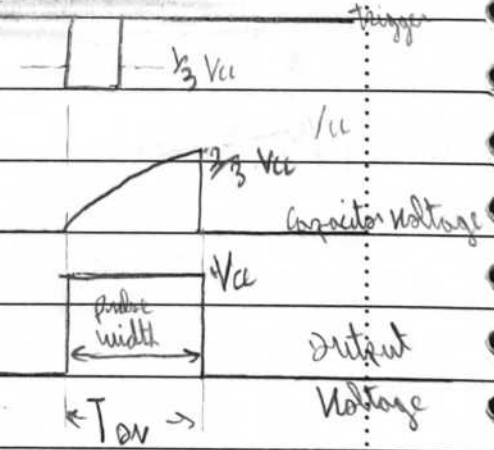
$S=0, R=0, Q=0, Q'=1$, transistor is on, C will fully discharge. This is the stable state of the circuit

$$V_C(t) = V_C(\infty) - [V_C(\infty) - V_C(0)]e^{-t/\tau}$$

$$\rightarrow V_C(t) = V_{CC} [1 - e^{-t/\tau}], \tau = RC$$

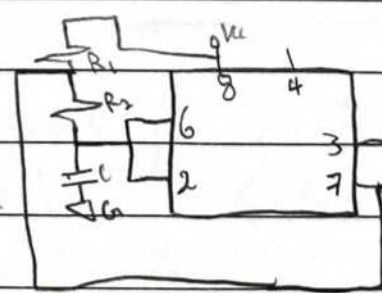
$$V_C(T) = \frac{2}{3} V_{CC} = V_{CC} (1 - e^{-T/\tau})$$

$$\rightarrow -\ln(1/3) = \frac{-T}{\tau} \rightarrow T = RC \ln(3) \approx 0.7$$



* Astable MV using 555 timer:

1- assuming the fb is initially reset, then the transistor will be saturated and the capacitor will discharge to less than $\frac{1}{3}V_{CC}$, which will cause the fb to be set



2- When the fb is set, $\bar{Q} = 0$, and the transistor is off hence the capacitor will charge.

at $V_C = \frac{1}{3} V_{CC}$, the output of the S comparator will go low.

and at $V_C = \frac{2}{3} V_{CC}$, the output of the R comparator will go high and the fb will be reset.

3- if the fb is reset, the transistor will be set and the capacitor will discharge to repeat the process.

$$\infty \quad V_{C,c}(t) = V_C(\infty) - [V_C(\infty) - V_C(0)] e^{-t/\tau_c}$$

$$\rightarrow V_{C,c}(t) = V_{CC} - V_{CC} \left[\frac{2}{3} \right] e^{-t/\tau_c} = 1 - \frac{2}{3} e^{-t/\tau_c}$$

where $\tau_c = (R_1 + R_2)C$

$$\wedge \quad V_{C,d} = V_{CC} - [V_{CC} - \frac{2}{3} V_{CC}] e^{-t/\tau_d}$$

$$\rightarrow V_{C,d} = V_{CC} [1 - \frac{1}{3} e^{-t/\tau_d}] \rightarrow \tau_d = R_2 C$$

$$\tau_{on} = (R_1 + R_2)C \cdot \ln(2) \quad \wedge \quad \tau_{off} = R_2 C \cdot \ln(2)$$

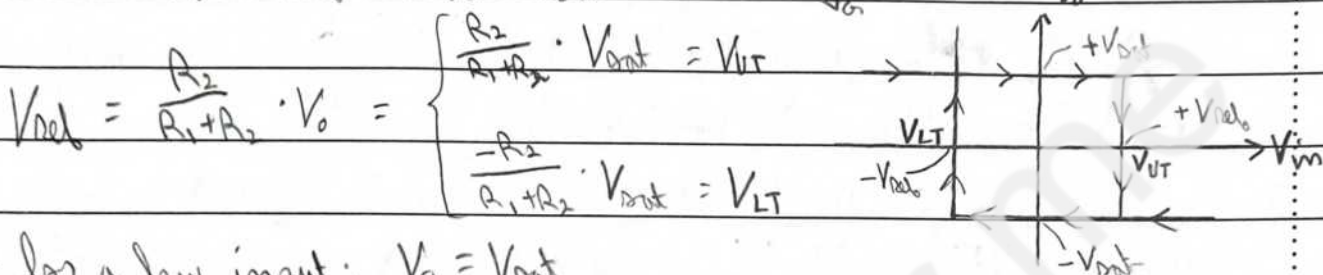
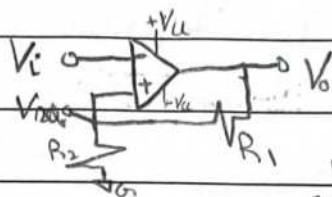
$$T = \tau_{on} + \tau_{off} \rightarrow D = \frac{\tau_{on}}{T} > 40\%$$

* Multivibrator using Schmitt trigger:

+ Schmitt trigger operation (inverting)

$$V_i > V_{oh} \rightarrow V_o = -V_{sat}$$

$$V_i < -V_{oh} \rightarrow V_o = V_{sat}$$



- for a low input: $V_o = V_{sat}$

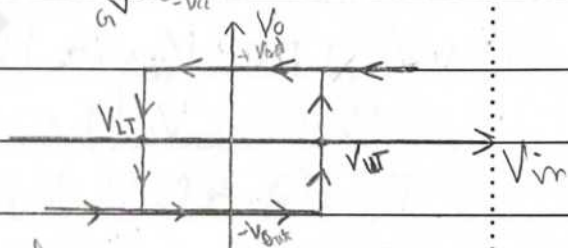
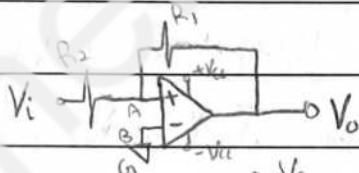
for a high input: $V_o = -V_{sat}$

+ Non-inverting Schmitt trigger:

$$V_A = \frac{R_1}{R_1 + R_2} V_i + \frac{R_2}{R_1 + R_2} V_o$$

$$V_B = 0$$

$$V_o = \pm V_{sat}$$



- assuming $V_o = -V_{sat}$ implies that V_A is less than zero for low input, then increase the input until $V_A = 0V$

$$\rightarrow V_A = \frac{R_1}{R_1 + R_2} V_i + \frac{R_2}{R_1 + R_2} (-V_{sat}) = 0 \rightarrow V_i = V_{UT} = \frac{R_2}{R_1} V_{sat}$$

- hence V_{UT} is our high input that gives $V_o = V_{sat}$

- next, set $V_o = V_{sat}$ and decrease the input until $V_A = 0$:

$$V_i = V_{LT} = -\frac{R_2}{R_1} V_{sat}$$

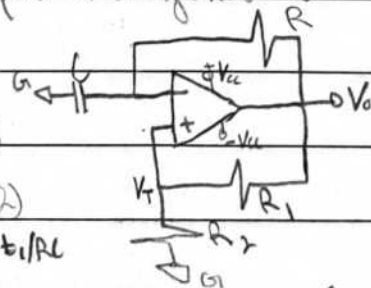
- therefore, V_{LT} input is the low input that gives $V_o = -V_{sat}$

* Astable multivibrator:

$$V_{c, sch} = V_{sat} - [V_{sat} - (-V_T)] e^{-t/RC} \quad \text{--- (1)}$$

$$\text{at } t = t_1, V_c = V_T = \frac{R_2}{R_1 + R_2} V_{sat} \quad \text{--- (2)}$$

$$\text{sub (2) in (1)} \rightarrow V_{sat} - [V_{sat} + \frac{R_2}{R_1 + R_2} V_{sat}] e^{-t_1/RC} = \frac{R_2}{R_1 + R_2} V_{sat} \therefore t_1 = RC \ln(1 + \frac{2R_2}{R_1})$$



$$V_{C, \text{dir}} = -V_{\text{out}} - [-V_{\text{out}} - V] e^{-t/RC} = -V = \frac{-R_2}{R_1 + R_2} V_{\text{out}}$$

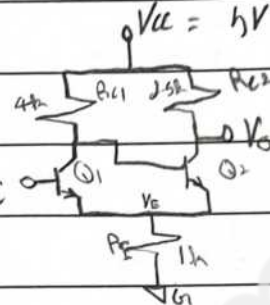
$$\therefore t_2 = RC \ln \left(1 + \frac{2R_2}{R_1} \right)$$

& Schmitt trigger using BJT:

- for $V_i = 0$, Q_1 is off and Q_2 must be on

$$I_E = I_B + I_C = I_{RC2} + I_{RC1} = I_{RF}$$

$$\rightarrow \frac{V_E}{R_E} = \frac{V_{CC} - V_{BE_{\text{sat}}} - V_E}{R_{C1}} + \frac{V_{CC} - V_{CE_{\text{sat}}} - V_E}{R_{C2}}$$



$$V_{CE_{\text{sat}}} = 0.2V$$

$$V_{BE_{\text{sat}}} = 0.7V$$

$$\therefore V_E = 1.8V \quad \wedge \quad V_o = 2V$$

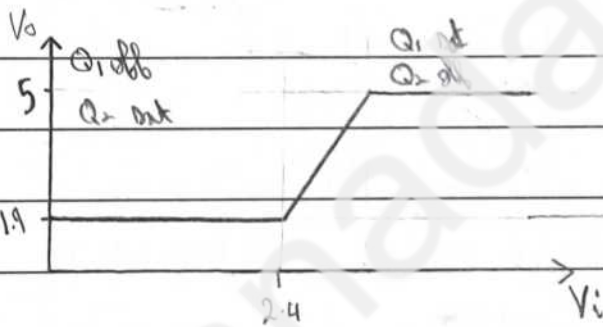
if Q_1 is placed at the edge of conduction by

increasing V_i until $V_{BE1} = 0.6V$:

for $V_i > 2.4$, Q_1 will be forward then

not causing Q_2 to go off

$$V_o = V_{CC}$$

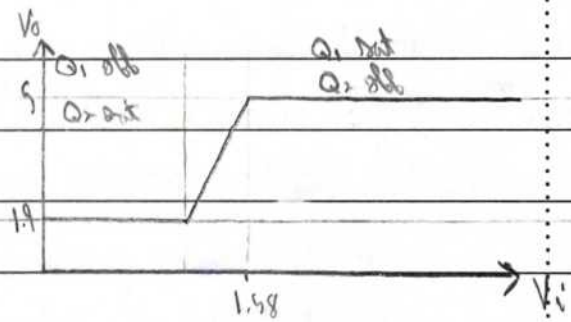


if Q_2 is placed at the edge of conduction

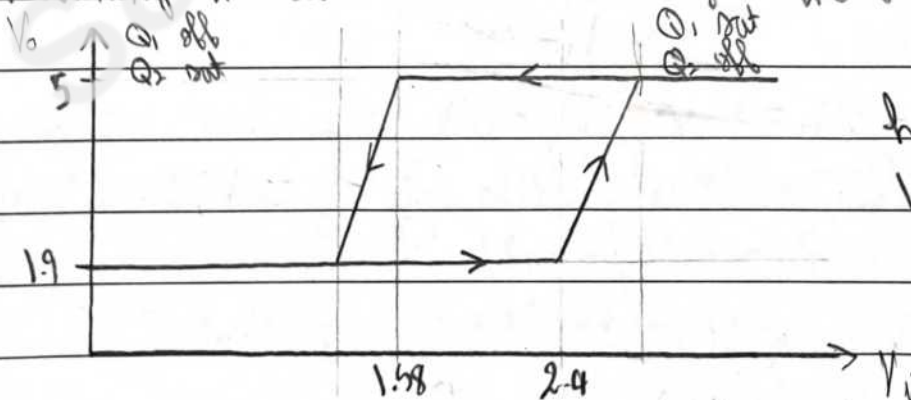
by decreasing V_i until $V_{CE1} = 0.6V$

neglecting $I_B \rightarrow I_C = I_E \rightarrow \frac{V_E}{R_E} = \frac{V_{CC} - 0.6 - V_E}{R_{C1}}$

$$\therefore V_E = 0.88V \quad \wedge \quad V_i = 1.58V$$



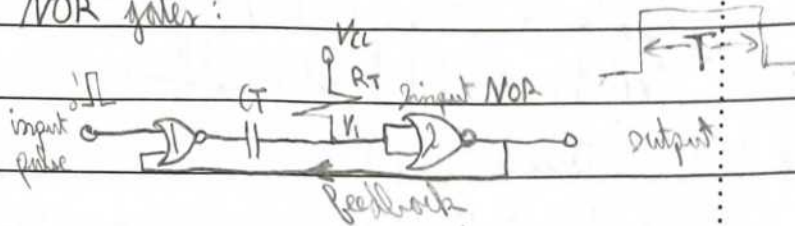
- Combining the above two VTC will give the following hysteresis



hysteresis width = $2.4 - 1.58$

Monostable MV using TTL NOR gates:

- if the trigger input is initially low:



Voltage across capacitor $V_{OH} \parallel V_{cc} \parallel V_1 \rightarrow$ the output from 2 will be low.

- this low input - low output state represents the circuit's stable state.

- when a positive trigger is applied, the output of the first NOR gate will go low and the capacitor will begin to charge through R_T :

$V_{OL} \parallel V_{cc} - (V_{OH} - V_{OL}) \therefore V_1(t) = V_1(\infty) - [V_1(\infty) - V_1(0)] e^{-t/\tau}$, $\tau = R_T C_T$

$\rightarrow V_1(t) = V_{cc} - [V_{cc} - (V_{cc} - V_{OH} + V_{OL})] e^{-t/R_T C_T}$

$\therefore V_1(t) = V_{cc} - [V_{OH} - V_{OL}] e^{-t/\tau}$

- define T : time for V_1 to equal $V_{IH} \rightarrow T = R_T C_T \ln \left[\frac{V_{OH} - V_{OL}}{V_{cc} - V_{IH}} \right]$

- the large jump in V_1 voltage

occurs when V_1 reaches V_{IH}

hence turning the output of NOR2

low, which is input into NOR1

causing it to go high. Therefore, the sudden increase in voltage occurs from the capacitor charging through NOR1.

$V_1 \lesssim V_{IH}$

$V_{OH} \parallel V_{IH}$

$V_1 > V_{IH}$

$V_{OH} \parallel V_{IH} + (V_{OH} - V_{IH})$

example: assume $V_{OH} = 3.8V$, $V_{OL} = 0.1$, $V_{IH} = 1.6V$, $R = 15k\Omega$, $C = 1nF$

pulse width: $T = 15 \times 10^{-6} \cdot \ln \left[\frac{3.7}{3.4} \right] \approx 1.27 \times 10^{-6} s$

Monostable MV using CMOS NOR gates:

$V_{OH} = V_{DD}$, $V_{IH} = V_{DD}/2$, $V_{OL} = 0$

$$\rightarrow V_1(t) = V_1(\infty) - [V_1(\infty) - V_1(0)] e^{-t/\tau}$$

$$\rightarrow V_1(t) = V_{OH} - [V_{OH} - (V_{OH} - (V_{OH} - 0))] e^{-t/\tau}, \tau = RC$$

$$\therefore V_1(t) = V_{OH} [1 - e^{-t/\tau}]$$

$$\text{if } t = T \rightarrow V_1 = V_{IH} \rightarrow T = RC \ln \left(\frac{V_{OH}}{V_{OH} - V_{IH}} \right)$$

* Astable MV using inverters:

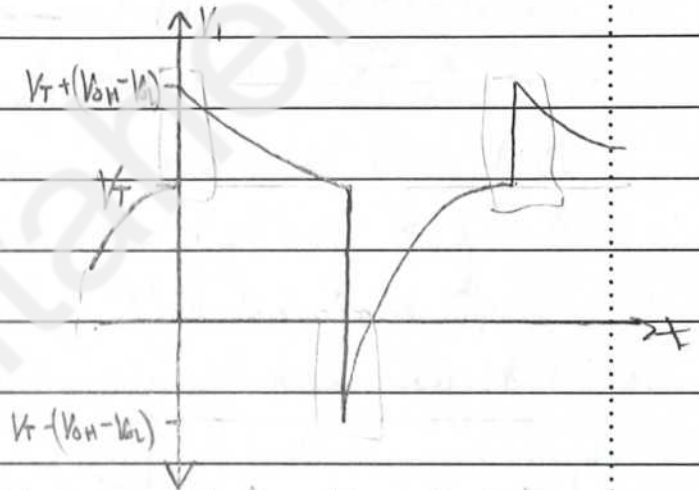
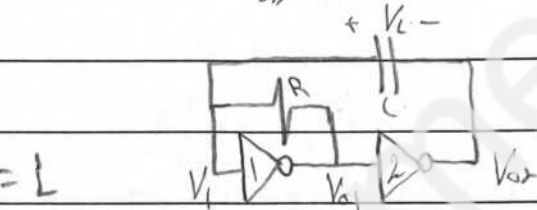
+ two possible states:

$$1) V_1 = L \rightarrow V_2 = H \rightarrow V_1 = L$$

$$2) V_1 = H \rightarrow V_2 = L \rightarrow V_1 = H$$

- both states are unstable

- assuming $V_1 = H$, C will start to charge through R until the turn-on voltage (V_T) of inverter 1 is reached, which is registered as a high input and causes V_1 to turn low.



- following the large jump in voltage, caused by the output of inverter 2 suddenly turning high, the capacitor will begin to discharge through R, until the voltage (V_1) is low enough to turn the output V_1 high. Another jump is seen here as V_2 suddenly goes low.

- discharging: $V_{1d}(t) = V_{1d}(\infty) - [V_{1d}(\infty) - V_{1d}(0)] e^{-t/\tau}$

$$\rightarrow V_{1d}(t) = V_{OL} - [V_{OL} - (V_T + (V_{OH} - V_{OL}))] e^{-t/\tau}$$

$$\therefore V_{1d}(t) = V_{OL} - [2V_{OL} - V_T - V_{OH}] e^{-t/\tau}$$

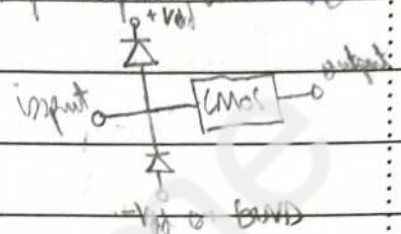
define T_1 : time at which $V_{1d} = V_T \rightarrow V_{1d}(T_1) = V_T$

$$\therefore T_1 = \tau \ln \left[\frac{2V_{OL} - V_{OH} - V_T}{V_{OL} - V_T} \right]$$

- charging: $V_{ic}(t) = V_{OH} - [2V_{OH} - V_{OL} - V_T] e^{-t/\tau}$

- T_2 : time at which $V_{ic}(t) = V_T \therefore T_2 = \tau \ln \left[\frac{2V_{OH} - V_{OL} - V_T}{V_{OH} - V_T} \right]$

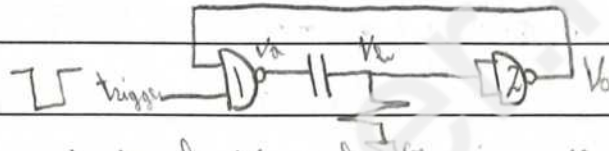
- If CMOS is used, then the maximum voltage input will be limited to V_{DD} by the protection diodes:



* Monostable using NAND gates:

- the circuit is triggered by a negative pulse and gives a negative pulse at the output.

- the circuit's stable



state occurs when the output of V_o is low since the inputs at NAND 1 will be low and high respectively, which will give no change to the output when the circuit is triggered [(Low * Low) = high]

- the circuit must be started at the quasi-stable state ($V_o = \text{high}$) so that will allow the circuit to stabilize.

* DAC: digital to analog converter.

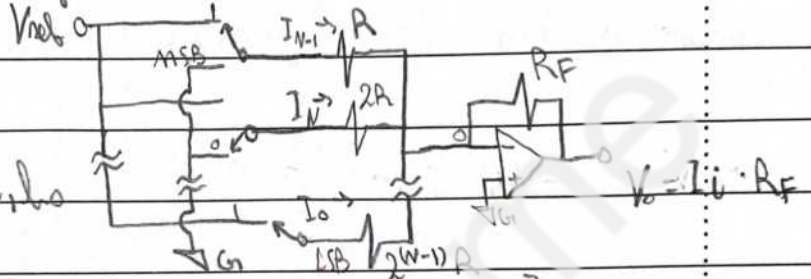
* ADC: analog to digital converter.

+ types of DACs: Binary weighted, 1) R-2A ladder.

1) Binary weighted DAC:

- if $N=4 \rightarrow$ $b_3 b_2 b_1 b_0$

$b_{N-1} b_{N-2} \dots b_1 b_0$



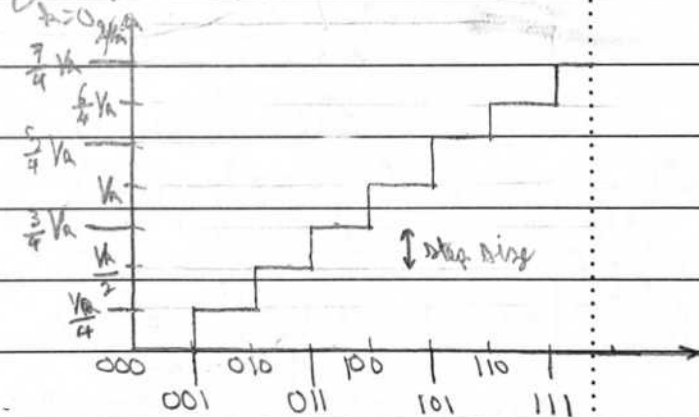
$$V_o = -I_i \cdot R_F = -R_F [I_0 + I_1 + \dots + I_{N-1}]$$

$$\therefore V_o = -V_{ref} \cdot \frac{R_F}{R} \cdot \frac{1}{2^{N-1}} \left[\frac{b_0}{2^0} + \frac{b_1}{2^1} + \dots + \frac{b_{N-2}}{2^{N-2}} + \frac{b_{N-1}}{2^{N-1}} \right]$$

example: $N=3, R_F=R, V_{ref} = -5V$

$$\rightarrow V_o = 5 \cdot \frac{1}{4} \cdot \sum_{k=0}^{N-1} 2^k \cdot b_k$$

b_2	b_1	b_0	V_o
0	0	0	$\frac{1}{4} V_{ref} = 0$
0	0	1	$\frac{1}{2} V_{ref}$
0	1	0	3.75
1	0	0	5
1	0	1	6.25
1	1	0	7.5
1	1	1	8.75

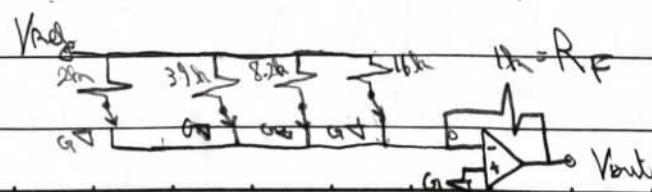


- Binary weighted DACs require a wide range of precision matching resistors: if $N=8 \rightarrow$ resistors needed: $2k, 4k, \dots, 128k$

- resistors should also have low tolerances and low temperature variations

8- Constant reference current

- another issue with binary weighted DACs is the



large jumps in current passing, which might cause a overshoot.

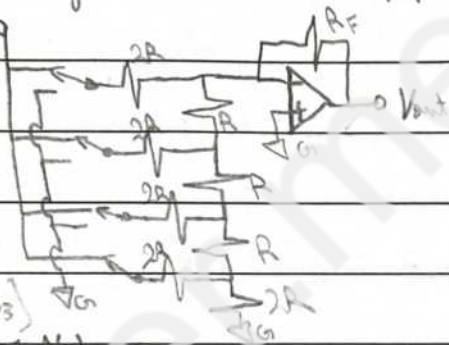
The circuit shown on the previous page will have a constant current passing through the resistors, based on the switch position, the current will either flow from/to the ground or the R_F .

② R-2R ladder DAC:

$$\rightarrow V_o = -V_{ref} \cdot \frac{R_F}{R} \left[\frac{B_0}{16} + \frac{B_1}{8} + \frac{B_2}{4} + \frac{B_3}{2} \right]$$

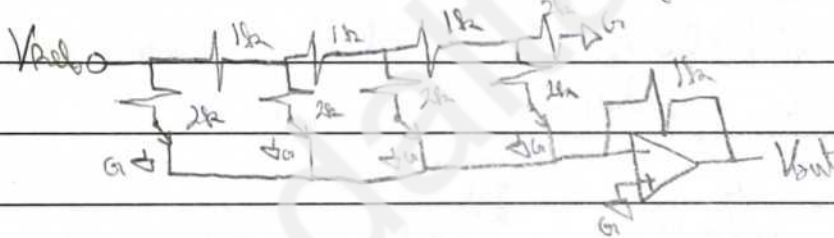
$$\therefore V_o = -V_{ref} \cdot \frac{R_F}{R} \cdot \frac{1}{16} [2^0 B_0 + 2^1 B_1 + 2^2 B_2 + 2^3 B_3]$$

$$\rightarrow V_o = -V_{ref} \cdot \frac{R_F}{R} \cdot \frac{1}{2^N} \sum_{k=0}^{N-1} 2^k B_k$$



- the same current issue still stands with the standard R-2R circuit.

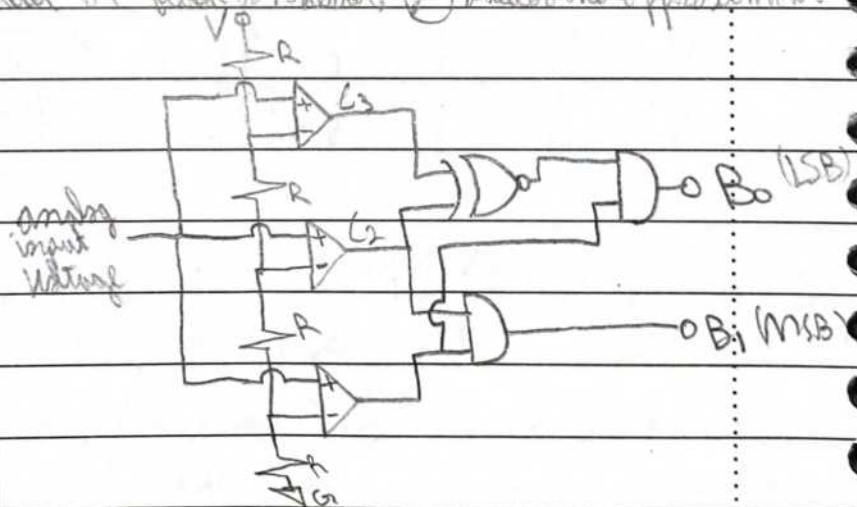
* Modified R-2R circuit: (constant reference current)



+ Types of ADCs: ① parallel or flash converter, ② successive approximation

① 2-bit flash converter:

Analog input condition	Comparator outputs			Digital output	
	C_1	C_2	C_3	B_1	B_0
$0 \leq V_{in} < \frac{V}{4}$	0	0	0	0	0
$\frac{V}{4} \leq V_{in} < \frac{V}{2}$	1	0	0	0	1
$\frac{V}{2} \leq V_{in} < \frac{3V}{4}$	1	1	0	1	0
$\frac{3V}{4} \leq V_{in} \leq V$	1	1	1	1	1



- for an N-bit output, $2^N - 1$ comparators must be used

+ advantages of flash converters:

- fastest type of ADC since the conversion is performed simultaneously typically taking 100 ns
- easy to design and construction is simple.

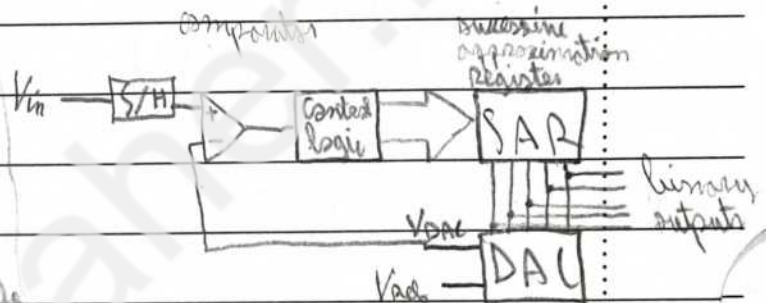
+ disadvantages of flash ADCs:

- not suitable for large numbers of bits as the number of comparators nearly doubles for every added bit.

(2) Successive approximation ADC:

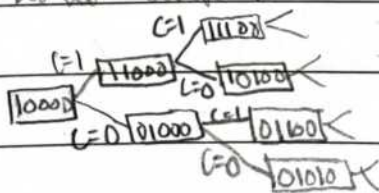
+ conversion procedure:

- 1) start of conversion (SOC) input is set to (1), then the SAR will set the MSB equal to 1 and the rest equal to zero as an initial guess (i.e., for 5 bits: 10000)



- 2) the V_{DAC} output is then compared with V_{in}
 - if $V_{in} > V_{DAC}$, the output of the comparator will be (1) and the SAR will maintain the MSB as (1) and set the next bit equal to (1) (i.e., 11000)
 - if $V_{in} < V_{DAC}$, the output of the comparator becomes (0) and the MSB is changed to zero while the second is set to (1) on the second guess (i.e., 01000)

- 3) the cycle repeats for all other bits until they have all been tested and set.



example: $N=4$, $V_{ref} = -5V$, $V_{in} = 3.45V$, DAC: binary weighted, $R=R_0$

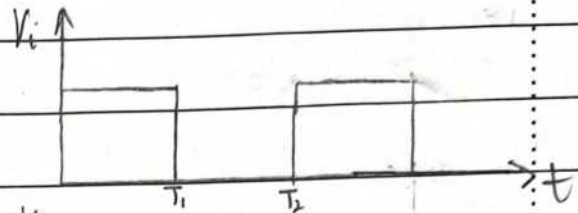
$$V_{DAC} = -\frac{V_{ref}}{2^{N-1}} \cdot \frac{R_F}{R} \sum_{k=0}^{N-1} 2^k b_k = \frac{5}{8} \sum_{k=0}^3 2^k b_k$$

$$\therefore 1) 1111 = 1000 \rightarrow V_{DAC} = 5V > V_{in} \quad \times$$

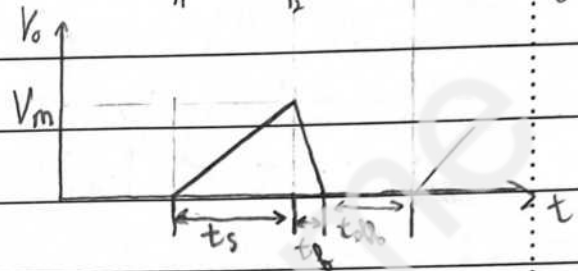
$$2) 0100 = 2.5V < V_{in} \quad \times$$

$$4) 0101 \approx V_{in} \rightarrow 3.45 = 0101 \quad (\text{closest})$$

* Sweep circuits:



- V_o should be linear as a function of time in interval $T_1 - T_2$

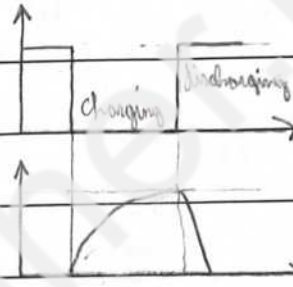
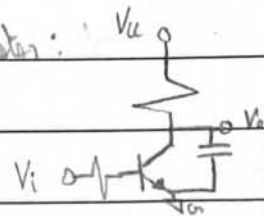


* t_s : sweep time, t_d : flyback time

- t_d should be very small.

* Sweep circuit with resistor:

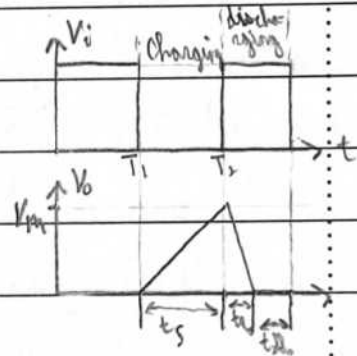
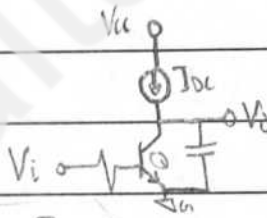
- Since the capacitor is charging through a resistor, the output will be exponential.



* Constant current sweep circuit:

• When Q is off: $I_{DC} = C \frac{dV}{dt}$

$$\rightarrow V_o(t) = \frac{I_{DC} t}{C}$$



- If Q is saturated: $\beta I_B > I_{DC} + I_{C,sat}$

• When charging: $I_{DC} = C \frac{dV_o}{dt}$

$$\rightarrow V_o(t) = \frac{I_{DC} t}{C} \rightarrow V_m = \frac{I_{DC}(T_2 - T_1)}{C} \text{ (max)}$$

• When discharging, Q is forward active:

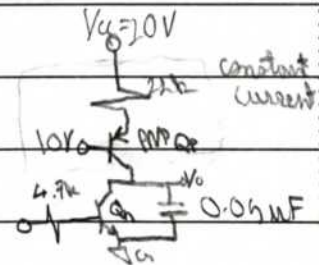
$$I_{DC} = \beta I_B + C \frac{dV_o}{dt} \rightarrow I_{DC} = \beta I_B + C \frac{(0 - V_m)}{t_d}$$

$$\therefore t_d = \frac{C V_m}{\beta I_B - I_{DC}}$$

Example: QP is forward active

∵ QP is FA \rightarrow Voltage on collector terminal is always 0.9V higher than that on the base.

$$\rightarrow I_{EP} = \frac{20 - 10.9}{22k} = 0.42293 \text{ mA}$$



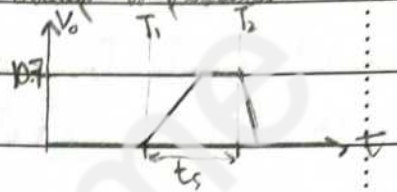
$$I_{CP} = \frac{\beta + 1}{\beta} I_{CP} \rightarrow I_{CP} = 0.4185 \text{ mA constant}$$

$$I_{CP} = C \frac{dV_o}{dt} \rightarrow V_o = \frac{I_{CP} \cdot t}{C} = 8.39 \times 10^2 t$$

- The maximum output voltage is set to be 10.9V, so the voltage drop across the OP will become zero then and cause the ramp to plateau.

$$\rightarrow V_{om} = 10.9 > \frac{I_{CP}}{C} \cdot t_s \quad \wedge \quad t_s = T_2 - T_1$$

$$\therefore t_s < 1.2784 \text{ ms}$$



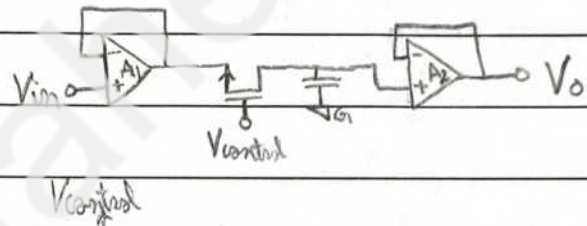
- for discharging: Qn1 first goes to forward active

$$I_{CP} = \beta n I_{Bn} + C \frac{dV_o}{dt} \rightarrow 0.4185 \text{ mA} = 100 \cdot I_B + C \cdot \frac{-V_m}{t_f}$$

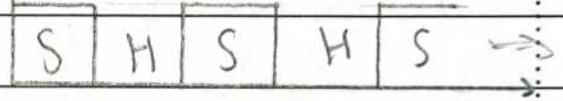
$$\therefore t_f = \frac{C \cdot V_m}{\beta n I_{Bn} - I_{CP}}$$

* Sample & hold:

- Voltage follower op-amp are used as buffers to eliminate loading effect.



- When active, the MOSFET can be modeled as a resistor and it will allow the capacitor to charge and sample the input.



- When the control input is low, the fet will go off, and the input will not be sampled.

$$Q1) T = t_2 + t_1, \quad t_1 = R_1 \cdot \ln\left(1 + \frac{2R_2}{R_1}\right) = 1k \cdot 1\mu F \cdot \ln(1+2) = 1.0986 \text{ ms}$$

$$t_2 = R_2 \ln(1+2) \rightarrow f = 0.4551 \text{ kHz}$$

$$Q2) Q_0 \text{ is always FA} \rightarrow V_{OH} = -0.9V$$

$$V_{OL} = -0.9 - I_E \cdot 300 \quad I_E = \frac{V_E + 5}{R_E} \quad V_E = V_{ref} - 0.9$$

$$\rightarrow V_{OL} = -0.9 - \frac{300}{1240} \left[\frac{V_{OL} - 0.9}{2} - 0.9 + 5 \right]$$

$$\rightarrow V_{OL} = -0.9 - \frac{15}{124} V_{OL} + \frac{15}{124} \cdot 0.9 + \frac{15}{124} \cdot 0.9 - \frac{15}{124} \cdot 5$$

$$\rightarrow V_{OL} \left[1 + \frac{15}{124} \right] = \frac{-2093}{1240} \rightarrow V_{OL} = -1.479V$$

$$\rightarrow V_{ref} =$$

$$I_E = \frac{V_E + 5}{1240} \rightarrow V_E = V_{ref} - 0.9$$

$$\rightarrow I_E = \frac{V_{ref} + 4.3}{1240}$$

$$-\frac{15}{124} V_{OL} + \frac{15}{124} \cdot 0.9 - \frac{15}{124} \cdot 4.3$$

$$Q2) T_{on} = R_1 \ln(3) = 11 \text{ ms}$$

$$Q5) V_{IA}, \quad V_E = -1.5942 \rightarrow I_E = 2.963 \text{ mA}$$

$$\rightarrow I_{EB} = 1.381 \text{ mA}$$

$$Q6) V_E = 4.9 - 0.9 \rightarrow I_E = 2.923 \text{ mA}$$

$$Q9) V_{SS} = \frac{1 + \frac{R_2}{R_1} [V_{DD} - 1]}{1 + \frac{R_2}{R_1}} = 2.242641V$$

$$I_{DD} = 60 \mu A = I_{in} (V_{in} - 1)^2 \rightarrow V_{in} = 1.9946$$

$$Q1) 60 \mu A = I_{in} (V_{DD} - V_{in} - 1)^2 \rightarrow V_{in} = 2.90449 \quad 75$$

$$\text{Q12) } V_{ce} = -0.699$$

$$V_{ce} = \frac{V_{ce0}}{A} = -0.69495$$

$$V_{ce} = \frac{V_{ce0}}{2}$$

$$\text{Q13) } V_{in} < V_i < V_{ss} \rightarrow I_{DQ} = \frac{1}{2} \mu_n (V_{GSQ} - V_{TN})^2 = \frac{1}{2} \mu_n (1.5 - 1)^2 = 25 \mu A$$

$$\text{Q14) } 0.8 - (V_{ce} - 0.1) = -6.1V$$

Lab's examples:

$$2) V_i = 1.5 \rightarrow V_{TN} < V_i < V_{SS}$$

$$V_{SS} = \frac{1 + \frac{1}{\sqrt{2}} [4]}{1 + \frac{1}{\sqrt{2}}} = -2 + 3\sqrt{2} V \approx -2.24264$$

$$\rightarrow I_{DD} = \beta_n (V_{GSn} - V_{TN})^2 = 0.25 \cdot 100 \mu A/V^2 = 25 \mu A$$

$$3) V_{DD} - V_{TN} = 4.4 V$$

$$4) R_R = \frac{I_{drain}}{I_{load}} \Rightarrow V_{SS} = V_{TN} + \sqrt{\frac{I}{\beta_n}} V_{TL}$$

$$\rightarrow R_R = 9$$

$$5) V_{OL} = 0.2, V_{OH} = 5 \rightarrow P_{OH} = 0$$

$$V_{OL} = 0.2 \rightarrow I_{OL} = \frac{5 - 0.2}{10} = 4.8 \text{ mA}$$

$$\rightarrow P_{OL} = 24 \text{ mW} \rightarrow P_{avg} = \frac{P_{OL} + P_{OH}}{2} = 12 \text{ mW}$$

6) linear mode of operation: $V_{GS} > V_{TN}$

$$V_{DS} < V_{GS} - V_{TN}$$

$$\rightarrow V_{GS1} > -2 \wedge V_{GS1} - -2 = 0 - -2 = 2V$$

$$V_{GS1} - V_{TN} > V_{DS1} \quad \circ \circ \quad V_{GS1} = 5 - V_o = 1V$$

$$\rightarrow \text{linear} \quad \therefore I_{DD} = \beta_n [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

$$\rightarrow I_{DD} = 25 \mu \cdot [2(2) \cdot 1 - 1^2] = 75 \mu A$$

$$7) V_o = V(\infty) - [V(\infty) - V(0)] e^{-t/\tau}$$

$$= 5 - (5 - 0.6) e^{-2} = 4.4045 V$$

$$\rightarrow I = \frac{5 - 4.4045}{10 \text{ k}\Omega} = 59.55 \mu A$$

$$8) V_{SS} = 5V$$

$$V_o = 0.5V \rightarrow V_{SS} < V_i < V_{DS} = V_{TL} \rightarrow \text{M.P. sat}$$

$$\rightarrow \text{Mn. ohmic} \wedge \text{M.P. sat} \quad V_i \neq 0$$

$$\rightarrow I_D = \beta_p (V_{GSP} - V_{TP})^2 = \beta_p (5 - 0 - 1.5)^2 = 0.245 \text{ mA}$$

example 8.8: $T_1 = \tau \ln \left[\frac{2V_{OL} - V_{OH} - V_T}{V_{OL} - V_T} \right]$
 $V_{OL} = 0, V_{OH} = V_{DD} \rightarrow \frac{2 \cdot 0 - V_{DD} - V_T}{0 - V_T} = \frac{V_{DD} + V_T}{V_T}$

$\therefore T_1 = RC \ln \left(\frac{V_{DD} + V_T}{V_T} \right)$

$\wedge T_2 = \tau \ln \left[\frac{2V_{OH} - V_{OL} - V_T}{V_{OH} - V_T} \right] \wedge V_{OH} = V_{DD}$

$\wedge V_{OL} = 0 \rightarrow \tau \ln \left[\frac{2V_{DD} - V_T}{V_{DD} - V_T} \right]$

$\therefore R = 10k\Omega \wedge C = 1nF$

$\rightarrow T = 10\mu s \rightarrow T_1 = 10\mu \ln \left(\frac{5+1}{1} \right) = 12.53\mu s$

$\wedge T_2 = 10\mu \ln \left(\frac{10-1}{5-1} \right) = 9.81\mu s$

\therefore the period is $T = T_1 + T_2 = 22.34\mu s$ \wedge duty cycle = $\frac{T_1}{T} = 56.1\%$

if $R = 12k \rightarrow T_1 = 12\mu \ln(3.5) = 15.03\mu s$

$T_2 = 11.77\mu s \rightarrow T = 26.8\mu s$

duty cycle = $\frac{15.03}{26.8} = 56.1\%$ same

1) $T_{off} = R_2 C \ln 2$

$T_{off} \rightarrow V_{C,d} = V(\infty) - (V(\infty) - V(0))e^{-t/\tau}$

$V(\infty) = \frac{5 \cdot 2}{3} = \frac{10}{3} \wedge V(0) = 0$

$\rightarrow V_{C,d} = \frac{10}{3} - \left[\frac{10}{3} e^{-t/\tau} \right] \wedge T_{off} = 20k \cdot C$

$V_{C,d} = 0 \rightarrow t = T$

$\rightarrow \frac{10}{3} = \frac{10}{3} e^{-t/\tau}$

$V(0) = \frac{2}{3} \cdot V_{CC} = \frac{10}{3} \wedge V(\infty) = V_{CC} \cdot \frac{20}{70} = \frac{10}{7}$

or $V(T) = \frac{5}{3}$

$\rightarrow \frac{5}{3} = \frac{10}{7} - \left(\frac{10}{7} - \frac{10}{3} \right) e^{-T/\tau}$

$\rightarrow \frac{\frac{5}{3} - \frac{10}{7}}{\frac{10}{3} - \frac{10}{7}} = \frac{1}{8} = e^{-T/\tau}$

$\rightarrow T_{off} = \tau \cdot \ln(8) = R_2 \cdot C \cdot \ln(8)$

$\wedge R_{eq} = R_1 // R_2 = 14.285k$

$$2) V_{out} = V_{CE2}$$

$$V_{CE2} = V_{CC} - (V_{CC} - V_{CE1}) e^{-t/\tau}$$

$$\tau = R \cdot C = 1k \cdot 1\mu, \quad V_{CE1} = 10$$

$$V_{CE1} = V_{CE \text{ sat}} = 0 \rightarrow V_{CE2} = 10 - 10e^{-t/\tau} = 8.66V$$

$$3) V_C = \frac{R_2}{R_1 + R_2} V_{out} = 2V$$

$$4) t_1 = RC \ln \left(1 + \frac{2R_2}{R_1} \right) = RC \cdot \ln(1+1) = RC \ln 2$$

$$5) V_{UT} = \frac{R_2}{R_1 + R_2} V_{out} = \frac{R_2}{R_1} V_{out} = 3V$$

$$\wedge V_{LT} = -\frac{R_2}{R_1} V_{out} = -3V$$

\rightarrow we are in the hysteresis region

\because Voltage is decreasing $\rightarrow + V_{out} = 6V$

examples from slides:

$$1) V_{DD} = 3V, \quad k_n = 150 \mu A/V^2, \quad V_{TN} = 0.5V$$

$$a) V_o = 0.1 \text{ for } V_i = 3V$$

$$\text{assume linear: } V_o = V_{DD} - R_D k_n [2(V_{GS} - V_{TN}) V_{DS} - V_{DS}^2]$$

$$V_{GS} = V_o \rightarrow 0.1 = 3 - R_D \cdot 150 \mu [2 \cdot (3 - 0.5) \cdot 0.1 - 0.1^2]$$

$$\rightarrow \frac{2.9}{150 \mu} = 0.4 R_D - (0.1)^2 R_D$$

$$\rightarrow R_D = 39.465 \text{ k}\Omega$$

$$b) \text{ transition point: } V_{in} = V_s \quad \wedge \quad V_o = (V_{GS} - V_{TN})$$

$$V_{GS} = V_o = V_{DD} - I_D \cdot R_D \quad \wedge \quad V_{GS} = V_s = V_{in}$$

$$\rightarrow (V_s - V_{TN}) = 3 - 5.91825 (V_s - V_{TN})^2$$

$$\text{define } x = (V_s - V_{TN}) \rightarrow 5.91825 x^2 + x - 3 = 0$$

$$\rightarrow x = 0.632484 \quad \text{or} \quad -0.80145 \quad x = V_o$$

$$\rightarrow V_s = 1.132484 V$$

example 2):

V_t : transition from sat to linear, $V_t = \frac{V_T + \frac{V_{DD}}{\beta R}}{1 + \frac{1}{\beta R R_D}}$

$$1) \text{ or } V_t = \frac{\sqrt{I_D R_D} + V_{DD}}{1 + \frac{1}{\beta R}} \quad \wedge \quad \beta R = \frac{\beta_{min}}{\beta_{max}} = \frac{\beta_{n1}}{\beta_{n2}} = \frac{1}{5.34}$$

$$\rightarrow V_t = \frac{1.7149859 + 5}{1 + \frac{1}{5.34}} = 2.14933042 \text{ V}$$

$$\wedge V_{sat} = V_t - \frac{1}{\beta R} = 1.4933 \text{ V}$$

$$2) V_{in} = 5 - 1 = 4 \text{ V} \rightarrow I_D = \beta_{n1} [2(4 - V_o) V_o - V_o^2]$$

$$\rightarrow I_D = \beta_{n2} (V_{DD} - V_o - V_{Tn})^2$$

$$\rightarrow 0.34 \cdot (4 - V_o)^2 = 6V_o - V_o^2$$

$$\rightarrow 0.34 (16 - 8V_o + V_o^2) = 6V_o - V_o^2$$

$$\rightarrow 5.44 - 2.72V_o + 0.34V_o^2 - 6V_o + V_o^2 = 0$$

$$\rightarrow V_o = 0.698919 \text{ V}$$

example 3):

$$\text{or } \beta_{n1} (V_{GS1} - V_{Tn})^2 = 3 \cdot \beta_{n2} \cdot [2(V_{GS1} - V_{Tn}) V_{GS1} - V_{GS1}^2]$$

$$1) \wedge V_{GS1} = V_o \rightarrow \beta_{n1} (V_{DD} - V_o - V_{Tn})^2 = 3 [6V_o - V_o^2]$$

$$\rightarrow 0.34 (16 - 8V_o + V_o^2) = 18V_o - 3V_o^2$$

$$\rightarrow 5.44 - 2.72V_o + 0.34V_o^2 - 18V_o + 3V_o^2 = 0$$

$$\rightarrow V_o = 0.2747134 \text{ V}$$

$$2) m=1$$

$$\rightarrow 5.44 - 2.72V_o + 0.34V_o^2 - 6V_o + V_o^2 = 0$$

$$\rightarrow V_o \approx 0.9 \text{ V}$$

example 4): all inputs high for low output, $I_D = 1 \text{ mA}$

$$V_{GS} = V_{GS1} + V_{GS2} + V_{GS3}$$

assume chonic:

$$1 \text{ mA} = \beta_{n1} \cdot [2(V_{GS1} - V_{Tn}) V_{GS1} - V_{GS1}^2] \rightarrow \frac{1}{2} = 2(V_{GS1} - 1) \cdot V_{GS1} - V_{GS1}^2$$

$$\wedge V_{GS1} = V_A = V_{DD} - V_T = 4 \text{ V}$$

$$\rightarrow -V_{DS1}^2 + 6V_{DS1} - 0.5 = 0 \rightarrow V_{DS1} = 0.084924 \text{ V}$$

$$\wedge \frac{1}{2} = \frac{1}{2} [2(V_{DS2} - 1) \cdot V_{DS2} - V_{DS2}^2]$$

$$V_{DS2} = 4 - V_{DS1} = 3.915476$$

$$\rightarrow -V_{DS2}^2 + 5.830952V_{DS2} - 0.5 = 0$$

$$\therefore V_{DS2} = 0.087049 \text{ V}$$

$$\wedge 2(V_{DS3} - 1) \cdot V_{DS3} - V_{DS3}^2 - 0.5 = 0$$

$$\wedge V_{DS3} = 4 - (V_{DS2} + V_{DS1}) = 3.828427 \text{ V}$$

$$\rightarrow -V_{DS3}^2 + 5.656854V_{DS3} - 0.5 = 0$$

$$\therefore V_{DS3} = 0.0898143 \text{ V} \rightarrow V_{OL} = 0.2614 \text{ V}$$

$$\therefore \text{look in always part} \rightarrow I_{nA} = \beta_n \cdot (V_{DS1} - V_T)^2$$

$$\wedge V_{DS1} = V_{DD} - V_O = 4.7386 \text{ V}$$

$$\rightarrow I_{nA} = 71.545 \text{ uA/V}^2$$

final exam practice:

$$D) V_C = \frac{R_2}{R_1 + R_2} \cdot 6 = 2 \text{ V} \quad \boxed{D}$$

$$2) T = RC \ln \left(\frac{V_{OH} - V_{OL}}{V_C - V_{OL}} \right) = RC \cdot \ln \frac{3.8 - 0}{5 - 1.6}$$

$$V_1(\infty) = 0 \quad V_1(0) = 0 + (V_{OH} - V_{OL})$$

$$\rightarrow V_1(t) = V_1(\infty) - [V_1(\infty) - V_1(0)] e^{-t/RC}$$

$$\rightarrow V_1(T) = V_{IL} = 0 - [0 - (V_{OH} - V_{OL})] e^{-T/RC}$$

$$\rightarrow V_{IL} = [V_{OH} - V_{OL}] e^{-T/RC}$$

$$\rightarrow \ln \left[\frac{V_{IL}}{V_{OH} - V_{OL}} \right] = -\frac{T}{RC}$$

$$\rightarrow T = RC \cdot \ln \left[\frac{V_{OH} - V_{OL}}{V_{IL}} \right] \quad \text{NAND ASTABLE}$$

$$\rightarrow T = \ln(2.375) \text{ ms} = \boxed{D}$$

3) $V_0 = 2 \rightarrow V_{GS} = 2V$, $V_{GS} - V_{Th} = 4V$
 \rightarrow linear $\rightarrow I_D = \mu_n [2 \cdot 4 \cdot 2 - 2^2] = \mu_n \cdot 12$
 $\rightarrow I = I_C = 1.2 \text{ mA}$

4) $V_0 = 4 \rightarrow$ Mn sat & mp linear $\rightarrow I = \mu_n (V_i - V_{Th})^2$
 $V_0 = 0 \rightarrow I = 50 \text{ mA}$
 A mp linear $\rightarrow I_p = \mu_n (2(V_{GSMP} - V_{Th}) - V_{GS}^2)$
 $\wedge V_{GS} = 1 \rightarrow I_p = 20 \mu (2 \cdot 3.5 \cdot 1 - 1^2) = 140 \mu \text{ A}$
 $V_{GS1} = V_{SS} - V_{Th}$ $\wedge V_{SS} = \frac{1 + \frac{\mu_n}{\mu_p} [5 - 1.5]}{1 + \frac{\mu_n}{\mu_p}} = 1.96865 \text{ V}$
 $\rightarrow V_{GS1} = 0.9686 \text{ V}$
 $\rightarrow I_C = 0.12 \text{ mA} = \text{D}$

5) M_1 : linear $\rightarrow I_D = \mu_n [2(0 - V) V_{BSL} - V_{BSL}^2] = 0.04393 \text{ mA}$
 $\rightarrow \text{D}$

6) $V_0 = 3V$ $\wedge V_{GS1} = V_{SS} - V_{Th}$ $\wedge V_{SS} = \frac{1 + \frac{\mu_n}{\mu_p} [3.5]}{1 + \frac{\mu_n}{\mu_p}}$
 $\rightarrow V_{SS} = 1.96865 \rightarrow V_{GS2} \approx 3.5 \text{ V}$
 $\rightarrow V_0 < V_{GS2} \rightarrow$ Mp out. Mn linear
 $\therefore I_D = \mu_n (V_{GS} - V_{Th})^2 = 20 \mu [3.5 - 1.5]^2 =$
 $I_D = \mu_n [2(V_{GS} - V_{Th}) V_{GS} - V_{GS}^2] = \mu_n [-6 - 9]$
 $= -750 \mu \text{ A} = \text{A}$

7) $-10 \cdot \frac{\mu_n}{\mu_p} \left[\frac{1}{8} + \frac{1}{4} + \frac{1}{2} \right] = -17.5 \text{ V} = \text{D}$

8) Mn sat $\rightarrow I_D = -\mu_n (5 - 1)^2 = -0.8 = \text{D}$

9) $1000 \rightarrow 1100 = 7.5 \text{ V} \rightarrow 1010 = \text{D}$

10) D

11) $I_D = C \frac{dV}{dt} \rightarrow V(0) - [V(0) - V(0)] e^{-t/\tau}$

12) $\rightarrow I = \frac{5 - (5 - 0.6) e^{-2}}{10 \times 10^{-3}} = 4 \cdot 4045 = 16180 \text{ A}$
 $\rightarrow I = 59.54 \mu \text{ A} \approx 60 \mu \text{ A}$

12) $M_{sat} \rightarrow I_D = k_n (5-1)^2 = 1.6 \text{ mA} = \boxed{1.6}$

13) $M_{n \text{ linear}} \wedge M_{p \text{ sat}} \rightarrow I_D = I_C = k_p (V_{SGP} - V_{TD})^2$
 $\rightarrow I_C = 0.245 \text{ mA} = \boxed{0.245}$

14) ?

15) $0 \rightarrow 1 \rightarrow 0 \quad \boxed{a}$

16) $V_o(t) = V_{cc} - (V_{cc} - (0.8 - V_{cc} - 0)) e^{-t/\tau}$
 $\tau = 0.25 \tau$

$\rightarrow 10 - [10 + 9.2] e^{-0.25} = -4.9624 \text{ V} = \boxed{-4.9624}$

17) $V_{UT} = \frac{10}{20} \cdot 6 = 3 \text{ V} \wedge V_{LT} = -3 \text{ V}$

still in hysteresis $\rightarrow \boxed{18}$

18) $V_T = \frac{3}{30} = 0.1 \text{ mA} \rightarrow V_{out} = V_x = 0.1 \text{ mA} \rightarrow V_{out} = 4 \text{ V}$

$\rightarrow 4 = 6 - [6 + 4] e^{-t/\tau} \rightarrow -T/\tau = \ln \frac{6-4}{6-6}$

$\rightarrow T = \tau \cdot \ln \left(\frac{10}{2}\right) = \tau \ln(5) = \boxed{19}$

19) $V_{oss} = 2.3094 \rightarrow M_2 \wedge M_D \text{ sat}$

$\rightarrow I = k_n (V_{GS1} - V_{T1})^2 = 0.1 \text{ mA} = \boxed{0.1}$

20) $-2.5 \quad \frac{-1.25}{1h} = 1.25 \quad \boxed{C}$

1) assume $V_{out} \Rightarrow V_{out} = 4 \quad \boxed{C}$

3) $M_{linear} \rightarrow I_D = k_n [2(5-1) \cdot 2 - 2^2] = 1.2 \text{ mA}$

$\wedge V \quad 1.2 \text{ mA} = C \frac{dV_o}{dt} + \frac{V_{DD} - V_o}{R_D}$

$\rightarrow C \frac{dV_o}{dt} = 1.2 \text{ mA} - 0.3 = 0.9$

20) $\frac{V_{cc}}{1h}$

$$\boxed{1} \quad \frac{C dv_o}{dt} + \frac{V_{DD} - v_o}{R_o} = I_{in} (G-1) \rightarrow \frac{C dv_o}{dt} = I$$

$$\boxed{1} \quad 0.11 \rightarrow \begin{array}{c} 0.11 \text{ } \swarrow \text{ } \searrow \\ \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \end{array} \quad I_{B1} = 83.8 \mu\text{A}$$

$\rightarrow I_{E1} =$ $83.8 \mu\text{A} \downarrow$

Lab exam practice:

$$\boxed{1} \quad t_1 = t_2 \rightarrow f = \frac{1}{2t_1} \quad \wedge \quad t_1 = RC \ln(1+2)$$

$$\wedge RC = 0.2 \text{ ms} \rightarrow t_1 = 0.2199 \text{ ms}$$

$$\rightarrow \boxed{2} \quad f = 2299.59$$

$$\boxed{2} \quad \text{pulse width} = RC \ln(3) \quad R = R_1 + R_2$$

$$\text{at } 1 \text{ ms: } R = 1820 \Omega$$

$$\text{at } 1.8 \text{ ms: } R = 3296 \Omega \rightarrow R_2 [0, 1.46 \text{ k}\Omega]$$

$$\boxed{3} \quad V_{in} = 9V \rightarrow \text{out}$$

$$\boxed{4} \quad Q_1 \text{ out: } V_{A1} = 5 - 0.8 = 4.3 \rightarrow I_{B1} = 83.8 \mu\text{A}$$

$$\rightarrow I_{C1} = \beta I_{B1} = 4.19 \text{ mA} = I_{B2}$$

$$\boxed{5} \quad \text{duty cycle} = T_{on} / T \quad V_{UT} = \frac{10}{20} \cdot V_{DD} = 6V$$

$$\wedge V_{LT} = -6V \rightarrow$$

$$2.4 \cdot 0.7 \text{ ms} \pm 8.04$$

$$\boxed{8} \quad V(t) = [V(\infty) - V(0)] e^{-t/\tau} \quad V(0) = 0, V(\infty) = 10V$$

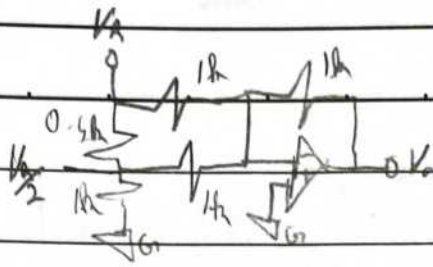
$$\rightarrow 10 = 10 e^{-0.5/\tau} = 5.4$$

$$\rightarrow \tau = 6.4389 \times 10^{-4}$$

$$\tau = R \times C$$

$$\tau = 1.58896 \text{ ms} \rightarrow R_x =$$

9



$$I = \frac{V_A}{2} + \frac{V_A}{1k}$$

$$\rightarrow V_A = 9.45V$$

$$10) f = 158 \text{ Hz} \rightarrow T_{on} + T_{off} = 66.667 \mu\text{s}$$

$$\rightarrow (10 + R_B) C \ln(2) + R_B C \ln(2) = 66.667 \mu\text{s}$$

$$\rightarrow \ln(2) \cdot C [10 + 2R_B] = 66.667 \mu\text{s}$$

$$\rightarrow 10 + 2R_B = 79.2359 \text{ k} \rightarrow R_B = 4600 \Omega$$

$$12) \ln(2) \cdot C \cdot [R_1 + 2R_2] = \frac{1}{8k}$$

$$\rightarrow R_1 + 2R_2 = 18.0337 \text{ k} \Omega$$

$$\wedge R_1 + R_2 = 10 \text{ k} \rightarrow R_1 = 1966.3 \text{ } \wedge R_2 = 8033.7$$

$$\rightarrow T_{on} = (R_1 + R_2) \cdot C \ln(2) = 6.9315 \times 10^{-5} \text{ s}$$

$$\rightarrow \frac{T_{on}}{T} = 8k \cdot T_{on} = 0.5545$$

13)

$$R_T = R + 50k$$

$$\rightarrow T_{max} =$$

14)

$$T_{on} = 0.64 \cdot T$$

$$\wedge T_{on} = 20k \cdot 0.05 \mu \cdot \ln(2)$$

$$\wedge T = T_{on} + R_B \cdot 0.05 \mu \cdot \ln(2)$$

$$\rightarrow \left(\frac{1}{0.64} - 1\right) T_{on} = R_B \cdot 0.05 \mu \cdot \ln(2)$$

$$\wedge T_{on} = 6.93149 \times 10^{-4} \text{ s}$$

$$\rightarrow R_B = 11260 \Omega$$

18)



$$\rightarrow \frac{5 - V_A}{5k} = \frac{V_A}{20k} + \frac{V_A + 12}{20k}$$

$$\rightarrow 20 - 4V_A = V_A + V_A + 12$$

$$\rightarrow 6V_A = 8 \rightarrow V_A = \frac{4}{3} \text{ V}$$

$$(19) V_{in} = 0 \rightarrow V_o = \text{high}$$

$$I_{RC} = n \cdot \frac{3.2294 - 0.8}{10k} = \frac{3.2294 - 0.8}{5 \cdot 10k}$$

$$\rightarrow n = 7$$

examples continued:

$$\text{example II: } V_{SS} = V_{T1} + \sqrt{\frac{\mu_{n1}}{\mu_{n2}}} \cdot V_{TL}$$

$$\wedge V_{SS}^2 = \frac{\mu_{n1}}{\mu_{n2}} V_{TL}^2$$

$$\rightarrow V_{SS} = 1 + \sqrt{\frac{0.2}{1}} \cdot (-1) = 1 \mp \frac{\sqrt{5}}{5}$$

$$\rightarrow V_{SS} = 0.553k \text{ or } 1.4492kV$$

$$\wedge V_{SS} = \pm \sqrt{0.2} (-1)^{-1} = 0.449V$$

$$I_D = \mu_{n1} (V_{SS} - V_{T1})^2 \approx 0.2 \text{ mA}$$

$$\text{for } V_{o2}: 2(V_i - V_{T1}) \cdot V_{o2} - V_{o2}^2 = 0.2 \cdot (V_{o2L} - V_{TL})^2$$

$$V_i = V_{DD} \wedge V_{o2L} = 0$$

$$\rightarrow 2(4) \cdot V_{o2} - V_{o2}^2 = 0.2$$

$$\therefore V_{o2} = 0.025079V$$

$$\square C \cdot 10k \cdot \ln(2) \cdot 8k =$$

Subject

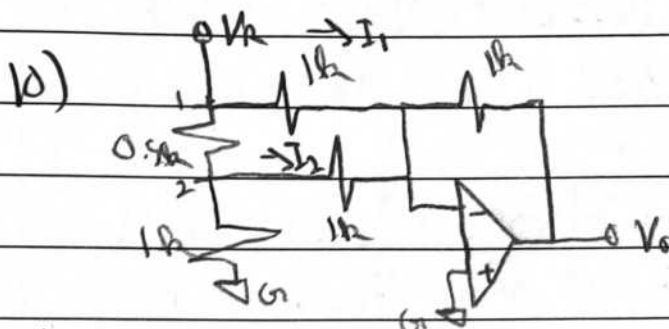
Date

No.

$$12) T_{on} = (R_1 + R_2) C \cdot \ln 2 \quad \wedge \quad R_1 + R_2 = 10k$$

$$T_{total} = \frac{1}{8k}, \quad \text{duty cycle} = \frac{T_{on}}{T_{total}} = T_{on} \cdot 8k$$

$$\rightarrow \text{duty cycle} = 0.59452$$



$$I_1 = \frac{V_A}{1k}$$

$$I_2 = \frac{V_2}{1k} \quad \wedge \quad V_2$$

11) $V_{UT} = 4V \quad \wedge \quad V_{LT} = -4V$

$$0 \rightarrow \frac{-4}{10}$$

$$0 \rightarrow 2.572 \times 10^{-3} \rightarrow t_{width} = 5.144 \text{ ms}$$

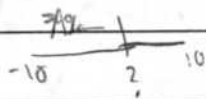
$$T = \frac{1}{f} \rightarrow T_{on} = \frac{5.144}{3000} = 1.715 \text{ ms}$$

$$\rightarrow D =$$

$$t_{-V_{UT}} = \sin^{-1}\left(\frac{4}{10}\right) / 160 \rightarrow t_{on} =$$

16) $V_A > V_{in}$

$$\frac{5 - V_{out}}{5k} = \frac{V_{out}}{20k} + \frac{V_{out} - V_{out}}{20k}$$



$$\rightarrow 20 - 4V_{out} = 2V_{out} - V_{out}$$

18) when $V_A = V_{in} \quad \wedge \quad V_o = -V_{out} \rightarrow 6V_{out} = 20 + V_{out}$

$$V_A = V_{in}$$

$$\frac{V_{in}}{20k} = \frac{V_x - V_{in}}{5k} - \frac{V_x - V_o}{20k}$$

$$\therefore V_{out} = \frac{20 + V_{out}}{6}$$

$$\text{If } V_{out} = 12$$

$$\rightarrow V_{out} = \frac{32}{6} =$$

$$V_{in} = 4V_x - 4V_{in} - V_x + V_{out} \rightarrow V_{in} =$$

$$V_{out} = 4V_x - 4V_{out} - V_x - V_o$$

$$\rightarrow V_{out} = \frac{5V_x - V_o}{5}$$

example:

$$1) \quad I_L (V_{DS1} - V_{TL})^2 = m \beta_{n1} V_{DS1} [2(V_{DS1} - V_{TL}) - V_{DS1}]$$

$$\rightarrow 0.34 = 3 \cdot [2(4) \cdot V_{OL} - V_{OL}^2]$$

$$\rightarrow 3V_{OL}^2 - 24V_{OL} + 0.34 = 0 \Rightarrow V_{OL} = 0.0142 \text{ V}$$

$$2) \quad V_{OL}^2 - 8V_{OL} + 0.34 = 0 \rightarrow V_{OL} = 0.04273 \text{ V}$$

example

$$1) \quad I_D = 1 \text{ mA}$$

$$\rightarrow 1 \text{ mA} = 2 \times 10^{-3} [2(V_{DS1} - V_T) V_{DS1} - V_{DS1}^2]$$

$$\rightarrow 0.5 = 2(5 - 1) \cdot V_{DS1} - V_{DS1}^2$$

$$\therefore V_{DS1} = 0.063 \text{ V}$$

$$1 \text{ mA} \rightarrow 0.5 = [2(5 - V_{DS1} - 1) V_{DS2} - V_{DS2}^2]$$

$$\rightarrow V_{DS2} = 0.066 \text{ V}$$

$$\rightarrow 0.5 = 2(5 - V_{DS1} - V_{DS2} - 1) V_{DS3} - V_{DS3}^2$$

$$\rightarrow V_{DS3} = 0.0651 \text{ V}$$

$$\therefore V_{OL} = 0.1921 \text{ V}$$

$$\therefore M_1 \text{ forward} \rightarrow 1 \text{ mA} = \beta_{n1} V_{DS1} [2(0+1) - V_{DS1}]$$

$$\therefore 1 \text{ m} = \beta_{n1} [2(4.8079) - 4.8079^2]$$

$$\rightarrow \beta_{n1} = \text{negative } \times$$

$$\therefore V_{DS1} > V_{DS2} - V_{TL} \rightarrow M_2 \text{ not}$$

$$\therefore 1 \text{ mA} = \beta_{n1} (V_{DS2} - V_{TL})^2 \Rightarrow \beta_{n1} = 1 \text{ mA}$$

Quiz 3:

$$1) \quad V_{OL} \rightarrow M_2 \text{ not } \rightarrow M_1 \text{ active}$$

$$I_D = \beta_{n1} (V_{DS1} - V_{TL})^2 = \beta_{n0} V_{OL} [2(V_{DS0} - V_{OL}) - V_{OL}]$$

$$\rightarrow 1 = 3 \cdot 2 [8V_{OL} - V_{OL}^2]$$

$$\rightarrow V_{OL} = 0.03924$$

$$5) \quad V_{UT} = \frac{10k}{10k+20k} \cdot V_{out} \rightarrow V_{UT} = 4V$$

$$\wedge V_{LT} = \frac{1}{3}(-V_{out}) \rightarrow V_{LT} = -4V$$

assume starting at $t=0$

V_{in} reaches V_{UT} then returns to 0

$$t_{UT}: \quad V_{UT} = V_{in} = 10 \sin(160t) \rightarrow t = \frac{\sin^{-1}(\frac{4}{10})}{160}$$

$$\rightarrow t_{UT} \approx 2.572 \text{ ms}$$

$$\therefore t_{on} = 2t_{UT}$$

$$\wedge t_{LT}: \quad V_{LT} = -4 = 10 \sin(160t) \Rightarrow t_{LT} = 2.572 \text{ ms}$$

$$\wedge t_{off} = 2t_{LT}$$

$$T = t_{off} + t_{on} \quad \wedge \text{ duty cycle} = \frac{t_{on}}{T}$$

$$\therefore T = 2t_{on} \rightarrow D = 50\%$$

1) $V_{in} = 5V$ M_1 is linear and M_2 is sat

$$\rightarrow \beta_{n1} \cdot V_{o1} [2(4) - V_{o1}] = \beta_{n2} (1)^2$$

$$\rightarrow \frac{\beta_{n1}}{\beta_{n2}} = 8V_{o1} - V_{o1}^2$$

$$2) \quad V_{o1} = V_{in} \quad \wedge \quad V_{o1} = V_{o2} \rightarrow V_{M1} = 3.41V$$

$$3) \quad V_{SS} = \frac{1 + \frac{\beta_{n1}}{\beta_{n2}} [V_{o1} - 1.2]}{1 + \frac{1}{3}} = \frac{1 + \frac{1}{3} [3.8]}{\frac{4}{3}} = \frac{3 + 3.8}{4} = 1.7$$

$\therefore V_{in} < V_{i1} < V_{SS} \rightarrow M_n$ sat $\wedge M_p$ linear

$$\therefore I_D = \beta_{n1} (V_{SS} - V_{in})^2 = 47.628 \mu A$$

$$\therefore P_{dissipated} = 238.4 \mu W$$

A) M_2 always sat

$$\therefore k_{n2} (V_{DS} - V_0 - 1.2)^2 = k_{n2} (V_{in} - 0.8)^2$$

$$\therefore k_{n1}/k_{n2} = 1.96 \rightarrow$$

$$\text{or } V_0 = V_{DD} - V_{T2} - \sqrt{\frac{1}{1.96}} \cdot (V_{in} - V_{T1})$$

$$\rightarrow V_0 = 4.2 - \frac{1}{2} [V_{in} - 0.8] \quad \text{at } V_{in} < V_{in} < V_{DD} + V_{in}$$

$$\therefore V_{T1} =$$

$$\therefore 2.45 < V_0 < 4.2$$

$$\text{assume } V_{in} = V_{DS} + V_{in} \rightarrow V_0 = 4.2 - \frac{1}{2} [V_{in}]$$

$$\rightarrow V_0 = 2.45$$

$$V_{DSL} = 5 - V_0, \quad V_{DS0} = V_0, \quad V_{GS0} = V_{in}$$

$$\therefore M_0 \text{ sat} \quad \rightarrow \quad 0.8 < V_{in} < V_{T1}$$

$$\text{find } V_{T1} : k_{n1} (V_{T1} - 0.8)^2 = k_{n2} (V_{DD} - V_{T1} - V_{T1})^2$$

$$\therefore \frac{k_{n1}}{k_{n2}} = \frac{k_{n1}}{k_{n2}} \rightarrow 1.96 (V_{T1} - 0.8)^2 = (V_{DD} - V_{T1} + 0.8 - 1.2)^2$$

$$\rightarrow \sqrt{1.96} (V_{T1} - 0.8) = (V_{DD} - V_{T1} - 0.4)$$

$$\rightarrow 2.4 V_{T1} - 1.12 = 5 - 0.4$$

$$\rightarrow V_{T1} = 2.38333 \dots V$$

$$\therefore V_{in} = V_{T1} \rightarrow V_0 = 1.9833$$

$$\frac{1}{2} V_0 - 1.12 = 4.6 - V_0$$

$$\rightarrow 2.4 V_0 = 6.72$$

④ M_1 always sat & M_D sat

$$\rightarrow V_{TD} < V_{in} < V_t \quad \text{check page 22 of prob's slides}$$

$$\wedge V_o = V_{DD} - V_{TL} - \sqrt{\beta_{AR}} (V_{in} - V_{TD}) \quad (\text{ch-7})$$

$$\circ \circ \beta_{AD} (V_t - V_{TD})^2 = \beta_{AL} (V_{DSSL} - V_{TL})^2$$

$$\wedge V_{DSSL} = V_{DD} - V_{ot} \wedge V_{ot} = V_t - V_{TD}$$

$$\rightarrow \beta_{AR} (V_t - 0.8)^2 = (V_{DD} - V_t + 0.8 - 1.2)^2$$

$$\rightarrow \frac{7}{5} (V_t - 0.8) = 4.8 - V_t$$

$$\therefore V_t = 2.38333 \text{ V}$$

$$\text{at } V_{in} = V_{TD}, V_o = V_{DD} - V_{TL} = 3.8 \text{ V}$$

$$\text{at } V_{in} = V_t, V_o = 1.58334 \text{ V}$$

$$\therefore 1.58334 \text{ V} < V_o < 3.8 \text{ V}$$

$$\text{① } \circ \circ V_{SS} = V_{TD} \pm \sqrt{\frac{\beta_{AL}}{\beta_{AD}}} \cdot V_{TL}$$

$$\rightarrow 1.6 = 1 \pm \sqrt{\frac{1}{4}} \cdot (-1.2)$$

$$\rightarrow \sqrt{\beta_{AR}} = 2 \rightarrow \beta_{AR} = 4$$

$$\text{③ } V_{in} < V_{TH} \rightarrow NM_L = V_{TH} - V_{OL} = V_{TH}$$

$$\text{④ } 2.4 = V_{TH} \wedge 1.2 = V_{TL}$$

$$NM_V = 1.2 - 0.2 = 1$$

$$\text{⑤ } V_{DD} - V_{TH} = 1 \text{ V}$$

$$\text{⑥ } V_{SS} = \frac{V_{TH} + \sqrt{\frac{1}{4}} [5 - 1]}{1 + \frac{1}{2}} = 2.2426 \text{ V}$$

$$\rightarrow V_{DD} - V_t < V_i < V_{DD} \rightarrow M_p \text{ off } \wedge V_o = 0$$

$$\rightarrow V_{DSSL} = -V_{SDDL} = -5$$

$$9 \quad P_{OH} = 0 \quad P_{OL} = \frac{4.5}{2} \cdot 5 = 22.5 \text{ mW}$$

$$\rightarrow P_{avg} = \frac{P_{OH} + P_{OL}}{2} = 11.25 \text{ mW}$$

$$10 \quad V_{in} = 0.3 \rightarrow M_1 \text{ off} \rightarrow V_o = V_{DD}$$

$$\rightarrow V_{DSL} = 0$$

$$11 \quad V_{DD} < V_{in} \quad \because V_{SS} = V_{TL} \pm \sqrt{\frac{I_D}{\beta_n}} V_{TL}$$

$$\rightarrow V_{SS} \quad \because V_{GS1} > V_{in} \text{ and } V_{GS1} = V_o > (V_{GS} - 1)$$

$$4.6 > 0.2 \rightarrow M_1 \text{ sat}$$

$$\because V_{GS2} = 0.4 < 0 - V_{TL} \rightarrow M_2 \text{ is linear}$$

$$13 \quad V_{in} = 1.8, \sqrt{\beta_n} (V_t - 0.8) = (V_{DD} - 0.4 - V_t)$$

$$\rightarrow \sqrt{1} 2V_t = 5.4 \text{ V} \rightarrow V_t = 2.7 \text{ V}$$

$$\therefore V_{TD} < V_{in} < V_t \rightarrow M_1 \text{ is sat}$$

No need to calculate V_t since we know it is larger than V_{in} so both transistors are sat

$$\rightarrow V_o = V_{DD} - V_{TL} - \sqrt{\beta_n} (V_{in} - V_{TD})$$

$$\rightarrow V_o = 2.8 \text{ V}$$

$$14 \quad \because V_{SS} = \frac{V_{Th} + \frac{1}{2} [4]}{1.5} = 2$$

$$\rightarrow V_{TD} < V_{in} < V_{SS} \rightarrow M_n \text{ sat, } M_p \text{ ohmic}$$

$$\rightarrow I_D = \beta_n (V_{in} - V_{Th})^2 = \frac{\beta_n}{4} = 50 \mu\text{A}$$

$$15 \quad V_{DD} - V_{TL} = 5 - 0.8 = 4.2 \text{ V}$$

$$16 \quad \frac{V_o - V_{rel_0}}{R_1} = \frac{V_{rel_0} - V_{sc}}{R_2} \rightarrow V_o - V_{rel_0} = 2V_{rel_0} - 2V_{sc}$$

$$\rightarrow V_o + 2V_{sc} = 3V_{rel_0} \rightarrow V_{rel_0} = 4$$

$$2) V_e(t) = V_e(\infty) - [V_e(\infty) - V_e(0)] e^{-t/\tau}$$

T: time until $V_e = V_{TL} = 1.6$

$$\rightarrow 1.6 = 0 - [0 - 3.8] e^{-T/RC}$$

$$\rightarrow \frac{1.6}{3.8} = e^{-T/RC} \rightarrow \ln\left(\frac{3.8}{1.6}\right) = \frac{T}{RC}$$

$$\therefore T = ms \ln(2.375)$$

$$3) \circ \circ V_{GS} = V_i = 5V \quad \wedge \quad V_{DS} = V_o = 2V$$

$\rightarrow 2 < 5 - V_{TN} \rightarrow$ linear

$$\therefore I_D = \beta_n V_o (2(V_{GS} - V_{TN}) - V_o)$$

$$\rightarrow I_D = 1.2 \text{ mA} = \left(\frac{\partial I_D}{\partial V_o} + \frac{V_{DS} - V_o}{R_D} \right)$$

$$\rightarrow C \frac{\partial I_D}{\partial V_o} = 0.9 \text{ mA}$$

$$4) \circ \circ V_{GS} = \frac{1 + \sqrt{\frac{2}{5}} (5 - 1.9)}{1 + \sqrt{\frac{1}{5}}} = 1.9685647$$

$$\rightarrow V_{GS2} = 3.06856$$

$\circ \circ V_{GS2} < V_o \rightarrow$ Mn odd \wedge Mp linear

$$\therefore I_D = I_L = I_{Dp} = V_o (2(V_{GS2} - V_{TN}) - V_o)$$

$$\rightarrow I_D =$$

$$I_D = \beta_p V_{SDP} [2(V_{GS2} - V_{TP}) - V_{SDP}]$$

$$\wedge V_{SDP} = 5V \quad \wedge \quad V_{SDP} = 1$$

$$\rightarrow I_D = \beta_p [7 - 1] = 120 \mu A$$

$$5) M_D \text{ linear} \quad \wedge \quad M_1 \text{ sat} \quad M_D \text{ sat} \quad \wedge \quad M_1 \text{ linear}$$

$$I_{DD} = \beta_n (V_{GS} - V_{TN})^2 = 75 \mu A$$

$$\wedge I_{D1} = \beta_{n1} (4 \cdot 0.5 - 0.5^2) = 0.04375$$

check! V_{GS} can M_D

$$4.5 > -V_{TN}$$

$$\circ \circ V_{GS2} = 0.9 < 0 \rightarrow \text{linear}$$

6) $V_{GS} = 1.964 \rightarrow < V_G < V_{GS}$
 M_n linear \wedge M_p sat \times M_p probb since $V_{SG1} < V_{TP}$

$$I_C = I_{Dn} - I_{Dp}$$

$$\wedge I_{Dn} = k_n (8.3 - 3)^2 = 750 \text{ mA}$$

$$\wedge I_C = I_{Dn} = -750 \text{ mA}$$

7) $V_{ref} = 10V \rightarrow V_G = -10 \cdot \frac{4k}{2k} \left(\frac{1}{8} + \frac{1}{4} + \frac{1}{2} \right)$
 $\rightarrow V_G = -17.5V$

8) M_p probb, M_n sat
 $\rightarrow I_D = k_n (5 - 1)^2 = -0.8 \text{ mA}$

9) $1000 : -V_{ref} \cdot \frac{R_2}{R_1} \cdot [1] = 5$
 $\therefore 6.5 > 5 \rightarrow$ end of first cycle = 1000

next 1100 : $-V_{ref} \cdot 1.5 = 7.5$
 $\therefore 6.5 < 7.5 \rightarrow$ comparator output = 0

\rightarrow end of second cycle: 1010

10) $V_C = \frac{V_{CC}}{3} = 1.667V$

11) $V_G(t) = V_G(\infty) - [V_G(\infty) - V_G(0)] e^{-t/\tau}$
 $\rightarrow 5 - (5 - 0.6) e^{-2t} \rightarrow V_G(t) = 4.4 \text{ V}$

$$\rightarrow I_C = 59.5 \mu\text{A} \approx 0.06 \text{ mA}$$

12) $I_D = I_C + \frac{V_{DD} - V_G}{R_D}$

$$\wedge \therefore V_{GS} = 4.5 > V_{GS} - 1 \rightarrow \text{sat}$$

$$\rightarrow I_D = k_n (4)^2 = 1.6 \text{ mA}$$

$$\rightarrow I_C = 1.55 \text{ mA}$$

13) M_n probb \wedge M_p sat $\therefore V_G < V_{TP}$

$$\rightarrow I_C = I_{Dp} = k_p (V_{SG1} - |V_{TP}|)^2$$

$$= 20 \mu [5 - 1.5]^2 = 0.245 \text{ mA}$$

$$\begin{aligned}
 \text{[16]} \quad V_c(t) &= V_c(\infty) - [V_c(\infty) - V_c(0)] e^{-t/\tau} \\
 \rightarrow V_c(0.25\tau) &= 10 - [10 - 0] e^{-0.25} \\
 \text{assuming trigger at } t=0^+ &\rightarrow 10(1 - e^{-0.25}) = \\
 V_c(0) &=
 \end{aligned}$$

at $t=0 \rightarrow Q_2$ switches from off to on

$\rightarrow t=0^- \rightarrow Q_1$ on

$$\therefore V_c(0) = 0.8 - 10 = -9.2 \text{ V}$$

$$V_{BE \text{ sat}} - (V_{CE \text{ sat}} - V_{cc}) \quad | \quad - (V_{CE \text{ sat}} - V_{cc})$$

$$\begin{aligned}
 \therefore V_c(0.25\tau) &= 10 - [10 + 9.2] e^{-0.25} \\
 &= -4.443 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 \text{[18]} \quad \frac{V_{rel} - V_{cc}}{R_2} &= \frac{V_0 - V_{rel}}{R_1} \rightarrow 2V_{rel} - 2V_{cc} = V_0 - V_{rel} \\
 &\rightarrow 3V_{rel} = 6 + 6 \rightarrow V_{rel} = 4 \\
 \rightarrow \text{turn} \Rightarrow V_c &= V_{rel} = V_{sat} - (V_{sat} + V_T) e^{-\text{turn}/RC} \\
 \rightarrow \frac{12 - 4}{12 + 4} &= e^{-\text{turn}/RC} \\
 &\rightarrow \text{turn} = RC \ln\left(\frac{16}{8}\right)
 \end{aligned}$$

$$\begin{aligned}
 \text{[19]} \quad \text{assume } M_1 \text{ off} &\rightarrow I_c = I_L \\
 \therefore V_{DSL} &= 3 \text{ V} > |V_{TL}| \rightarrow M_2 \text{ on} \\
 \rightarrow I_c = I_L &= I_{sat} (0 + 2)^2 = 100 \text{ mA}
 \end{aligned}$$

$$\begin{aligned}
 \text{[18]} \quad V_{rel} &= V_{cc} - [V_c(\infty) - V_c(0)] e^{-t/RC} \\
 \rightarrow V(\text{turn}) &= V_{rel} \quad \frac{V_{rel} - 3}{10k} = \frac{6 - V_{rel}}{20k}
 \end{aligned}$$

$$\rightarrow 2V_{rel} - 6 = 6 - V_{rel} \rightarrow V_{rel} = 4 \text{ V}$$

$$\rightarrow V_c \text{ at } t = 6 - [6 + 4] e^{-t/RC}$$

$$\rightarrow \ln\left(\frac{6-4}{6+4}\right) = \frac{-\text{turn}}{RC} \rightarrow \text{turn} = RC \ln\left(\frac{10}{2}\right)$$

Q1)
$$\frac{V_{neb} - V_x}{R_2} = \frac{V_0 - V_{neb}}{R_1}$$

$$\rightarrow \frac{V_{neb} - 4.5}{10.7} = \frac{9 - V_{neb}}{20.5}$$

$$\rightarrow 2.9158 V_{neb} = 9 + 4.5 \cdot \frac{20.5}{10.7}$$

$$\rightarrow V_{neb} = 6.04$$

Q2) $V_{os} < V_{os} - 1 \rightarrow$ linear

$$\therefore I_D = I_C + I_L \quad \text{and } M_1 \text{ is sat}$$

$$\rightarrow I_L = \beta_n (0 + 2)^2 = 0.1168 \text{ mA}$$

$$\wedge I_D = \beta_n (8 \cdot 1.8 - 1.8^2)$$

$$\rightarrow I_D = 8.91686 \times 10^{-4}$$

$$\rightarrow I_L = 974.88 \text{ nA}$$

Q3) $V_x = 0V \rightarrow t_{on} = RC \ln \left(1 + \frac{2R_2}{R_1}\right)$

$$\wedge t_{off} = t_{on}$$

$$\rightarrow t = 2 \cdot 1.43 \text{ ms}$$

$$\rightarrow f = 1.9460 \text{ kHz}$$

$$\rightarrow \beta =$$

Q4) $1.96V \quad 5.6V$

Q5) $-V_{neb} \cdot \frac{R_{eq}}{R} \cdot \left(\frac{1}{8} + \frac{1}{4} + \frac{1}{2}\right) =$

Q6) $I =$ (a) $I_{op \text{ amp}} =$

Q7) $Q_2(t=0) = \text{sat}$, $Q_2(t=0) = 10$



$$\rightarrow V_{o2}(t) = V(\infty) - [V(\infty) - V(0)] e^{-t/RC}$$

$V(\infty) = 0.8 \rightarrow V_{o2}(2.1T) = 0.8 - (0.8 - 10) e^{-2.1}$

$$\rightarrow V_{o2} =$$

$$V_o = 0.8 - V_{th} V_o$$

$$\wedge V_o(2.1 \tau) = 10 - (10 + 10 - 0.8) e^{-2.1}$$

$$\rightarrow V_o = 8 - (8 + 9.2) e^{-2.1} = 6.1389$$

Q2) M_D off ∞ $V_{out} = 3.2 \rightarrow M_D$ is out

$$I_D = I_C = R_L (0 + 2)^2 = 0.1148 \text{ mA}$$

Q9) $t = 2T \rightarrow V_C = V_{o1} - (V_{o1} - V_{o2}) e^{-2}$

$$\infty V_{o1} = 0.48 \text{ V}$$

$$\rightarrow V_C = 1.06597$$

$$\infty M \text{ is linear} \rightarrow I_D = R_{in} (8 V_o - V_o^2)$$

$$\rightarrow I_D = 0.739 \text{ mA}$$

$$I_D = I_C + I_A \quad \wedge \quad I_A = \frac{5 - V_o}{10.7 \Omega}$$

$$\rightarrow I_C = \dots$$

low \rightarrow high

$$V_C(\infty) = V_{o1} \quad V_C(\infty) - (V_C(\infty) - V_C(0)) e^{-2}$$

$$\rightarrow V_C(2T) = \dots$$

$$4.384 \text{ V} \rightarrow$$

Q10) $V_C(\infty) = 0.48 \text{ V}$

$$\rightarrow 0.48 - (0.48 - 0) e^{-T/\tau}$$

$$\rightarrow 0.48 e^{-T/\tau} = 0.48$$

$$\rightarrow T = \dots$$

$$0.48 V_{DD}$$

$$0.48 V_{DD} = 0 - (0 - V_{DD}) e^{-T/\tau}$$

$$\rightarrow e^{-T/\tau} = 0.48$$

$$\rightarrow T = \tau \ln \frac{1}{0.48} = 0.73$$

$$Q 11) M_D \text{ is sat} \rightarrow I_D = I_{D0} (4)^t = 1.312 \text{ mA}$$

$$I_L = I_{L0} (2 \cdot 0.2 - 0.2^2) = 10.388 \text{ mA}$$

$$Q 12) \rightarrow I_C = -I_{C0} (V_{DD} - V_{TN})^t$$

$$Q 13) V_{B2}(\infty) = 8$$

$$8 - (8+10)e^{-0.4} \\ 0.8 - 0.8+10$$

$$V_{B2}(0) = 0 \quad \wedge \quad V_{B2}(\infty) = 10$$

$$Q 14) T_D = T_C + V_{DD}$$

$$Q 15) T_{\text{on}} = (R_1 + R_2) C \ln 2$$

$$\wedge \quad \frac{R_1 + R_2}{R_1 + 2R_2} > 40\% \rightarrow \frac{(R_1 + R_2) \cdot \ln 2}{(R_1 + R_2 + R_2) \ln 2} > 40\%$$

$$19.6 \text{ k}\Omega \rightarrow R_1 + R_2 \geq 0.5 R_1 + R_2$$

$$\rightarrow 0.5 R_1 >$$

$$\frac{R_1}{R_1 + R_2} = \frac{2}{3}$$

$$Q 16)$$

$$V(\infty) = \frac{V_{CC}}{3}$$

$$\frac{V_{CC}}{3} = 5 - \frac{5}{3} - \frac{V_{CC}}{3} e^{-t/\tau}$$

$$\rightarrow \frac{V_{CC}}{3} = V_{CC} - V_{CC} \left(1 - \frac{1}{3}\right) e^{-t/\tau}$$

$$\rightarrow \frac{1}{3} = 1 - \frac{2}{3} e^{-t/\tau}$$

$$T = \frac{1}{3} = 1 - e^{-t/\tau} \rightarrow T =$$

$$\frac{2}{3} = 1 - (1 - \frac{1}{3})e^{-t} \rightarrow \frac{1}{3} = \frac{2}{3}e^{-t}$$

$$\text{Q18) at } V_{SS} : \quad V_{SS} = \frac{1 + \sqrt{\frac{65}{228}} (5 - 1.5)}{1 + \sqrt{\frac{65}{228}}} = 1.861 \text{ V}$$

$$\rightarrow I_0 = \beta n (V_{SS} - 1)^2 = 169 \text{ mA}$$

Q19)

Q20) \rightarrow $g \cdot (1) \rightarrow$ end of first: 1000
end of second \rightarrow

$$\text{Q21) } \frac{10.7}{I_{CP}} \cdot C > T_2 - T_1$$

$$\frac{10.7}{I_{CP}} C > 1.5 \text{ ms}$$

$$I_{CP} =$$

$$I_{CP} = 10.7 \rightarrow 0.23 \times 10^{-4}$$

$$25818 \cdot C > 1.5 \text{ ms}$$

$$C >$$

Q23) Mm linear \rightarrow

$$I_0 =$$

$$\text{Q24) } V_{omax} = \frac{I_{CP} \cdot t_s}{C}, \quad t_s = 10.7 \cdot \frac{C}{I_{CP}}$$

$$I_{CP} = 0.414 \text{ mA} \rightarrow t_s = 1.80729 \times 10^{-23}$$

$$\rightarrow V_{omax} =$$

$$C = 0.5$$

$$\text{Q25) } M_p \text{ sat} \rightarrow I_C = \beta n (V_{SS} - V_{TD})^2 =$$