mode theory:

- optical fibers are cylindrical dielectric waveguides
+ maxwell's equations for linear isotropic dielectric medium free of current and charges:

faraday's law: $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

Ampere's law: $\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$

gauss's law: $\nabla \cdot \bar{D} = 0$, $\nabla \cdot \bar{B} = 0$ | $\bar{D} = \epsilon \bar{E}$, $\bar{B} = \mu \bar{H}$

+ the wave equations are derived as follows:

$$\nabla \times (\nabla \times \bar{E}) = \nabla \times \left(-\mu \frac{\partial \bar{H}}{\partial t}\right) = -\mu \frac{\partial}{\partial t}(\nabla \times \bar{H})$$

not affected by curl

$$= -\epsilon\mu \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\therefore \nabla \times (\nabla \times \bar{E}) = \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E}$$ (vector identity), $\nabla \cdot \bar{E} \to 0$

$$\therefore \boxed{\nabla^2 \bar{E} = \epsilon\mu \frac{\partial^2 \bar{E}}{\partial t^2} , \quad \nabla^2 \bar{H} = \epsilon\mu \frac{\partial^2 \bar{H}}{\partial t^2}}$$

wave equations

- the cylindrical coordinate system is used since the optical fibers are cylindrical

+ve z direction

→ $E = E_0(r, \phi)\, e^{i(\omega t - \beta z)}$

& $H = H_0(r, \phi)\, e^{i(\omega t - \beta z)}$

$\rho = r$

subbing in wave equations and using $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

→ $\frac{\partial^2 \bar{E}}{\partial z^2} = -E_z \cdot \beta^2$ & $\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \bar{E}}{\partial r}\right) =$

Or

∵ $\nabla \times \bar{E} + \mu \frac{\partial \bar{H}}{\partial t}$

& $\nabla \times \bar{E} = \frac{1}{r}\left[\frac{\partial E_z}{\partial \phi} - r\frac{\partial E_\phi}{\partial z}\right]\bar{a}_r + \left[\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r}\right]\bar{a}_\phi$

$+ \frac{1}{r}\left[\frac{\partial r E_\phi}{\partial r} - \frac{\partial E_r}{\partial \phi}\right]\bar{a}_z = -j\mu\omega\bar{H}$

$\rightarrow -j\omega\mu H_r = \frac{1}{r}\left[\frac{\partial E_z}{\partial \phi} - r\frac{\partial E_\phi}{\partial z}\right]$

∵ $\bar{E} = E_0(r,\phi)e^{j(\omega t - \beta z)} \rightarrow \frac{\partial E_\phi}{\partial z} = -E_\phi \cdot j\beta$

∴ $-j\omega\mu H_r = \frac{1}{r}\left[\frac{\partial E_z}{\partial \phi} + E_\phi \cdot j r\beta\right]$

& $-j\omega\mu H_\phi = -E_r \cdot j\beta - \frac{\partial E_z}{\partial r} \rightarrow j\omega\mu H_\phi = E_r j\beta + \frac{\partial E_z}{\partial r}$

& $-j\omega\mu H_z = \frac{1}{r}\left[\frac{\partial (rE_\phi)}{\partial r} - \frac{\partial E_r}{\partial \phi}\right]$

---

after substitutions : (wave equations in cylindrical coordinates)

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \phi^2} + q^2 E_z = 0$$

&

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r}\frac{\partial H_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 H_z}{\partial \phi^2} + q^2 H_z = 0$$

where $q^2 = \omega^2\varepsilon\mu - \beta^2 = k^2 - \beta^2$

- separation of variables can be used to solve the above wave equations:

$$E_z = A\, F_1(r) F_2(\phi) F_3(z) F_4(t)$$

$$\rightarrow F_3(z)F_4(t) = e^{j(\omega t - z\beta)}$$ sinusoidal propagating in +z direction

- since the waveguide is symmetric about the central axis (circular symmetry) the field components must not change after a full ϕ rotation $\rightarrow F_2(\phi) = e^{j\nu\phi}$, ν is ± integer

$$\therefore \boxed{\frac{\partial^2 F_1}{\partial r^2} + \frac{1}{r}\frac{\partial F_1}{\partial r} + \left(q^2 - \frac{\nu^2}{r^2}\right)F_1 = 0}$$ differential equation for bessel function

- exact same equation can be derived for H_z

- above equation must be solved for two regions:
  - inside core $r < a$, solution must remain finite as $r \to 0$
  - outside core, in cladding $r > a$, solution decays to zero as $r \to \infty$

$\therefore$

$$\forall\ r < a: \quad E_z(r<a) = A J_\nu(ur)\, e^{j\nu\phi}\, e^{j(\omega t - \beta z)}$$

$$H_z(r<a) = B J_\nu(ur)\, e^{j\nu\phi}\, e^{j(\omega t - \beta z)}$$

(A, B: arbitrary constants; $u^2 = k_1^2 - \beta^2$, $k_1 = 2\pi n_1/\lambda$, n_1: index of refraction; ν: ± ve integer)

- $J_\nu(ur)$: bessel function of first kind and order ν

$\wedge$

$$\forall\ r > a: \quad E_z(r > a) = C K_\nu(wr)\, e^{j\nu\phi}\, e^{j(\omega t - \beta z)}$$

$$H_z(r > a) = D K_\nu(wr)\, e^{j\nu\phi}\, e^{j(\omega t - \beta z)}$$

(C, D: arbitrary constants; $w^2 = \beta^2 - k_2^2$, $k_2 = 2\pi n_2/\lambda$, n_2: index of refraction)

- $K_\nu(wr)$: modified bessel function of second kind and order ν

* cutoff condition : the point at which a mode is no longer bound to the core region ($\beta \approx k_2$)

$\therefore$ for bound solutions : $n_2 k = k_2 < \beta < k_1 = n_1 k$, $k = 2\pi/\lambda$

- next, boundary conditions must be applied at interface of core and cladding. boundary conditions necessitate the tangential components (to the radius) to be equal at the interface

→ $E_\phi^{cladding} = E_\phi^{core}$, $E_z^{cladding} = E_z^{core}$ (similarly for H fields)

→ $E_{z1} - E_{z2} = A J_\nu(ua) - C K_\nu(wa) = 0$

- E_ϕ can be found in terms of E_z and H_z, same is done for H_ϕ

- therefore, a set of four equations all equal to zero in terms of the Bessel functions are obtained. These equations exist only if the determinant of the unknown coefficients is zero:

$$\begin{vmatrix} J_\nu(ua) & 0 & -K_\nu(wa) & 0 \\ \frac{\beta\nu}{au^2}J_\nu(ua) & \frac{j\omega\mu}{u}J'_\nu(ua) & \frac{\beta\nu}{aw^2}K_\nu(wa) & \frac{j\omega\mu}{w}K'_\nu(wa) \\ 0 & J_\nu(ua) & 0 & -K_\nu(wa) \\ -\frac{j\omega\epsilon_1}{u}J'_\nu(ua) & \frac{\beta\nu}{au^2}J_\nu(ua) & -\frac{j\omega\epsilon_2}{w}K'_\nu(wa) & \frac{\beta\nu}{aw^2}K_\nu(wa) \end{vmatrix} = 0$$

- evaluation of the determinant yields the following eigenvalue equation for β:

$$(\mathcal{J}_\nu + \mathcal{K}_\nu)(k_1^2\mathcal{J}_\nu + k_2^2\mathcal{K}_\nu) = \left(\frac{\beta\nu}{a}\right)^2\left(\frac{1}{u^2}+\frac{1}{w^2}\right)^2$$

where $\mathcal{J}_\nu = \frac{J'_\nu(ua)}{uJ_\nu(ua)}$ and $\mathcal{K}_\nu = \frac{K'_\nu(wa)}{wK_\nu(wa)}$

- since β is bounded between k_2 and k_1, the solutions of the above equation will be discrete values within the range
- all modes in a dielectric fiber waveguide are hybrid, except when $\nu=0$ (in above eigenvalue equation for β) → $\frac{J_1(ua)}{uJ_0(ua)} + \frac{K_1(wa)}{wK_0(wa)} = 0$, corresponding to TE_{0m} modes ($E_z = 0$)

or → $\frac{k_1^2 J_1(ua)}{uJ_0(ua)} + \frac{k_2^2 K_1(wa)}{wK_0(wa)} = 0$, corresponding to TM_{0m} modes ($H_z = 0$)

- if $V \neq 0$, analytical solutions are not possible. instead, following approximations can be used when $n_1 - n_2 \ll 1$ (weakly guided modes condition)

ν	Mode	Cutoff condition
0	TE_{0m}, TM_{0m}	$J_0(ua) = 0$
1	HE_{1m}, EH_{1m}	$J_1(ua) = 0$
≥ 2	$EH_{\nu m}$	$J_\nu(ua) = 0$
	$HE_{\nu m}$	$\left(\frac{n_1^2}{n_2^2}+1\right)J_{\nu-1}(ua) = \frac{ua}{\nu-1}J_\nu(ua)$

* normalized frequency (V): parameter connected with cutoff condition that determines how many modes a fiber can support.

$$V^2 = (u^2+w^2)a^2 = \left(\frac{2\pi a}{\lambda}\right)^2(n_1^2 - n_2^2) = \left(\frac{2\pi a}{\lambda}\right)^2 \mathrm{NA}^2$$

- normalized propagation constant, b, is found from V: $b = \frac{a^2 w^2}{V^2} = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}$
  - modes are cutoff when $\beta/k = n_2$
  - single mode (HE_{11}) is realized at $V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} \leq 2.405$
    (value at which the lowest-order bessel function ($J_0 = 0$) is obtained)
  - number of modes in a multimode fiber can be approximated when M is large as follows: $M \approx \frac{V^2}{2}$

- modes for graded index fibers $M \simeq \frac{2A}{\lambda^2}\Omega = \frac{2\pi^2 a^2}{\lambda^2}(n_1^2 - n_2^2) = \frac{V^2}{2}$

- if Δ is larger than 1%, the cable supports multi-mode operation for both step and graded index fibers

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- total power in core and cladding for each mode:

$$P = 0.5 \, \mathrm{Re}\{E \times H^*\} \cdot \bar{e}_z$$

$$P_{core} = \frac{1}{2}\int_0^a \int_0^{2\pi} r(E_x H_y^* - E_y H_x^*) \, d\theta \, dr$$

$$\frac{P_{cladding}}{P} = 1 - \frac{P_{core}}{P}$$

$\wedge$

$$P_{cladding} = \frac{1}{2}\int_0^{a}\int_0^{2\pi} r(E_x H_y^* - E_y H_x^*) \, d\theta \, dr$$

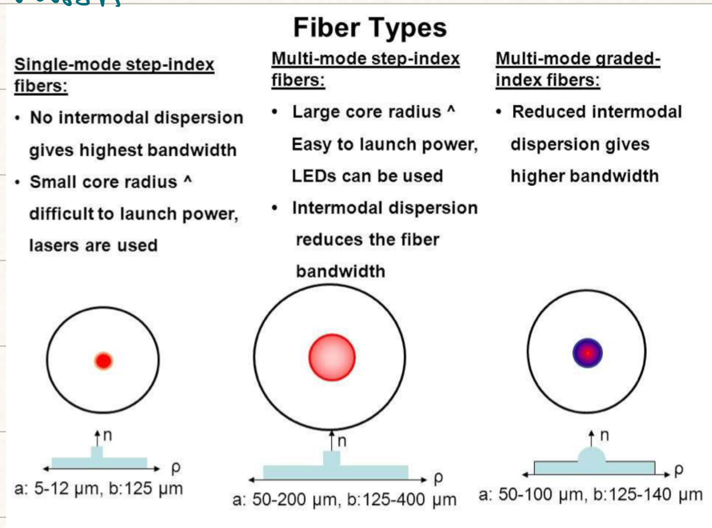
$$\wedge \quad \left(\frac{P_{cladding}}{P}\right)_{total} \approx \frac{4}{3} M^{-\frac{1}{2}}$$

where $P = P_{cladding} + P_{core}$

- single mode index difference ranges from 0.002 to 0.01
- HE_{11} is the dominant mode with distribution: $E(r) = E_0 e^{\left(\frac{-r^2}{W_0^2}\right)}$
  where W_0 is the mode field diameter, also referred to as d_n, such that:

$$d_n \; 2\sqrt{2}\left(\frac{\int_0^\infty E^2(r)\, r^3 \, dr}{\int_0^\infty E^2(r)\, r \, dr}\right)^{1/2}$$

+ fiber comparison



+ fiber materials must be:

1- ductile (can be made long, thin, and flexible)

2- must be transparent allowing for wide range of optical wavelengths with low losses

3- materials for core and cladding must be physically compatible

- silica is most used for fibers (SiO_2)
- refractive index of core or cladding can be varied with dopants such as GeO_2, B_2O_3, P_2O_5

Core	**Cladding**
SiO_2+GeO_2(dopant)	**SiO_2**
SiO_2+P_2O_5(dopant)	**SiO_2**
SiO_2	**SiO_2+B_2O_3 (dopant)**
SiO_2+[GeO_2+B_2O_3] (dopants)	**SiO_2+B_2O_3 (dopant)**

2.12: ∵ $NA = (n_1^2 - n_2^2)^{1/2} \approx n_1 (2\Delta)^{1/2}$, $\Delta = \frac{n_1 - n_2}{n_1}$

∵ $n_1 = 1.48$ & $n_2 = 1.46$

→ $NA = 0.2425 \approx 0.243$

& $\theta_{0,max} = \sin^{-1}[(n_1^2 - n_2^2)^{1/2}] = \sin^{-1}[NA] = 0.245$ rad (14°)

2.18: $NA = 0.2$, 1000 modes , 850-nm

∵ $V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2}$ → number of modes $\approx \frac{V^2}{2}$

(a: core radius; λ: wavelength)

a) ∵ $\frac{V^2}{2} \approx 1000$ → $V \approx 44.72 = \frac{2\pi a}{850n} \cdot 0.2 \rightarrow a = 30.25 \mu m$

∴ $d = 60.5 \mu m$

b) $M = 414.66$ → 414 modes

c) $M = 300.9$ → 300 modes

2.21: ∵ $V \leq 2.405$ for single mode

→ $2.405 \geq \frac{2\pi a}{1.32} \cdot NA$, $NA = (n_1^2 - n_2^2)^{1/2} = 0.079$

& $\theta_{0,max} = 4.41°$

∴ $a \lesssim 6.56 \mu m$

2.29: ∵ $M_g = \frac{\alpha}{\alpha + 2} a^2 k^2 n_1^2 \Delta \approx \frac{\alpha}{\alpha+2} \frac{V^2}{2}$

(α: core index profile, α = 2.0 for parabolic; k: propagation constant: $\frac{2\pi}{\lambda}$)

$V_{@820nm} = 46.45$, $V_{@1.3\mu m} = 29.29$

at 820 nm

→ $M_g = 539$ modes, at 1.3 μm → $M_g = 214$ modes

modes for this core index profile are half that of step index first 2021:

Q1: + three main types of fibers:

1- multimode step-index fiber (50 - 200 um core, 125 - 400 um cladding)
   - low bandwidth (disadvantage)
   - many modes can be supported (advantage)

graded index can't be single mode

2- multimode graded-index fiber: (50 - 100 um core, 125 - 140 um cladding)
   - longer bandwidth than multimode step-index
   - less modes than multimode step-index

3- single mode step-index fiber (8 - 12 um core, ~125 um cladding) monomode
   - high bandwidth
   - requires light source to be directed, very small max acceptance angle

Q2: $d = 50\,\mu$, step, $n_1 = 1.48$, $\Delta = 1\%$ at 1550n

$\therefore \Delta = \frac{n_1 - n_2}{n_1} = 1\% \rightarrow n_2 = 1.4652$, $\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = 81.9°$

$NA = n_1(2\Delta)^{1/2} = 0.21$, $\theta_{0,max} = \sin^{-1}(NA) = 12.1°$

$\therefore V = \frac{2\pi a}{\lambda} \cdot NA = 21.28 \rightarrow M = \frac{V^2}{2} = 226$ modes

for single mode: $V \leq 2.405 \rightarrow \frac{2.405}{0.21} \cdot 1.55 = 2\pi a \rightarrow a < 2.82$

$d < 5.64\,\mu m$

Q3: the four modes are TE_{0m}, TM_{0m}, HE_{vm}, and EH_{vm}

- dominant mode is the operating mode with the lowest cutoff frequency, HE_{11}
- the tangential components in the core and cladding E_z, E_ϕ, H_z, and H_ϕ must be equal at the interface

Q4: 1- modified chemical vapor deposition

2- outside vapor deposition

3- vapor axial deposition

- silica is the main material SiO_2 or special plastics
- GeO_2 raise refractive index, B_2O_3 drops it
  or P_2O_5

Q5: conservation of volume → $L \cdot \pi \frac{D^2}{4} = l \pi \frac{d^2}{4}$ → $\left(\frac{d}{D}\right)^2 = \frac{L}{l}$ → $l = 25600$ m

same core to cladding ratio → $\frac{D_c}{20\,mm} = \frac{50\mu}{125\mu}$ → $D_c = 8$ mm

drawing speed: 1 m → 25600 m → 20 mm → 512 m/min

or $D^2 \cdot L = d^2 l$ ∧ $L = S \cdot t$, $l = s \cdot t$ → $d^2 s = D^2 S$ → $s = \left(\frac{D}{d}\right)^2 S$

- the solution to the bessel equation inside the core is:

$$E_z(r<a) = A J_\nu(ur) e^{j\nu\phi} e^{j(\omega t - \beta z)}$$

$$H_z(r<a) = B J_\nu(ur) e^{j\nu\phi} e^{j(\omega t - \beta z)}$$

- - ——— - - —— —— in the cladding region:

$$E_z(r>a) = C K_\nu(wr) e^{j\nu\phi} e^{j(\omega t - \beta z)}$$

$$H_z(r>a) = D K_\nu(wr) e^{j\nu\phi} e^{j(\omega t - \beta z)}$$

2018 spring:

Q2: $n_2 = 1.4553$ → $\Phi_c = 81.4°$, $NA = 0.208$ → $\frac{V^2}{2} = m = 222$

$\theta_{0,max} \approx 12°$ $0 \le \Delta \le 0.03$ multimode

Q4: 1- modified chemical vapor deposition

2- outside vapor deposition

3- vapor axial deposition

| glass and special plastics are mainly used

GeO_2 and P_2O_5 to raise index
fluorine B_2O_3 to lower

Q5: to conserve volume: $L\pi \frac{D^2}{4} = l\pi \frac{d^2}{4} \rightarrow \frac{L}{l} = \frac{d^2}{D^2}$

$\frac{L}{5\,km} = \left(\frac{125\,\mu m}{10\,mm}\right)^2 \rightarrow L = 0.78125\,m$

diameter ratio must remain constant $\rightarrow \frac{D_c}{D} = \frac{d_c}{d} \rightarrow D_c = 4\,mm$

$\therefore L \cdot D^2 = l\,d^2$ & $L = St,\ l = \Delta t \rightarrow SD^2 = \Delta d^2$

$\therefore \Delta = S\left(\frac{D}{d}\right)^2 = 96000\,mm/min = 96\,m/min = 1.6\,m/s$

example 3 in notes:

$\therefore M = \frac{V^2}{2}$ & $V = \frac{2\pi a}{\lambda} NA$ & $NA = 1.48 \cdot (2 \cdot 0.01)^{1/2} \rightarrow M = 748$

$\therefore \left(\frac{P_{clad}}{P_{total}}\right) \approx \frac{4}{3} M^{-1/2} = 4.88\%\ P_{clad}$

& $P_{core} = 95.12\%$ of total power

Spring 2013:

Q1: optical advantages over coaxial:

- higher bandwidth
- smaller size → lower cost
- less attenuation → less repeaters required
- immune to noise
- no cross talk
- difficult to tap
- common ground not required

disadvantages:

- more fragile
- difficult splicing
- can't carry electric power to repeaters

10 Gbit/s for voice → 155520 channels

+ SDH vs PDH:

- PDH max rate of 566 Mbps, SDH max of 160 Gbps
- PDH not compatible with other signal
- SDH is simpler to implement

Q4: TE: transverse electric, E-field is perpendicular to reference plane and propagation direction $E_z = 0$

subscript is the possible mode of propagation, its value equals the number of field zeros across the waveguide

- hybrid modes exist when the z-components are non-zero (both E_z and H_z) called EH if $E_z > H_z$ and HE on the contrary.
- all modes are hybrid except when $v = 0$

$HE_{vm} \rightarrow v$ number of field zeros across length of waveguide

& m — — — — — width — —

Q5:

$$E_z(r<a) = A J_v(ur) e^{jv\phi} e^{j(wt-\beta z)}$$

$$H_z(r<a) = B J_v(ur) e^{jv\phi} e^{j(wt-\beta z)}$$

& $$E_z(r>a) = C K_v(wr) e^{jv\phi} e^{j(wt-\beta z)}$$

$$H_z(r>a) = D K_v(wr) e^{jv\phi} e^{j(wt-\beta z)}$$

Q6: $a = 50\mu m$, $n_1 = 1.47$ & $n_2 = 1.46$, $\lambda = 1300 nm$

step-index: $m = \frac{V^2}{2}$, $V = \frac{2\pi a}{\lambda} NA$, $NA = (n_1^2 - n_2^2)^{1/2} \rightarrow m = 213$

graded-index: $m = \frac{\alpha}{\alpha+2} \cdot m_{step}$ assuming parabolic: $m = 106$

Q7: for single mode: $0.002 \leq \Delta \leq 0.01$ ✓

∴ $V \leq 2.405$ & $V = \frac{2\pi a}{\lambda} \cdot NA$ & $NA = 0.1458$

for $\lambda = 850 n \rightarrow V = 4.31$ can't be used | $\lambda = 1300 n$ can't

if $\lambda = 1550 n \rightarrow V = 2.367$ can be used |

Q8: 1- can be drawn into thin and long wires + flexible

2- transparent for low losses and many wavelengths

3- core and cladding must be physically compatible

GeO_2 → increases index , B_2O_3 → decreases index, fluorine decrease

P_2O_5 → increases

Q9: 1- modified chemical vapor deposition (MCVD)

2- outside vapor deposition (OVD)

3- vapor axial deposition (VAD)

to produce silica : $SiCl_4 + O_2 \xrightarrow{\Delta} SiO_2 + 2Cl_2$

GeO_2 : $GeCl_4 + O_2 \xrightarrow{\Delta} GeO_2 + 2Cl_2$

$4POCl_3 + 3O_2 \xrightarrow{\Delta} 2P_2O_5 + 6Cl_2$

$4BCl_3 + 3O_2 \xrightarrow{\Delta} 2B_2O_3 + 6Cl_2$

Q10: $l d^2 = L D^2$ → $L = l\left(\frac{d}{D}\right)^2 = 0.98125$ m

$\frac{10}{125} = \frac{Dc}{D}$ → $Dc = 0.8$ mm

+ received signal is not the same as transmitted due to:

- attenuation - distortion - time delay 4- noise 5- ISI 6- multipath fading

- attenuation determines maximum repeaterless distance between Tx and Rx

$$P(z) = P(0)\, e^{-\alpha_p z}$$

where z is distance from transmitter, α_p is attenuation coefficient (nepers/km)

$$\alpha_p = \frac{1}{z} \ln\left(\frac{P(0)}{P(z)}\right) \rightarrow \alpha(dB) = \frac{10}{z} \log\left[\frac{P(0)}{P(z)}\right] = 4.343\, \alpha_p$$

- for a fiber of length L:

$$P_{out}(dBm) = P_{in}(dBm) - \alpha(dB/km) \cdot L(km)$$

+ attenuation is caused by:

- **absorption**
  - by atomic defects in glass composition
    - small compared to other two
    - due to imperfections in atomic structure
  - extrinsic absorption by impurity atom in glass material
    - due to impurities in the glass materials
  - intrinsic absorption by basic constituent atoms of fiber material
    - set a lower limit for attenuation
    - associated with basic manufacture material
    - UV absorption, IR absorption
- **scattering**
  - due to density fluctuations within the fiber which cause refractive index variations
- **radiative (bending)**
  - macrobending
    - bend is much larger than fiber diameter
  - microbending

- UV loss for a given mole fraction x of GeO_2 to SiO_2 is: $\alpha_{uv} = \frac{154.2x}{46.6x + 60} \cdot x \cdot 10^{-2} \cdot e^{\left(\frac{4.63}{\lambda}\right)}$ (dB/km)

- IR absorption for SiO_2 glass is: $\alpha_{IR} = 7.81 \cdot 10^{11} \cdot e^{\left(\frac{-48.48}{\lambda}\right)}$ (dB/km)

- Rayleigh scattering is inversely proportional to quadratic wavelength, given by:

$$\alpha_{scat} = \frac{8\pi^3}{3\lambda^4} (n^2 - 1)^2 \cdot k_B \cdot T_f \cdot \beta_T$$

(n: refractive index; k_B: boltzmann constant; T_f: fictive temperature; β_T: isothermal compressibility of material)

- higher order modes radiate out first in macrobending.
- curved fiber supports less modes than straight fiber.
- number of modes that a curved fiber can support is found from:

$$N_{eff} = N_\infty \left\{1 - \frac{\alpha+2}{2\alpha\Delta} \cdot \left[\frac{2a}{R} + \left(\frac{3}{2kRn_2}\right)^{2/3}\right]\right\}, \quad N_\infty = \frac{\alpha}{\alpha+2}(n_1 k a)^2 \Delta$$

(N_{eff}: effective number of modes; N_∞: number of modes in a straight fiber; R: radius of curvature)

- bending loss: occurs due to more of the signal tail traveling in the cladding

$$\therefore \frac{P_{clad}}{P} = \frac{4}{3}(V)^{-1/2} \rightarrow \text{bending loss} = \left[1 - \left(\frac{N_{eff}}{N_\infty}\right)^{1/2}\right]$$

- bit rate to minimize ISI is found from the rms pulse width ΔT: $B < \frac{1}{\Delta T}$ (bits/s) (B: bit rate)

- pulse broadening increases linearly with distance, hence the bit rate - distance product is a measure of capacity:

constant (depends on fiber type) = $B \cdot L$ (b·km/s) or bandwidth-distance product: $BW \cdot L$ (MHz·km)

- $BW \cdot L$ is prefered over $B \cdot L$ since BW is constant, whereas B depends on modulation scheme.

- BW·L for multi-mode step index ~ 20 MHz·km, for graded-index ~ 2.5 GHz·km, for single mode > 10 GHz·km

+ types of dispersion:
  - intermodal (modal delay) — due to delays between different modes carrying the same optical pulse at same wavelength
  - intramodal (chromatic) — due to delays between different wavelengths carrying the same optical pulse in same mode, increases with spectral width of optical source (50 nm for LED and 3 nm for LD)
    - material dispersion: due to variation of refractive index as function of wavelength
    - waveguide dispersion: due to waves propagating faster in cladding than in core
  - polarization — due to different polarizations carrying the same optical pulse

- for intermodal, worst pulse width in multimode step index: $\Delta T = T_{max} - T_{min} = \frac{n_1}{c}\left(\frac{L}{\sin(\theta_c)} - L\right) = \frac{L n_1^2}{c\, n_2}\cdot\Delta$

$$\rightarrow B\cdot L = \frac{L}{\Delta T} = \frac{c\, n_2}{n_1^2 \Delta} \approx \frac{c}{\Delta n_1} \quad b\cdot km/s$$

- rms pulse width (standard case):

$$(\Delta T)_{rms,\,step} = \frac{L \Delta n_1}{2\sqrt{3}\, c} \rightarrow B\cdot L = \frac{2\sqrt{3}\, c}{n_1 \cdot \Delta} \quad b\cdot km/s$$

$$(\Delta T)_{rms,\,graded} = \frac{L \Delta^2 n_1}{2\sqrt{3}\, c} \rightarrow B\cdot L = \frac{2\sqrt{3}\cdot c}{n_1\cdot \Delta^2} \quad b\cdot km/s$$

- group velocity: $V_g = \frac{\partial \omega}{\partial \beta}$
- phase velocity: $V_p = \frac{\omega}{\beta}$

$$V_p \cdot V_g = \left(\frac{c}{n}\right)^2$$

- group delay per unit length: $\tau_g = 1/V_g = \frac{\partial \beta}{\partial \omega} = -\frac{\lambda^2}{2\pi}\cdot\frac{\partial \beta}{\partial \lambda}$ ($\frac{\partial \lambda}{2\pi\,\partial \lambda^{-1}} = \frac{-\lambda^2\,\partial\beta}{2\pi\cdot\partial\lambda}$; $\partial \lambda^{-1} \rightarrow -\lambda^{-2}\,\partial\lambda$)
  - if spectral width is narrow, the delay difference per unit wavelength is $\frac{d\tau_g}{d\lambda}$
  - for spectral components which lie at $\frac{\delta\lambda}{2}$ above and below the central wavelength λ_0, the total delay difference over a distance L is: $\delta_\tau = \frac{d\tau_g}{d\lambda}\delta\lambda = -\frac{L}{2\pi c}\left(2\lambda\cdot\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right)\delta\lambda$

$$D = -\frac{1}{2\pi c}\left(2\lambda\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right)$$

- pulse dispersion by its rms pulse width:

$$\sigma_g = D\cdot L\cdot\sigma_\lambda = -\frac{L\sigma_\lambda}{2\pi c}\left(2\lambda\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right)$$

(D: dispersion factor; σ_λ: optical source pulse width)

- material dispersion: $\beta(\lambda) = \frac{2\pi n(\lambda)}{\lambda}$, $\beta(\lambda)$ is not linear
  - substituting $\beta(\lambda)$ into the equation for the dispersion factor D: $D_{mat} = \frac{\lambda}{c}\frac{d^2 n}{d\lambda^2} \rightarrow \sigma_{mat} = D_{mat}\cdot L\cdot\sigma_\lambda$

$$B\cdot L = \frac{1}{\sigma_{mat}}$$

- Chromatic dispersion (waveguide intramodal), due to β depending nonlinearly on freq.

$$\beta = n_2 k\left(b\Delta + 1\right), \quad b = \frac{\left(\frac{\beta}{k}\right)^2 - n_2^2}{n_1^2 - n_2^2} \quad \text{normalized propagation constant}$$

(k: propagation constant in free space; b: normalized propagation constant; Δ: index difference)

- group delay can be found in terms of kb or Vb as:

$$\tau_{wg} = \frac{L\,\partial\beta}{c\,\partial k} = \frac{L}{c}\left[n_2 + n_2\Delta\frac{\partial(kb)}{\partial k}\right] \quad \text{or} \quad \tau_{wg} = \frac{L}{c}\left[n_2 + n_2\Delta\frac{\partial(Vb)}{\partial k}\right], \text{ where } \frac{d(Vb)}{dV} = b\left[1 - \frac{2J_v^2(ua)}{J_{v+1}(ua)\,J_{v-1}(ua)}\right]$$

$\rightarrow \sigma_{wg}^{rms} = \sigma_\lambda \frac{d\tau_{wg}}{d\lambda} = -\frac{L\Delta n_2 \sigma_\lambda}{c\lambda} \cdot V \frac{d^2(Vb)}{dV^2}$, $V = \frac{2\pi a}{\lambda} \cdot n_1 (2\Delta)^{1/2}$

(V: normalized frequency (V-parameter))

or

$\sigma_{wg} = D_{wg} \cdot L \cdot \sigma_\lambda$, $D_{wg} = -\frac{\Delta n_2}{c\lambda} \cdot V \frac{d^2(Vb)}{dV^2}$ (found from graph)

- the normalized propagation constant b as a function of normalized frequency for HE_{11} is: $b(V) = 1 - \frac{(1+\sqrt{2})^2}{[1+(4+V^4)^{1/4}]^2}$
- total intramodal (chromatic) dispersion: $D_{chrom} = D_{mat} + D_{wg}$
- polarization mode dispersion (PMD): time delay between two polarization modes: $\Delta\tau_{PMD} = \left| \frac{L}{V_{gx}} - \frac{L}{V_{gy}} \right| \approx D_{PMD} \cdot \sqrt{L}$ (D_{PMD}: 0.05 to 1 ps/$\sqrt{km}$; V_{gx}, V_{gy}: group velocity of modes x & y)

+ benefits of operating at 1550 nm:

1- minimum fiber loss, 2- erbium-doped amplifier operating frequency, 3- zero dispersion can be shifted here

- material dispersion is fixed, but waveguide dispersion can be varied by changing the size of the core, refractive indices, etc.

Quiz practice: step & graded: $n_1 = 1.48$, $\Delta = 0.02$

a) $(B \cdot L)_{step} = 35.11$ Mbps·km , $(B \cdot L)_{graded} = 1.75$ Gbps·km

b) $(B)_{graded} = 17.55$ Mbps , $(B)_{step} = 0.35$ Mbps

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EE555: homework #2

3.2: ∵ $P_{out}(dBm) = P_{in}(dBm) - \alpha(dB/km) \cdot L(km)$

& $P_{in, at\ 1310} = -8.239$ dBm , $P_{in, at\ 1550} = -10$ dBm

a) $L = 8$ km ∴ $P_{out, 1310} = -13.039$ dBm $= \boxed{49.67\ \mu W}$

& $P_{out, 1550} = -12.4$ dBm $= \boxed{57.54\ \mu W}$

b) $L = 20$ km ∴ $\boxed{P_{out, 1310} = 9.46\ \mu W}$ & $\boxed{P_{out, 1550} = 25.12\ \mu W}$

3.13: ∵ $\sigma_{mat} = D_{mat} \cdot L \cdot \sigma_\lambda$ & D_{mat} found from graph.

a) $D_{mat, 850} \approx -75$, assuming quenched SiO_2 → $\boxed{\sigma_{mat} = 3.375\ ns/km}$ LED

& $\boxed{\sigma_{mat} = 0.15\ ns/km}$ LD

b) $D_{mat, 1550} \approx 20$ → $\boxed{\sigma_{mat} = 1.5\ nm/km}$

3.16: ∵ $\Delta = \frac{n_1 - n_2}{n_1}$ & eq.: $\sigma_s \approx \frac{L n_1 \Delta}{2\sqrt{3}\, c}$

→ σ_s from approximate expression: $\boxed{\frac{\sigma_s}{L} \approx 21.36\ ns/km}$

exact eq.: $\frac{\sigma_{mod}}{L} = \frac{n_1 - n_2}{c}\left(1 - \frac{\pi}{V}\right)$, $n_1 - n_2 = n_1 \Delta$, $V = \frac{2\pi a}{\lambda} \cdot n_1 (2\Delta)^{1/2} = 76.844$

→ $\boxed{\frac{\sigma_{mod}}{L} = 70.97\ ns/km}$

- These two equations produce significantly different results.

3.22: ∵ $D_{wg} = -\frac{n_2 \Delta}{c\lambda} \cdot V \cdot \frac{d^2(Vb)}{dV^2}$, at $\lambda = 1320$ nm $V = \frac{2\pi a}{\lambda} n_1 (2\Delta)^{1/2} = 4.205$

not single mode! at $\lambda = 1320$ nm, $a < 5.15$ um for single mode.

assuming $a = 5$ um → $V = 2.336$ → $V \cdot \frac{d^2(Vb)}{dV^2} \approx 0.3$

∴ $D_{wg} = -\frac{n_2 \Delta}{c\lambda} \cdot 0.3$ & $n_2 = 1.4769$ → $D_{wg} = -2.46$ ps/(nm·km)

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EE555 : homework #3

4.3: for $Ga_{1-x}Al_xAs$, $E_g = 1.424 + 1.266x + 0.266x^2$

(a) if $E_g = 1.54$ eV →

$\boxed{x = 0.0899}$ by solving $0.266x^2 + 1.266x + 1.424 - E_g = 0$

$E_g = h\nu \rightarrow \nu = \frac{E_g}{h} \rightarrow \lambda = \frac{ch}{E_g} = \boxed{0.805\,\mu m}$

(b) for $x = 0.015$, $\boxed{E_g = 1.443\text{ eV}}$, $\boxed{\lambda = 0.859\,\mu m}$

4.6: (a) : $\eta_{int} = \frac{\tau_{nr}}{\tau_r + \tau_{nr}} = \frac{90}{90+25} = \boxed{78\%}$

$P_{int} = \eta_{int} \cdot \frac{I}{q} \cdot h\nu$, $\nu = \frac{c}{\lambda}$ → $\boxed{P_{int} = 29.8\text{ mW}}$

(b) $P = P_{int} \cdot \eta_{ext}$, $\eta_{ext} = \frac{1}{n_1(n_1+1)^2} = 14.1\%$ → $\boxed{P = 0.364\text{ mW}}$

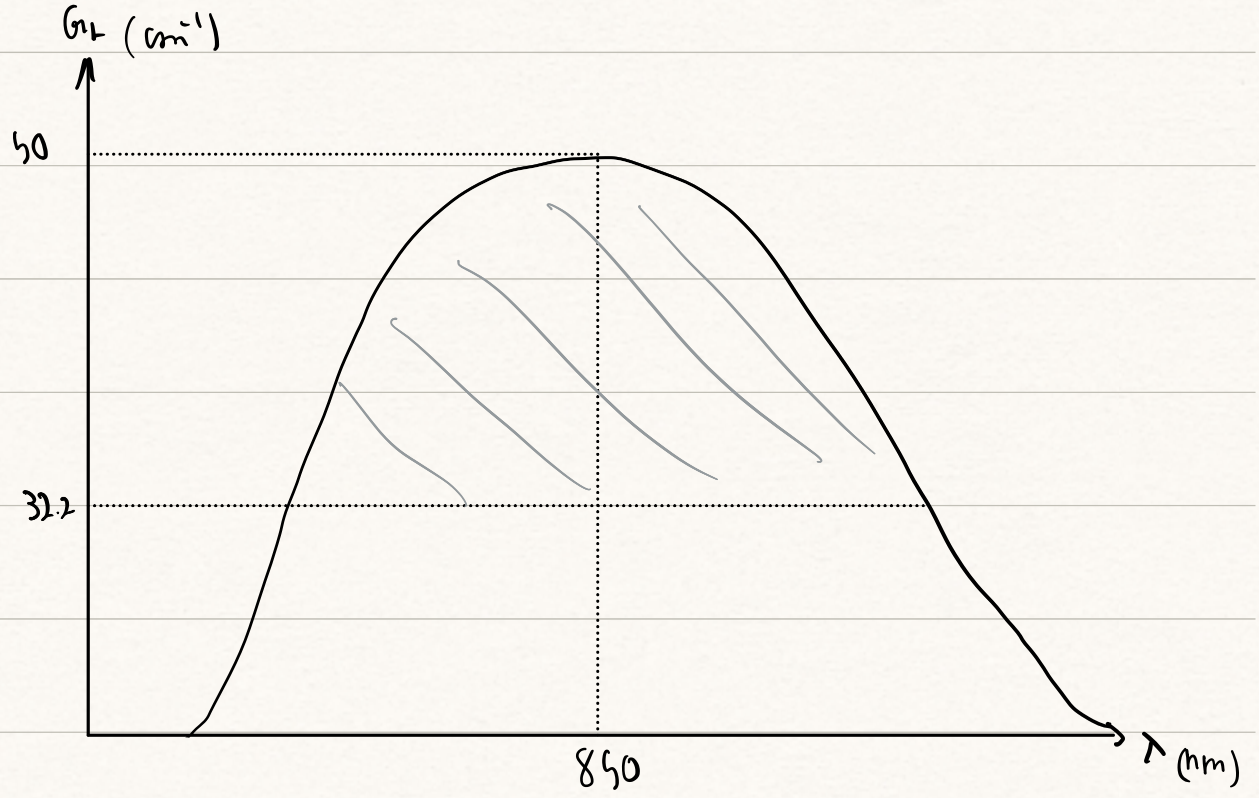
4.9: (a) : $g_{th} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$ → $g_{th} = 32.78\text{ cm}^{-1}$ ($\Gamma = 1$)

(b) $g_{th} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{0.32 \cdot 0.9}\right) = 22.45\text{ cm}^{-1}$

(c) $\eta_{ext} = \frac{\eta_i(g_{th} - \alpha)}{g_{th}}$ → $\eta_{ext}(a) = 45.17\%$

→ $\eta_{ext}(b) = 36.04\%$

4.13:



$$\because\ g_{th} = g(0)\cdot e^{\left(-\frac{(\lambda_1-\lambda_0)^2}{2\sigma^2}\right)} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

$$\rightarrow \frac{-(\lambda_1-\lambda_0)^2}{2\sigma^2} = \ln\left[\frac{\alpha}{g(0)}\right] \quad R_1=R_2=0.32$$

$$\rightarrow \lambda_1-\lambda_0 = 30\ nm \quad \rightarrow \Delta\lambda = \frac{(850)^2}{2\cdot 408000\cdot 3.6} = 0.25$$

$$\therefore \frac{2(\lambda_1-\lambda_0)}{\Delta\lambda} = 239\ \text{modes}$$

4.15: (a) $\because\ g_{th} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right) = \beta J_{th}$ $R_1=R_2=0.32$

$\rightarrow J_{th} = 2646.5\ A/cm^2$

$\therefore I_{th} = J_{th}\cdot \text{Area} = 0.661\ A$

(b) $I_{th} = \frac{0.661}{10} = 0.0661\ A$

quiz practice :

characteristics of light sources :

1) high switching speed, 2) long lifetime, 3) low cost

Coherent light is directional same freq. same phase same polarization

$y = 0.88$

$\therefore E_g = 0.809\ eV \rightarrow \lambda(\mu m) = 1.532\ \mu m$

Second preparation:

+ required characteristics of optical source:

1- driven by current 2- wavelength in low loss window
3- beam must be directional 4- size compatible
5- small spectral width 6- high switching speed
7- high optical power (radiance) 8- linear (with input power)
9- long lifetime 10- high efficiency

- LED: spontaneous emission, incoherent, not directional, large spectral width
- LD: stimulated emission, coherent, directional, small spectral width

- concentration of free electrons in the valence band (concentration of holes in the conduction band) is:

$$n = p = n_i = K \exp\left(-\frac{E_g}{2k_BT}\right), \quad K = 2\left(2\pi k_B T/h^2\right)^{3/2} \cdot \left(m_e m_h\right)^{3/4}$$

$$E_g = h\nu = \frac{hc}{\lambda}$$

+ LED characteristics required:

1- high radiance output, 2- fast emission response time,
3- high quantum efficiency

+ characteristics can be achieved by confining one or more of the following: 1- carrier, 2- optical, 3- current

1- carrier: by giving recombination region minimum bandgap energy

2- optical: refractive index of recombination region is greater than others

3- current: by making the depletion region narrower such that the current is forced through a certain area.

+ types of LEDs:

SLED	ELED
1- easy to fabricate, 2- easy to mount	1- difficult to fabricate, 2- difficult to mount
3- less critical tolerance, 4- less reliable	3- need critical tolerances, 4- highly reliable
5- low performance, 6- smaller BW	5- high performance, 6- better BW
7- less power coupled	7- more power coupled

$In_{1-x}Ga_xAs_yP_{1-y}$: 1000 - 1700 nm

$Ga_{1-x}Al_xAs$: 800 - 900 nm

GaAs, GaSb, InAs, etc. : fixed wavelength

- ternary : $E_g = 1.424 + 1.266x + 0.266x^2$ eV

- quaternary : $y = 2.2x \rightarrow E_g = 1.35 - 0.72y + 0.12y^2$ eV

$n(t) = n_0 e^{-t/\tau}$, n_0 initial value of electron concentration

recombination rate : $R = \frac{dn}{dt} = -\frac{1}{\tau} n_0 e^{-t/\tau} = -\frac{n(t)}{\tau}$ $\frac{n}{\tau}$

injection rate : $\frac{J}{qd}$

carrier recombination : $\frac{dn}{dt} = \frac{J}{qd} - \frac{n}{\tau}$ (photons/second)

equilibrium : $\frac{dn}{dt} = 0 \rightarrow n = \frac{J\tau}{qd}$ electrons/cm^3

- quantum efficiency is ratio of radiative recombination to total (rad + nonrad)
  (light) (heat)

internal quantum efficiency : $\eta_{int} = \frac{R_r}{R_r + R_{nr}} = \frac{\tau_{nr}}{\tau_r + \tau_{nr}}$

total recombination rate : $R_r + R_{nr} = I/q$ electrons/s

internal power generated : $P_{int} = \eta_{int} \cdot \frac{I}{q} \cdot h\nu = \eta_{int} \frac{hcI}{q\lambda}$ W

$\eta_{ext} = \frac{n_2}{n_1(n_1+n_2)^2}$, if interface is air, $n_2 = 1 \rightarrow \eta_{ext} = \frac{1}{n_1(n_1+1)^2}$

output power = $\eta_{ext} \cdot P_{int} = \eta_{ext} \cdot \eta_{int} \cdot \frac{hcI}{q\lambda}$ W

- commonly used modulation techniques cannot be used with LED optical sources due to the large spectral width (not single wavelength), but may be used with coherent sources (like LDs)

+ LED response time depends on:

1- injected carrier lifetime τ_i, 2- parasitic capacitance

3- doping level of active region

Output power as function of frequency: $P(\omega) = P_o [1 + (\omega \tau_i)]^{-1/2}$

+ LED advantages:

1- simple to design and manufacture

2- low cost, 3- high reliability, 4- long lifetime

5- less sensitive to temperature than LDs

+ LED disadvantages:

1- low coupling efficiency, 2- large spectral width (chromatic dispersion)

3- low switching speed

+ LED uses:

1- short range optical links, 2- digital systems up to 200 Mb/s

3- multimode optical links

+ laser characteristics:

- spatial and temporal coherence
- small spectral width (monochromatic)
- high directivity and intensity
- small beam cross section

+ conditions for lasing:

1- active gain, 2- carrier population inversion, 3- positive feedback

more electrons in conduction band than valence

leads to higher number of electrons producing photons

* fabry-perot cavity: two parallel mirrors positive feedback
* Bragg gratings: distributed feedback by selecting spacing between gratings for specific wavelength gain.

Fabry-perot:

light intensity; optical field confinement factor; absorption coefficient

$$I(z) = I(0)\exp([\Gamma g(h\nu) - \alpha(h\nu)]z)$$

gain coefficient

for a round trip:

length of cavity

$$I(2L) = I(0) R_1 R_2 e^{2L[\Gamma g(h\nu) - \alpha(h\nu)]}$$

n_2 inside active medium

$$R_1 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2, \quad R_2 = \left(\frac{n_3 - n_2}{n_3 + n_2}\right)^2$$

power reflection coefficients

* gain: number of photons generated from one photon's round trip.

at lasing threshold: $I(2L) = I(0) \rightarrow \ln\left(\frac{1}{R_1 R_2}\right) = [\Gamma g(h\nu) - \alpha(h\nu)]\cdot 2L$

$$\therefore \Gamma g_{th} = \alpha + \frac{1}{2L} \cdot \ln\left(\frac{1}{R_1 R_2}\right)$$

- for lasing, $g_{th} < g$

to determine L — $\because E(z,t) = A(z) e^{j(\omega t - \beta z)} \rightarrow e^{-j2\beta L} = 1$

phase condition : $2\beta L = 2\pi m$, $m = 0, 1, 2, \ldots$

$g_{th} = \gamma J_{th} \rightarrow g_{th} \propto J_{th}$ (g_{th} in cm^{-1})

(γ: some constant)

$I_{th} = J_{th} \cdot \text{Area}$

- if $I < I_{th} \rightarrow$ LED, if $I > I_{th} \rightarrow$ LD
- I_{th} should be minimized to reduce heating and power

rate equation : (photon density) $\frac{d\phi}{dt} = Cn\phi + R_{sp} - \frac{\phi}{\tau_{ph}}$ (ϕ = photon density)

(stimulated, spontaneous, absorption)

(electron density) $\frac{dn}{dt} = \frac{J}{qd} - \frac{n}{\tau_{sp}} - Cn\phi$

- if operating above I_{th}, both rate equations equal zero

$$R_{sp} - \frac{\phi}{\tau_{ph}} = \frac{J}{qd} - \frac{n}{\tau_{sp}} \rightarrow \phi = \left(R_{sp} + \frac{n}{\tau_{sp}} - \frac{J}{qd}\right) \cdot \tau_{ph}$$

$$\rightarrow \phi_s = \frac{\tau_{ph}}{qd}(J - J_{th}) + \tau_{ph} R_{sp}$$

$\therefore P_{int} = \phi_s \cdot h\nu \cdot$ volume of laser

$$\rightarrow P_{int} = \frac{\tau_{ph}}{qd}(J - J_{th}) \cdot h\nu \cdot L \cdot W \cdot d = \frac{\tau_{ph}}{q} \cdot (I - I_{th}) \cdot h\nu$$

$$\therefore P = \eta_{ext} P_{int} = \frac{\eta_{int}(g_{th} - \alpha)}{g_{th}} P_{int}, \quad \eta_{int} \sim 0.6 - 0.9$$

- Resonant modes must satisfy the phase condition

$\therefore 2\beta L = 2\pi m$ & $\beta = \frac{2\pi n}{\lambda} = \frac{2\pi n \nu}{c}$

$\rightarrow \frac{4\pi n \nu L}{c} = 2\pi m$

$\therefore$ Resonant frequencies: $\nu_m = m \frac{c}{2Ln} \rightarrow \Delta\nu = \frac{c}{2L}$

$\Delta\lambda = \frac{\lambda_0^2}{c} \Delta\nu = \frac{\lambda_0^2}{2Ln}$

σ: spectral width of gain

Optical gain: $g(\lambda) = g(0) \exp\left[\frac{-(\lambda-\lambda_0)^2}{2\sigma^2}\right]$

for lasing modes: $g_{th} = g(0) \exp\left[\frac{-(\lambda_1-\lambda_0)^2}{2\sigma^2}\right]$

number of modes: $\frac{2(\lambda_1-\lambda_0)}{\Delta\lambda}$

+ for single mode:

1- L is reduced to make $\Delta\nu$ large $\therefore \Delta\nu = \frac{c}{2Ln}$

2- g_{th} increased, but requires more current thus impractical

3- Using distributed feedback to filter out modes

Bragg wavelength: $\lambda_B = \frac{2n_e\Lambda}{k}$ (n_e: effective refractive, Λ: period, k: order of grating)

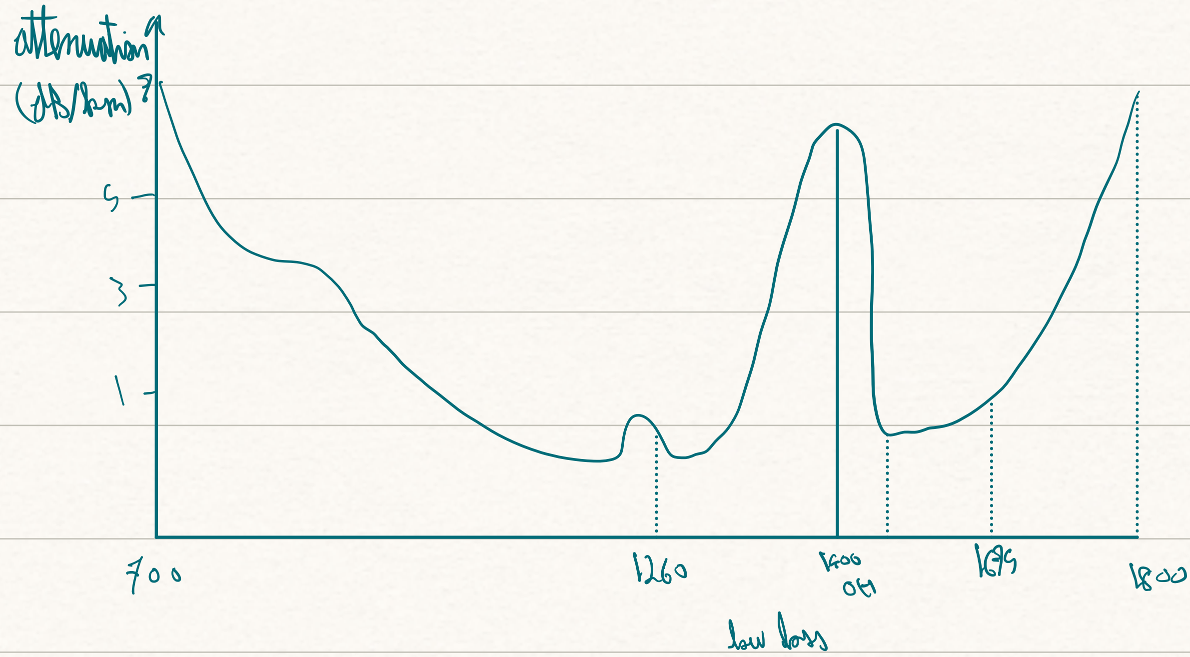
LD modulation:

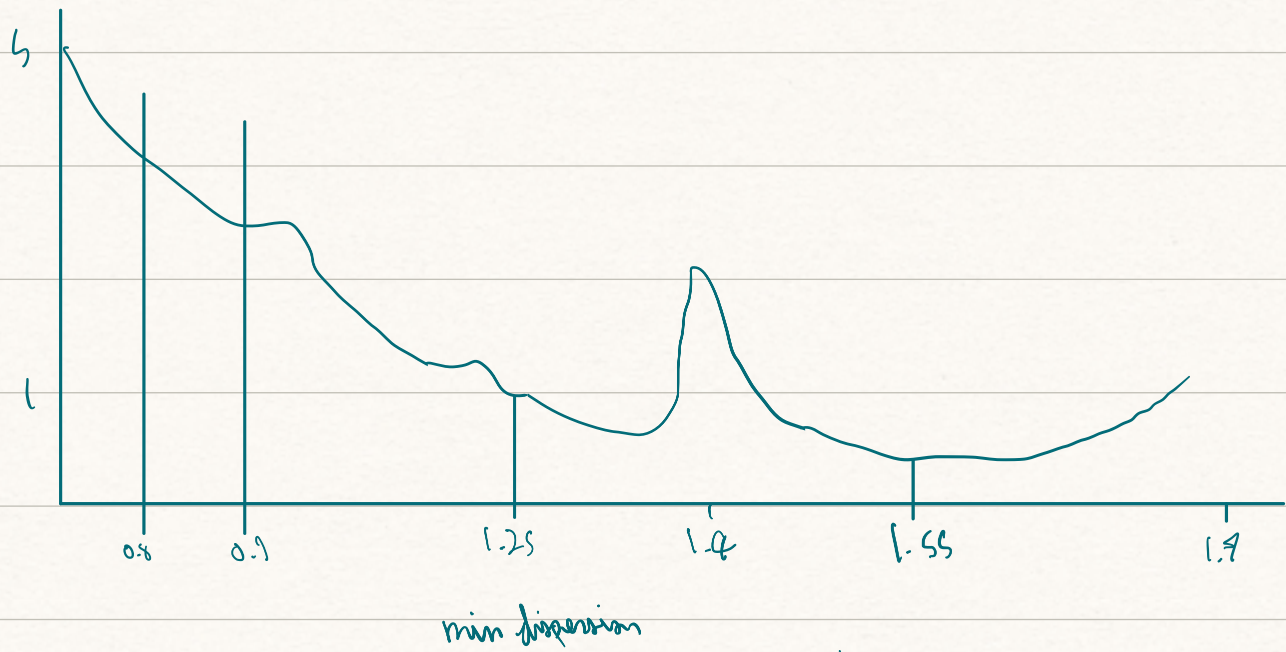
1- direct (with current switching) — used for BW < 3 GHz 2- external (external on-off switch) — with beam splitters and combiners and phase shifters

- Temperature affects LDs greatly, for a certain output power more current is needed for higher temperatures

$I_{th} = I_z e^{T/T_0}$ (T_0: relative temperature insensitivity) mid 2021

Q6





O original band : 1260 - 1360 nm | extended E Band : 1360-1460
min loss

S short band : 1460 - 1530 | C conventional band : 1530 - 1565

L long band : 1565 - 1625 | U ultra long : 1625 - 1675

Q7: P_{out} dBm = P_{in} dBm − 0.2 · 50 =

−3 − 10 = −13 dBm = 50 μW

Q8: 1- intermodal dispersion (modal delay)

2- intramodal dispersion (chromatic dispersion)

3- polarization mode dispersion

+ types of single mode fibers:

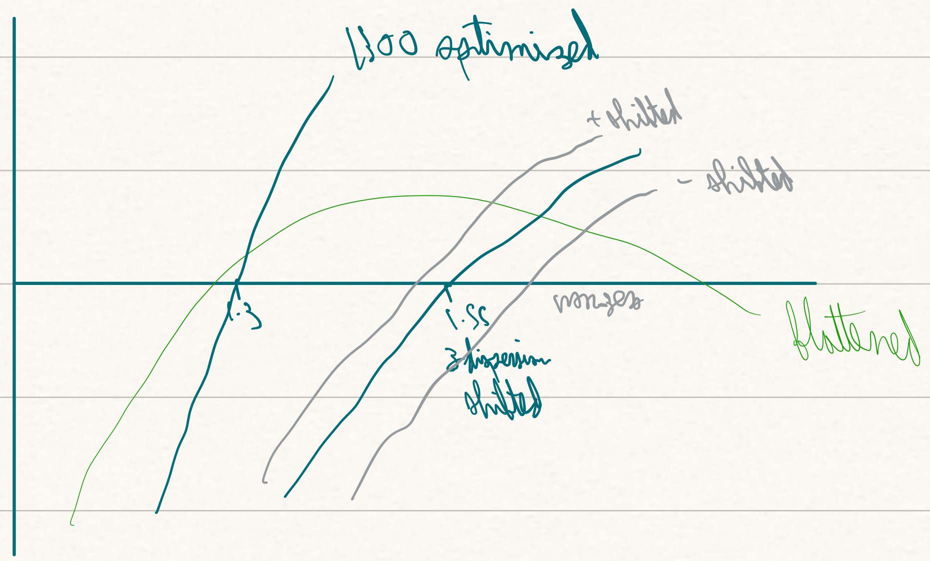
1- 1300 nm optimized single-mode zero dispersion at 1300 nm but high loss

2- zero dispersion shifted : zero dispersion and low loss at 1550 — intermodulation distortion

3- non-zero dispersion shifted : zero dispersion shifted from 1550 nm — lower four wave mixing

4- dispersion-flattened : very low dispersion over second and third windows

5- dispersion compensating fiber : negative dispersion to compensate



Q9: standard $B \cdot L = \frac{2\sqrt{3}\, c}{n_1 \cdot \Delta}$ step $= 3.534 \times 10^7$

$\Delta = \frac{n_1 - n_2}{n_1}$ $B \cdot L$ graded $= \frac{2\sqrt{3}\, c}{n_1 \Delta^2} = 1.767 \times 10^9$

$B_{step} = 706.8$ kbps , $B_{graded} = 35.34$ Mbps

$n_2 = n_1 \left(1 - \frac{0.5}{100}\right)$
$\rightarrow n_2 = 1.4597$

Q10: $\sigma_{chrom} = L \sigma_\lambda \left[D_{wg} + D_{mat} \right]$

$D_{mat} = 5$, $D_{wg} = - \frac{\Delta n_2}{c \lambda} V \frac{d^2(Vb)}{dV^2}$

$\rightarrow D_{wg} = - \frac{\frac{0.5}{100} \cdot n_2}{c \lambda} \cdot 0.4 = -6.25$ ps/nm·km

$\sigma_{chrom} = 100 \cdot 1 \cdot (-1.25) = -125$ ps

max bit rate $= \frac{1}{|\sigma_{chrom}|} = 8$ Gbps , $B \cdot L$ max $= 800$ Gbps·km

Second 2015

Q1: 1- light source must be driven by current

2- wavelength must be in low loss window

3- beam must be directional

4- size of light source must be compatible with fiber

5- optical carrier must have small spectral width

LED light is incoherent, whereas LD light is coherent.

Q2: GaAlAs : 800-900 nm

InGaAsP : 1000-1700 nm

GaAl, InAs, etc. : fixed wavelength

- Ge and Si have indirect bandgaps and cannot be efficiently used.

Q3: 1- carrier population inversion, 2- active gain

3- positive feedback

- double heterostructure is used for carrier confinement.

Q4: $\because$ $y = 2.2x$ & $E_g = 1.35 - 0.72y + 0.12y^2$

$\rightarrow E_g = 1.125$ eV

$\because$ $\lambda(\mu m) = 1.24 / E_g(eV)$ $\rightarrow$ $\lambda = 1.102\ \mu m$

$\rightarrow$ photon energy $= h\frac{c}{\lambda} = 1.804 \times 10^{-19}$ J

$\rightarrow$ external power $= \eta_{ext} \cdot P_{int} = \eta_{ext} \cdot \eta_{int} \cdot \frac{I}{q} \cdot E_g$

$\because$ $\eta_{ext} = \frac{1}{n_1(n_1+1)^2}$ & $\eta_{int} = \frac{\tau_{nr}}{\tau_{nr} + \tau_r}$

$\therefore$ external power $= 0.254$ mW

Q5: $\because$ $\Gamma g_{th} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$ (0.85; 20 cm^{-1}; 0.05 cm; 0.5·0.5)

$\rightarrow g_{th} = 39.84$ cm^{-1}

$\because$ $g_{th} = \beta_g \cdot J_{th} = \beta_g \cdot \frac{I_{th}}{L \cdot W}$ (0.02; 0.05; 10×10^{-4}) $\rightarrow I_{th} = 99.6$ mA

note: $\beta_g = 0.02$ cm/A

number of excited modes: $\frac{2(\lambda_1 - \lambda_0)}{\Delta\lambda}$

$\because$ $\Delta\lambda = \frac{\lambda_0^2}{2Ln}$ ($(1550 \times 10^{-9})^2$; 500×10^{-6}, 3.5) & $g_{th} = g(0)\, e^{\frac{-(\lambda_1 - \lambda_0)^2}{2\sigma^2}}$ (39.84; 50; 5×10^{-9})

$\rightarrow \Delta\lambda = 0.686$ nm $\rightarrow$ $\lambda_1 - \lambda_0 = 3.37$ nm

$\therefore$ number of excited modes $= 9$ (round down)

for single mode: $\Delta\lambda = 2(\lambda_1 - \lambda_0)$ $\rightarrow$ $2Ln = \frac{\lambda_0^2}{2(\lambda_1 - \lambda_0)}$

$\therefore L = 50.9\ \mu m$

Q6 → 8: N/A

3 dB → half power

Q9: $\because B(\theta, \phi) = B_0 \cos(\theta) \rightarrow \frac{1}{2} B_0 = B_0 \cos(\theta) \rightarrow \theta = 60^\circ$

beamwidth $= 2\theta = 120^\circ$

lateral $\rightarrow \phi = 0$, $\frac{1}{B(\theta,\phi)} = \frac{\sin^2(\phi)}{B_0 \cos^T(\theta)} + \frac{\cos^2 \phi}{B_0 \cos^L(\theta)}$

half power: $5^\circ \rightarrow \theta = 2.5^\circ$ $\therefore \frac{1}{B(2.5, 0)} = 0 + \frac{1}{B_0 \cos^L(\theta)}$

$\rightarrow \frac{1}{2} B_0 = B_0 \cos^L(\theta) \rightarrow L = \log_{\cos\theta}\left(\frac{1}{2}\right)$

$\therefore L = 727.9$ not included, I think

Q10: N/A

Second 2017

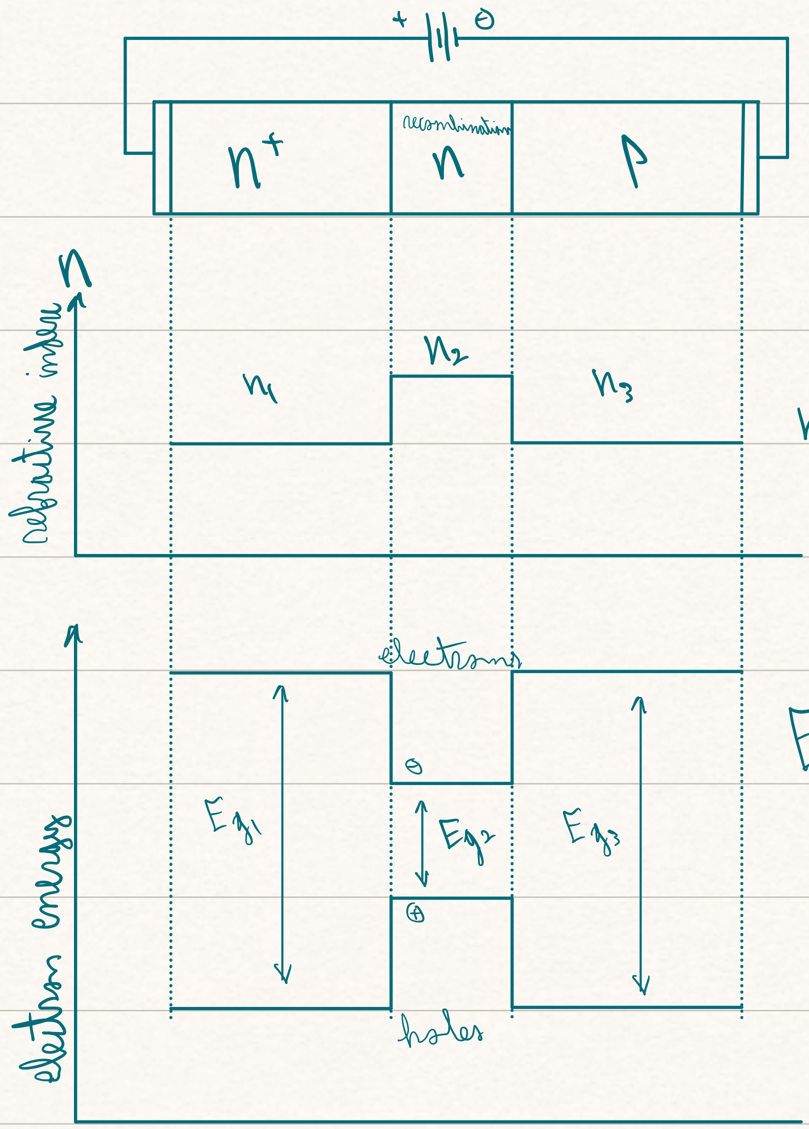
Q1: LED:

- advantages: simple to design and manufacture, low cost, less sensitive to temp.
- disadvantages: low coupling efficiency, large chromatic dispersion

LDs:

- advantages: low chromatic dispersion, high coupling efficiency, high bandwidth
- disadvantages: high temperature sensitivity, difficult to manufacture

Q2:



This structure is used to confine the carrier within the recombination region.

$n_2 > n_1, n_3$

$E_{g2} < E_{g1}, E_{g3}$

Q3: 1- active gain, 2- carrier population inversion, 3- positive feedback

confinement methods: 1- carrier, 2- optical, 3- current

Q4: $\because E_g = 1.35 - 0.72y + 0.12y^2$ & $y = 2.2x \rightarrow E_g = 1.1255$ eV

$\because E_g = h\nu = h\frac{c}{\lambda} \rightarrow \lambda = \frac{hc}{E_g}$ (6.626×10^{-34}, 3×10^{8}, $1.1255\times1.602\times10^{-19}$) $\rightarrow \lambda = 1.102$ um

$\because \eta_{int} = \frac{\tau_{nr}}{\tau_{nr}+\tau_r}$ & $P_{int} = \eta_{int} \cdot \frac{I}{q} \cdot E_g \rightarrow P_{int} = 22.51$ mW

& $\eta_{ext} = \frac{n_2}{n_1(n_1+n_2)^2}$, emitted to air implies $n_2 = 1$ $\therefore \eta_{ext} = \frac{1}{n_1(n_1+1)^2}$

$\rightarrow P_{ext} = \eta_{ext} \cdot P_{int} = 317.6$ uW

Q5: $\because g_{th} = \frac{1}{\Gamma}\left[\alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)\right] = 55.67\ cm^{-1}$

$\because g_{th} = \beta \cdot J_{th}$ & $J_{th} = \frac{I_{th}}{L \cdot W}$, $\beta = 0.02$ cm/A

$\rightarrow I_{th} = 111.3$ mA

$\because g_{th} = g(0)\, e^{\frac{-(\lambda_1-\lambda_0)^2}{2\sigma^2}} \rightarrow \frac{-(\lambda_1-\lambda_0)^2}{2\sigma^2} = \ln\left(\frac{g_{th}}{g(0)}\right)$

$\rightarrow \lambda_1 - \lambda_0 = 2.32$ nm

& $\Delta\lambda = \frac{\lambda_0^2}{2Ln}$ (850nm, 400um), $n = 3.5 \rightarrow \Delta\lambda = 0.258$ nm

$\therefore$ number of modes: $\frac{2(\lambda_1-\lambda_0)}{\Delta\lambda} = 17$

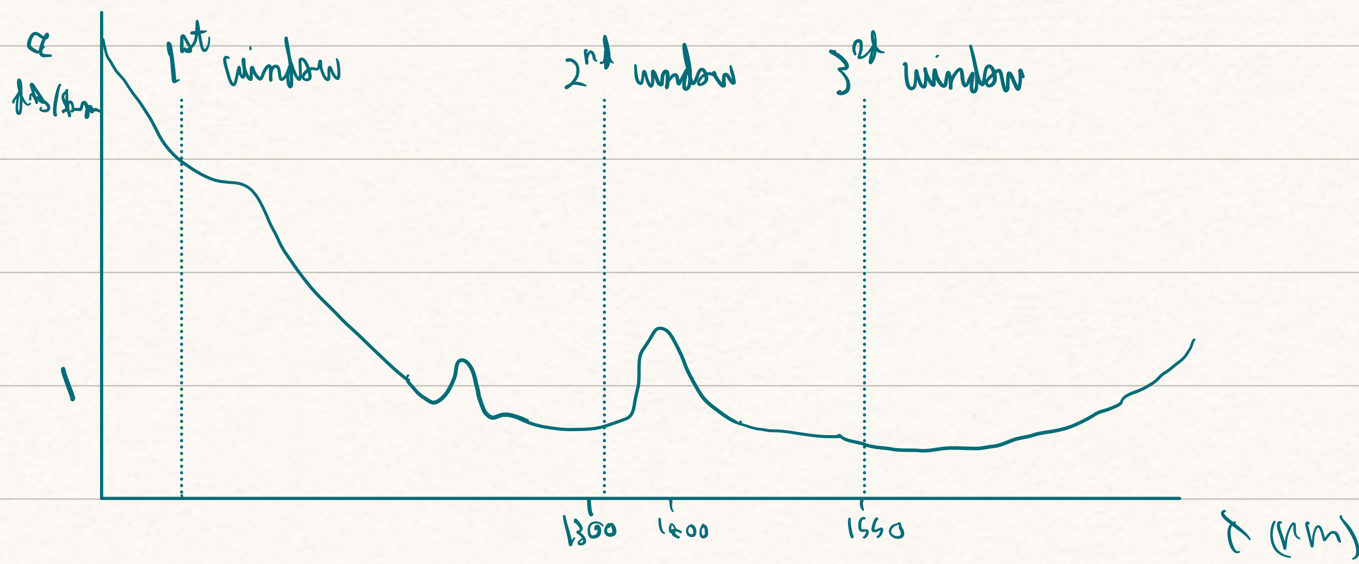
Q6→10: N/A

Second 2012

Q1: 1- absorption, 2- scattering, 3- radiative

extrinsic absorption: caused by impurities in the glass.

intrinsic absorption: caused by normal fiber material.



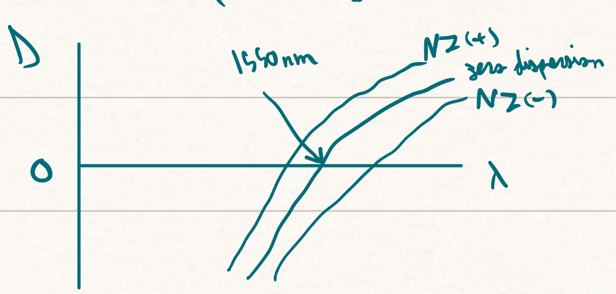
Q2: $P_{out} = P_{in} - \alpha \cdot L \rightarrow P_{in} = P_{out} + \alpha \cdot L$

$\therefore P_{in} = 1\ mW$

Q3: 1- intermodal dispersion (modal delay)

2- intramodal (chromatic) dispersion

3- polarization mode dispersion



designed by double cladding and careful selection of refractive indices of cladding material.

Q4: $D_{chrom} = D_{mat} + D_{wg} \rightarrow \sigma_{chrom} = D_{chrom} \cdot L \cdot \sigma_{\lambda}$

$D_{wg} = -4.29 \text{ ps}/(\text{nm} \cdot \text{km}) \rightarrow D_{chrom} = 5.71 \text{ ps}/(\text{nm} \cdot \text{km})$

$\therefore \sigma_{chrom} = 571 \text{ ps}$

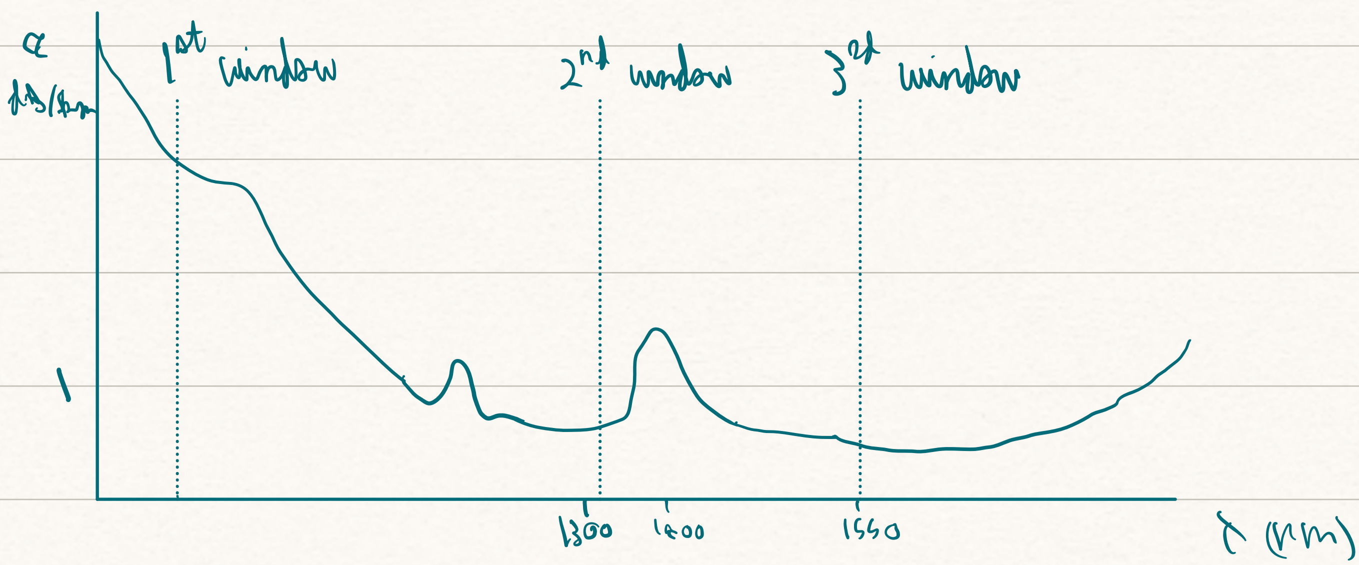
$B \cdot L = \dfrac{L}{\sigma_{chrom}} = 175 \text{ Gbps} \cdot \text{km}$

Second 2012

Q1: 1- absorption, 2- scattering, 3- Radiative

extrinsic absorption: caused by impurities in the glass.

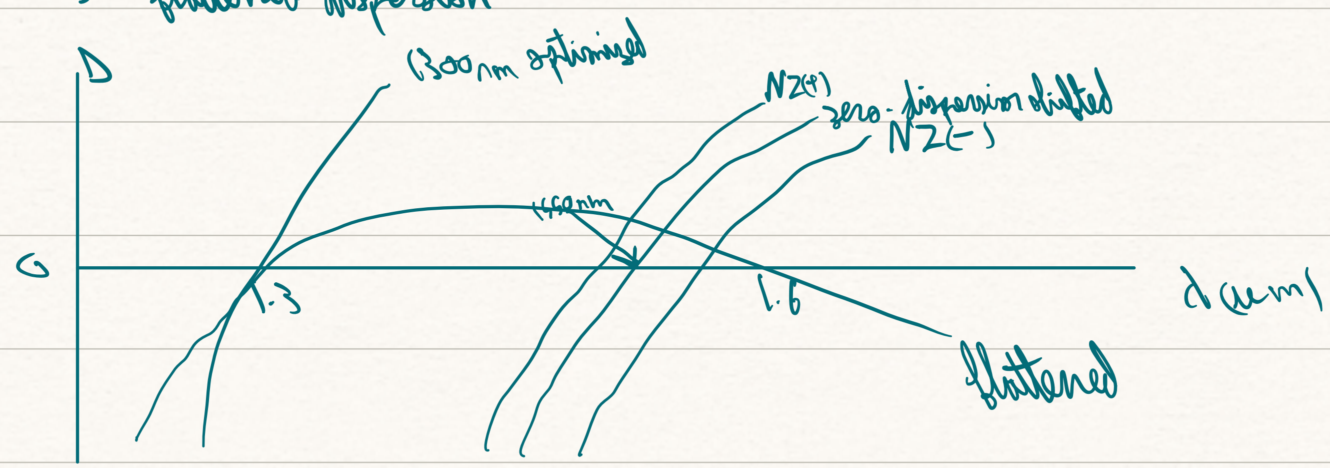
intrinsic absorption: caused by normal fiber material.



Q2: $P_{out} = P_{in} - \alpha \cdot L = 1.58 \ uW$

Q3: 1- 1300 nm optimized, 2- dispersion shifted,

3- flattened dispersion



Q4: $D_{chrom} = D_{mat} + D_{wg}$, $D_{wg} = \frac{-n_2 \Delta}{\lambda c} \cdot 0.5$

→ $\sigma_{chrom} = D_{chrom} \cdot L \cdot \sigma_\lambda = -72$ ps

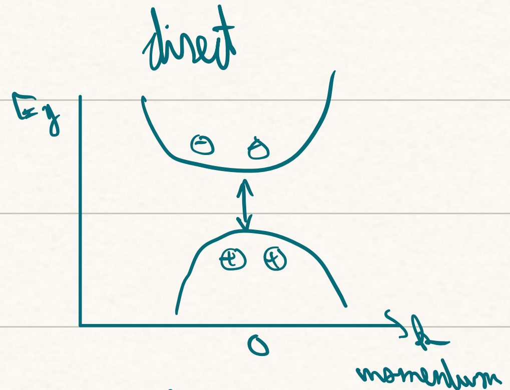
∵ single mode step → $V < 2.405$ → $\frac{2\pi a}{\lambda} \cdot n_1 (2\Delta)^{1/2} < 2.405$

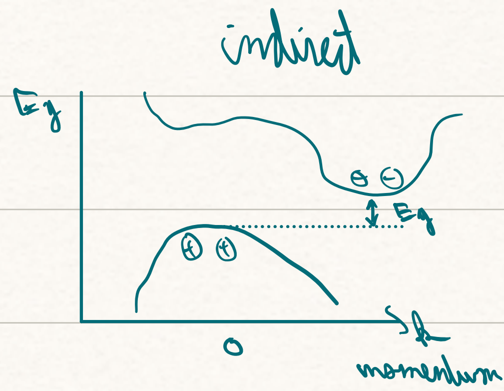
∴ $\lambda_c = \frac{2\pi a \cdot n_1 \cdot (2\Delta)^{1/2}}{2.405} = 1.59$ μm

$B \cdot L = \frac{L}{\sigma_{chrom}} = 1.38$ THz · km

→ B for 50 km $= \frac{1.38}{50} = 27.8$ GHz

Q5: direct / indirect





GaAlAs   Si

InGaAsP   Ge

- direct bandgap material are used

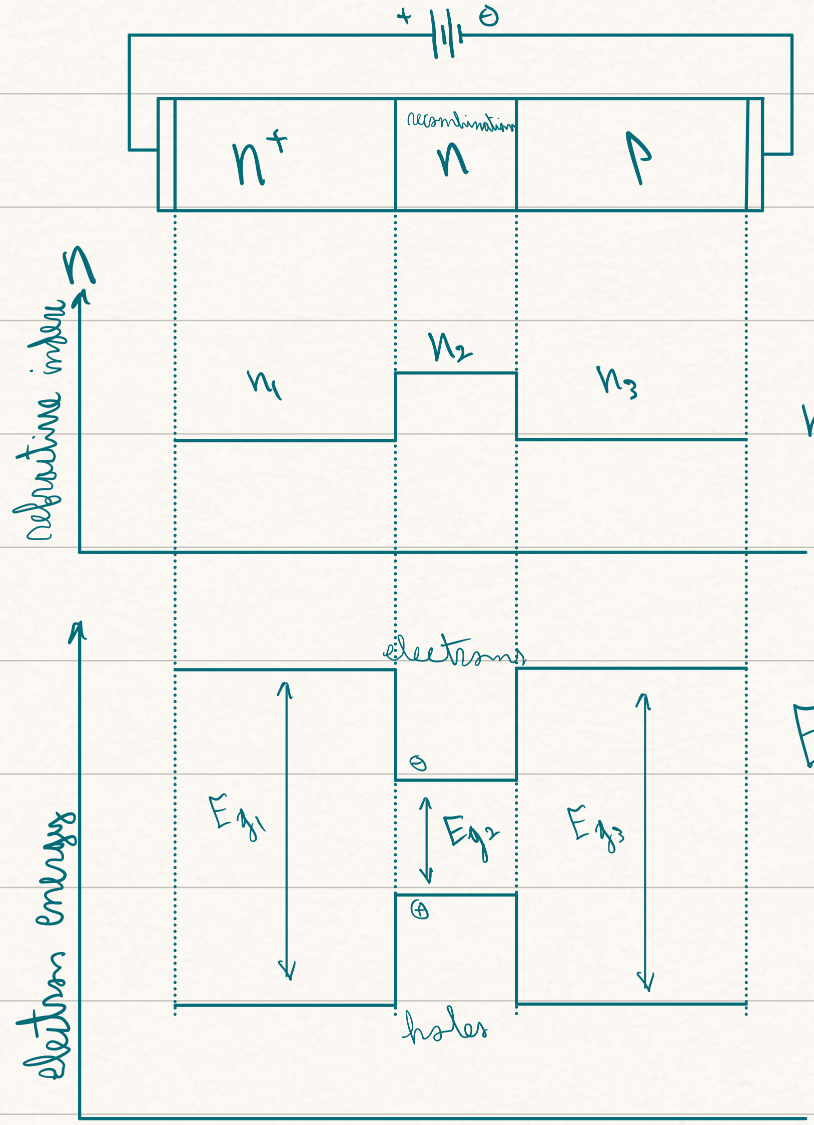
Q7: 1- active gain, 2- carrier population inversion, 3- positive feedback

1- carrier, 2- optical, 3- current · confinement

Q8: 1- LED emits incoherent light (LD coherent)

2- LED emits spontaneously, 3- LED large spectral width

Q6:



$n_2 > n_1, n_3$

$E_{g2} < E_{g1}, E_{g3}$

Q9:

$$E_g = \frac{1.24}{\lambda(\mu m)} = 0.8\ eV$$

$\therefore E_g = 1.35 - 0.72y + 0.12y^2 \rightarrow y = 0.898$

$\therefore y = 2.2x \rightarrow x = 0.408$

$P_{int} = \eta_{int} \cdot \frac{I}{q} \cdot E_g \quad \wedge \quad \eta_{int} = \frac{\tau_{nr}}{\tau_{nr} + \tau_r} \quad \therefore P_{int} = 12.8\ mW$

$P_{ext} = \eta_{ext} \cdot P_{int}, \quad \eta_{ext} = \frac{1}{n(n+1)^2} \quad \therefore P_{ext} = 180.6\ \mu W$

Q10: $\Gamma g_{th} = \alpha + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) \rightarrow g_{th} = 25.99 \ cm^{-1}$

$g_{th} = \delta \cdot J_{th} \rightarrow I_{th} = 1.299 \ A$

$\ln\left(\frac{g_{th}}{g(s)}\right) = \frac{-(\lambda_1 - \lambda_0)^2}{2\sigma^2} \rightarrow \lambda_1 - \lambda_0 = 4.58 \ nm$

$\sim \Delta\lambda = \frac{\lambda_0^2}{2Ln}$, n not given. can be solved with assumptions

$R_2 = 0.5 = \left(\frac{1-n}{1+n}\right)^2 \rightarrow n = 5.828$

$\therefore \Delta\lambda = 0.124 nm \rightarrow$ number of modes: 73

chapter 6:

+ photodetector requirements:

  1- high responsivity and sensitivity in low-loss region

  2- high response time or bandwidth to handle required data rate.

  3- low noise

  4- high linearity and dynamic range

  5- small size, long lifetime, low cost.

Types of detectors:

- photodiodes: compact, fast, linear, quantum efficient, high dynamic range
- metal-semiconductor-metal (MSM): faster than photodiodes
- phototransistors: similar to photodiodes but internally amplify photocurrent
- photoconductive detectors: cheaper than PDs, slower, less sensitive, nonlinear
- phototubes: photoelectric effect

- Two types of photodiodes used exclusively as optical Receivers: PIN and APD

  x: distance, α_s: absorption coefficient

- incident optical power absorbed: $P(x) = P_{in} e^{-\alpha_s x}$
  - for depletion region of length w: $P(w) = P_{in}(1 - e^{-\alpha_s x})$
- direct bandgap gives lower transit time → higher bandwidth

- quantum efficiency: $\frac{I_p/q}{P_{in}/h\nu}$ = $\frac{h\nu}{q} \cdot \frac{I_p}{P_{in}} = \eta$

number of generated electron

number of incident photons

- responsivity: $R = \frac{I_p}{P_{in}} = \frac{\eta q}{h\nu}$ (A/W)

$\rightarrow I_p = \frac{\eta q \lambda}{hc} \cdot P_{in}$

- multiplication gain: $M = \frac{I_m}{I_p}$

- responsivity of APD = responsivity of PIN multiplied by gain

$R_{APD} = M \cdot R = M \cdot \frac{\eta q \lambda}{hc}$ (A/W)

- primary photocurrent: $i_p(t) = \underset{\text{responsivity}}{R} \cdot P(t)$

$(i_p(t))^2 \propto \sigma^2_{s,PIN}$

$(i_p(t))^2 \cdot M^2 \propto \sigma^2_{s,APD}$

- shot noise power: $\sigma^2_{shot} = 2 q I_p M^2 \cdot F(M) B_e$

$F(M) = M^x \rightarrow \sigma^2_{shot}$ for PIN $= 2 q I_p B_e$

- dark current power: $\sigma^2_{DB} = 2 q I_D M^2 F(M) B_e$ (A^2)

- dark surface current: $\sigma^2_{DB} = 2 q I_L B_e$, not multiplied

- Thermal noise: $\sigma^2_T = \frac{4 k_B T}{R_L} \cdot B_e$ (A^2)

MOHAMMAD SANAD ALTAHER 130806

EE555: homework #4

6.6: compare noise from dark current, dark surface current, and shot noise.

$\because P_{in} = 500\,nW$ & $\eta = 0.95 \rightarrow I_P = 0.593\,uA$

$\therefore \sigma^2_{shot} = 2q I_P \cdot B_e = 2.85 \times 10^{-17}\ A^2$

& $\sigma^2_{DB} = 2q I_D \cdot B_e = 4.8 \times 10^{-20}\ A^2$

& $\sigma^2_{DS} = 0$

& assuming $T = 300$ (27°C)

$\rightarrow \sigma^2_T = \frac{4 k_B T}{R_L} \cdot B_e = 4.97 \times 10^{-15}\ A^2$

- σ^2_T is the largest and therefore dominant.

6.12: $\because \tau_{RC} = R_T C_T = R_T \cdot \frac{\varepsilon A}{W}$, $\varepsilon = \varepsilon_0 \cdot \varepsilon_r$

ε_r given equal to 11.9 $\therefore R_T \cdot C_T = 2.59\,ns$

$\because$ carrier drift: $t_d = \frac{W}{V_d}$

& $V_d = 4.4 \times 10^4$ m/s for electrons } $t_d = 0.45\,ns$

- RC time constant is dominant, diffusion time negligible

7.5: a) $\because P_0(V_{th}) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{V_{th}-V_{off}}{\sqrt{2}\,\sigma_{off}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{V_1}{2\sqrt{2}\cdot 0.2V_1}\right)$

$\therefore P_0(V_{th}) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{5\sqrt{2}}{4}\right) \approx 0.0162$

& $P_1(V_{th}) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{25\sqrt{2}}{24}\right) \approx 0.0339$

b) $P_e = \frac{1}{2}\left(0.65\cdot P_0(V_{th}) + 0.35\,P_1(V_{th})\right) = 0.0112$

c) $P_e = 0.0125$

7.7: $\because P_r(n) = N^n \frac{e^{-N}}{n!}$, $n = 5$

$\because P_{in} = 25\,\mu W$ & $\alpha = 40\,dB \rightarrow P_{received} = 2.5\,nW$

received energy in $1\,ns = 2.5\,nW \cdot 1\,ns = 2.5\times10^{-18}\,J$

received photons in $1\,ns = 16.35$

$\therefore P_r(5) = 16.35^5 \cdot \frac{e^{-16.35}}{5!} = 7.72\times10^{-4}$

Chapter 5:

- Coupling efficiency: $\eta = \frac{P_F}{P_S}$ fiber / source

SLED

- radiation pattern of lambertian source: $B(\theta, \phi) = B_0 \cos(\theta)$ (radiance along normal to radiating surface)
- ELEDs and LDs do not have a lambertian radiation pattern

$$\frac{1}{B(\theta,\phi)} = \frac{\sin^2(\phi)}{B_0 \cos^T(\theta)} + \frac{\cos^2(\phi)}{B_0 \cos^L(\theta)}$$

$(\phi = 0°)$ $(\phi = 90°)$

  - integers L and T are the lateral and transversal power distribution coefficients of the radiation pattern (very large for LDs)

- for a SLED to step-index fiber, coupled power:

$$P_{SLED,\,step} = \pi^2 r_s^2 (NA)^2 \cdot B_0$$

(NA: numerical aperture)

- any optical source with circular emitting surface and uniform radiance has a total emitted power of $P_S = \pi^2 \cdot r_s^2 B_0$

$\rightarrow P_{SLED,\,step} = (NA)^2 \cdot P_S \rightarrow \eta = (NA)^2$ for $r_s < a$

  - if $r_s > a$, then $P_{SLED,\,step} = \left(\frac{a}{r_s}\right)^2 \cdot (NA)^2 \cdot P_S$ (η for $r_s > a$)

- for graded-index, power coupled from SLED for $r_s < a$:

$$P_{LED,\,graded} = 2\pi^2 \cdot r_s^2 \cdot B_0 \cdot n_1^2 \cdot \Delta \cdot \left[1 - \frac{2}{\alpha+2}\left(\frac{r_s}{a}\right)^\alpha\right]$$

(profile $\alpha \simeq 2$)

$\because n_1\sqrt{2\Delta} = NA \rightarrow P_{SLED,\,graded} = (NA)^2 \cdot P_S \cdot \left[1 - \frac{2}{\alpha+2}\left(\frac{r_s}{a}\right)^2\right]$

- if $r_s > a$: $P_{sLED,\ graded} = \left(\frac{a}{r_s}\right)^2 \cdot (NA)^2 \cdot P_s\left(1 - \frac{a}{\alpha+2}\right)$
- due to differences in refractive indices of the source and fiber, some power will be reflected

$$\rightarrow P_{coupled} = (1-R)\, P_{emitted}\ , \quad R = \left[\frac{n_F - n_{LED}}{n_F + n_{LED}}\right]^2$$

(R: power reflectivity)

$$loss = -10 \log\left(\frac{P_{coupled}}{P_{emitted}}\right) = -10 \log (1-R)$$

+ lensing schemes: small lens placed between source and fiber to improve coupling efficiency by magnifying the emitting area to match the fiber core.
  - round-ended fiber, - small nonimaging glass sphere,
  - larger spherical lens, - cylindrical lens, - taper-ended fiber
  - system of spherical-surfaced LED and spherical-ended fiber

+ fiber-to-fiber joints:
  - needed at Tx, Rx, repeaters, Amps., add-drop points, between fibers
  - types: splices (permanent, between fibers), connectors (demountable)
  - fiber-to-fiber coupling efficiency: $\eta_F = \frac{M_{comm}}{M_E}$ (M_{comm}: number of modes in both fibers; M_E: number of modes in the emitter)
    - fiber coupling loss: $L_F = -10 \log(\eta_F)$

+ mechanical misalignment:

1 - lateral: power coupled proportional to common

(offset) (introduces most losses)

area of fibers

$$A_{comm} = 2a^2 \cos^{-1}\left(\frac{d}{2a}\right) - d\left[a^2 - \frac{d^2}{4}\right]^{1/2}$$

$$\rightarrow \eta_{F,step} = \frac{A_{comm}}{\pi a^2} = \frac{2}{\pi}\cos^{-1}\left(\frac{d}{2a}\right) - \frac{d}{\pi a}\left[1 - \left(\frac{d}{2a}\right)^2\right]^{1/2}$$

$$\& \quad \eta_{graded} \approx \left[1 - \frac{8d}{3\pi a}\right]$$

2- longitudinal: ends separated by distance s

$$\eta_F = \left(\frac{a}{a + s\tan(\theta_{0,max})}\right)^2$$

maximum acceptance angle

3- Angular: $\eta_F \approx 1 - \frac{\theta}{\pi (NA)}$ (θ: radians, misalignment angle)

$$\text{loss} = L_F = -10\log(\eta_F)$$

+ other coupling losses:

- due to different core diameters: $L_F(a) = -10\log\left(\frac{a_R}{a_E}\right)^2$ for $a_R < a_E$, 0 if $a_R > a_E$ (a_R: receiver, a_E: emitter)
- due to different NA: $L_F(NA) = -10\log\left(\frac{(NA)_R}{(NA)_E}\right)^2$ for $NA_R < NA_E$, 0 if $NA_R > NA_E$
- due to different refractive indices: $L_F(\alpha) = -10\log\left(\frac{\alpha_R(\alpha_E+2)}{\alpha_E(\alpha_R+2)}\right)$ for $\alpha_R < \alpha_E$, 0 if $\alpha_R > \alpha_E$ (α: refractive index profile)

+ single-mode fibers:

- axial (lateral) misalignment: $L_{SM,lat} = -10\log\left[\exp\left\{-\left(\frac{d}{W}\right)^2\right\}\right]$ (d: offset)

where $W = a\left(0.65 + 1.619V^{-3/2} + 2.879V^{-6}\right)$ (W: mode-field diameter; a: radius; V: V number; $V \leq 2.405$ for single mode)

- angular: $L_{SM,ang} = -10\log\left[\exp\left\{-\left(\frac{\pi n_2 W\theta}{\lambda}\right)^2\right\}\right]$ (λ in μm)

- longitudinal: $L_{SM,gap} = -10 \log \left\{ \dfrac{64 \cdot n_1^2 \cdot n_3^2}{(n_1+n_3)^4 \cdot (G^2+4)} \right\}$ (n_3: material between fibers)

where $G = \dfrac{s}{k W^2}$ (s: separation), $k = \dfrac{2\pi}{\lambda}$

+ splicing steps: 1- losing: removing coating, keeping core and cladding

2- cleaning: cutting fiber to produce flat surface (no angle)

3- polishing surface

+ splicing techniques: 1- fusion, 2- V- groove mechanical, 3- elastic-tube

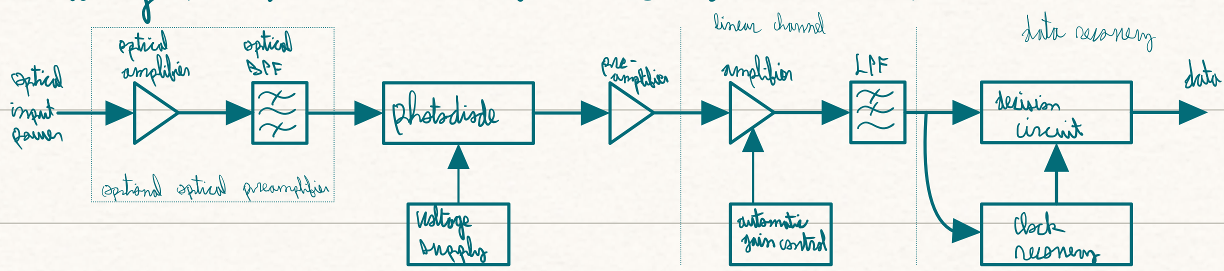
- return loss: $RL = -10 \log \left[2R \left(1 - \cos\left(\dfrac{4\pi n d}{\lambda} \right) \right) \right]$

(R: reflectivity; n: refractive index of matching material on fiber core)

Chapter 7:

- optical receiver consists of photodetector, preamplifier, and signal processing circuit
- receiver converts optical EM wave to electrical signal
- modulation scheme considered is unipolar (on-off-keying)
- data voltage signal → electric current signal → intensity modulation → optical power. logic 1: light pulse with duration T_b no light, same duration for 0

- the decision circuit compares the amplified signal with some voltage threshold.
- intensity modulation and direct detection are most commonly used



- receiver converts optical signals to electrical and recovers data.
- photodiode directly converts signal to baseband electric.
  → no need for RF section

- Random arrival rate of signal photons produces quantum (or shot) noise.
- APD will have additional shot noise (excess noise) caused by the statistical nature of the multiplication process.
- Thermal noise generated in the photodetector's load resistance and the input resistance of the preamplifier.
- Primary current is time-varying poisson process. This is a property of EM radiation.
- Average number of electron-hole pairs generated is:

$$N = \frac{\eta}{h\nu}\int_0^T P(t)\,dt = \frac{\eta E}{h\nu}$$

(η: quantum efficiency; $\frac{\eta E}{h\nu}$: number of photons per pulse)

- probability that n electrons are generated in T is a poisson distribution given as:

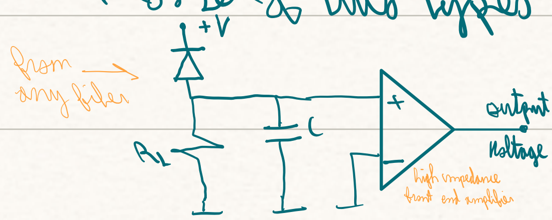
$$P_r(n) = N^n \frac{e^{-N}}{n!}$$

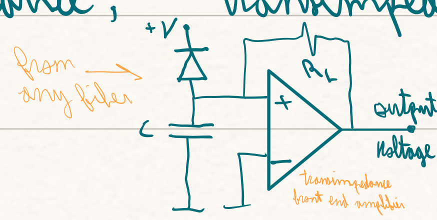
- for an APD with mean gain M, the excess noise factor is:

$$F(M) \approx M^x$$

x : factor that ranges from 0 to 1 (depends on material)

- front-end amplifier amplifies the weak signal generated by the photodiode. it must be low noise and large bandwidth + one of two types: high impedance, Transimpedance





- high impedance amplifier requires a high load resistance R_L to minimize the thermal noise, but at the expense of bandwidth.

$$B_e = \frac{1}{2\pi R_L C}$$

$\frac{1}{2\pi (R_L // R_{in}) C_{eq}}$, $R_L // R_{in} = R_L$ ∵ $R_{in} \approx \infty$

- negative feedback in the transimpedance amplifier reduces effective resistance seen by the PD by a factor G, where G is the gain of the amplifier.

$$B_e = \frac{G}{2\pi R_L C}$$

∵ $R_{eff} = \frac{R_L}{G}$

- If same R_L used for transimpedance as high impedance, the transimpedance will have double the thermal noise and G times the bandwidth.

→ Transimpedance amplifier preferable.

- received pulse train: $P(t) = \sum_{n=-\infty}^{+\infty} a_n h_p(t - nT_b)$
- output photocurrent: $i(t) = M \cdot R \cdot \sum_{n=-\infty}^{+\infty} a_n h_p(t - nT_b)$ (M: multiplication factor; R: responsivity)
- converted, amplified, and filtered voltage: $V(t) = \sum_{n=-\infty}^{+\infty} b_n h_p(t - nT_b) + n(t)$ (n(t): sum of all types of noises)
- bit-error probability: $BER = P_e = a P_1(V_{th}) + b P_0(V_{th})$ ($a + b = 1$)
- The probability of error given logic 0: $P_0(V_{th}) = \frac{1}{2} \operatorname{erfc}\left[\frac{V_{th} - b_{off}}{\sqrt{2}\, \sigma_{off}}\right]$ (b_{off}: mean voltage for off)
- error function complementary: $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp[-y^2]\, dy$

$$\rightarrow \operatorname{erfc}(x) \approx \frac{e^{-x^2}}{2\pi}$$

- If logic 1 was sent: $P_1(V_{th}) = \frac{1}{2} \text{erfc}\left[\frac{b_{on} - V_{th}}{\sqrt{2}\,\sigma_{on}}\right]$

- average bit error probability: $\overline{BER} = \overline{P_e} = \frac{1}{2} \cdot [P_1(V_{th}) + P_0(V_{th})]$

- The optimum threshold giving the minimum BER is found as: (point at which functions are equal)

$$V_{th,opt} = \frac{\sigma_{on} \cdot b_{on} + \sigma_{off} \cdot b_{off}}{\sigma_{on} + \sigma_{off}}$$

$\therefore Q = \frac{V_{th} - b_{off}}{\sigma_{off}} = \frac{b_{on} - V_{th}}{\sigma_{on}}$

- BER at optimum threshold: $BER = P_e = \frac{1}{2} \text{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{1}{4\pi} \exp\left[-\frac{Q^2}{2}\right]$ (at $a = b = 0.5$)

  $\wedge$ Q at $V_{th,opt} = \frac{b_{on} - b_{off}}{\sigma_{on} + \sigma_{off}}$

* receiver sensitivity: minimum (received) power required to achieve a certain performance measure for some bit rate

- the Q number can be written as: $\frac{I_1 - I_0}{\sigma_1 + \sigma_0} \approx \frac{I_1}{\sigma_1 + \sigma_0}$ ($I_0 = 0$) $\therefore Q = \frac{b_{on} - b_{off}}{\sigma_{on} + \sigma_{off}}$

- average received optical power is: $P = \frac{P_1 + P_0}{2} \approx \frac{P_1}{2}$ ($P_0 = 0$)

- average photocurrent from received power: $I \approx I_1 = RMP_1$ (R: responsivity, M: multiplication factor)

- Therefore, power to achieve Q: $P_{sensitivity} = \frac{P_1}{2} = \frac{I_1}{RM} = \frac{Q(\sigma_1 + \sigma_0)}{RM}$

- Since the assumed power received in 0 pulse is 0, there will only be thermal noise $\rightarrow \sigma_0 = \sigma_T$, whereas the 1 pulse will have thermal and shot noise $\rightarrow \sigma_1 = \sigma_T + \sigma_{shot}$

- average thermal noise power received in electrical bandwidth B_e:

$$\sigma_T^2 = \frac{4 k_B T}{R_L} \cdot F_n \cdot B_e$$

(k_B: boltzmann constant, T: temperature in kelvin, F_n: noise figure)

- average shot noise power in B_e:

$$\sigma_{shot}^2 = 2 q I_1 M \cdot F(M) \cdot B_e = 2 q P_1 R M^2 F(M) \cdot B_e$$

$\therefore \sigma^2_{shot} = 4q\, P_{sensitivity} \cdot R \cdot M^2 \cdot F(M) \cdot B_e$

- somehow:

$$P_{sensitivity} = \frac{Q}{RM} \cdot \left[q \cdot M \cdot F(M) \cdot Q \cdot B_e + \sqrt{\frac{4k_BT}{R_L} F_n B_e} \right]$$

- for a low-pass nyquist channel: $B_e = \frac{B}{2}$ bit rate

* quantum limit: minimum received power required for a specific BER

- error occurs if no electron hole pairs are generated when logic 1

$$\therefore P_r(n) = N^n \frac{e^{-N}}{n!}$$

(n: number of electrons; N: average number of electron hole pairs generated in T one pulse duration)

$$\therefore P_r(0) = e^{-N} = P_e$$

Chapter 8:

- Two optical fiber comms. systems categories: point-to-point or distributed net.
  - point-to-point: electronic repeaters or optical amplifiers
    - electronic repeaters: overcome attenuation and dispersion + other disadvantages but only used in single wavelength systems
    - optical amplifiers: amplify signal, dispersion, and noise
      - dispersion solved by using dispersion compensating fibers at different locations (pre-, inline-, or post-compensation)

+ key design requirements:
  1- transmission distance, 2- data rate or bandwidth,
  3- BER or output SNR

+ design choices: three components: source, fiber, receiver
  - optical source: LED (SLED or ELED) or LD
  - optical fiber channel: single mode step, multimode step, multimode graded
  - optical receiver: PIN or Avalanch photodiodes

+ considerations for source:
  1- emission wavelength, 2- spectral line width, 3- output power
  4- effective radiating area, 5- emission pattern, 6- number of modes

7- switching speed, 8- lifetime

+ considerations for fiber:

1- core and cladding diameters, 2- core refractive index profile (step or graded)

3- bandwidth (dispersion), 4- attenuation, 5- numerical aperture (or mode-field diameter)

+ considerations for receivers:

1- Responsivity (quantum efficiency), 2- operating wavelength

3- response time, 4- sensitivity, 5- lifetime

* power budget: difference between power from source and power received

- must be equal to or greater than sum of all losses plus some margin:

$$P_T = P_S - P_R = \alpha_f \cdot L + M \cdot l_c + N \cdot l_{sp} + \text{margin}$$

(P_S: transmitted, P_R: received, α_f: fiber attenuation, L: length, M: number of connectors, l_c: connector loss, N: number of splices, l_{sp}: splices loss, margin: usually 6 dB)

* rise-time: time in which an output changes from 10% to 90% of its maximum value when the input is a step function.

- rise-time budget deals with the allowed intersymbol interference.

- the system rise time must be smaller than some fraction of the bit period. This fraction depends on the line coding scheme used.

- system rise-time is: $t_{sys} = \sqrt{t_{tx}^2 + t_{mod}^2 + t_{chrom}^2 + t_{rx}^2}$

(t_{tx}: transmitter rise-time, t_{mod}: rise-time due to intermodal dispersion, t_{chrom}: rise-time due to chromatic dispersion, t_{rx}: receiver rise-time)

- the rise-times are squared because pulses are assumed gaussian shaped
- for NRZ : $t_{sys} = 0.7\,T_b = \frac{0.7}{B}$
- for RZ : $t_{sys} = 0.35\,T_b = \frac{0.35}{B}$ (half since RZ has half NRZ width)
- receiver rise-time : $\frac{350}{B_{rx}(MHz)} = t_{rx}(ns)$ (receiver bandwidth)

- rise-time due to intermodal dispersion:

$$\frac{440\,L^{q}}{B_0\,(MHz \cdot km)} = t_{mod}\,(ns)$$

(L: length in km; q: mode mixing parameter ranges from 0.5 to 1; B_0: bandwidth-distance product of fiber due to intermodal dispersion)

- rise-time due to chromatic dispersion:

$$t_{chrom} = D\,\sigma_\lambda\,L = (D_{mat} + D_{wg}) \cdot \sigma_\lambda \cdot L$$

(D: chromatic dispersion factor in ns/(nm·km); σ_λ: spectral width of source in nm)

- maximum link distance for a given bit rate is found from:

$$t_{sys} = \sqrt{t_{tx}^2 + \left(\frac{440\,L^q}{B_0}\right)^2 + (D\,\sigma_\lambda\,L)^2 + \left(\frac{350}{B_{rx}}\right)^2} \leq \frac{0.7}{B} \text{ (NRZ) or } \frac{0.35}{B} \text{ (RZ)}$$

- intermodal dispersion is zero for single mode fibers ($t_{mod} = 0$)

final 2016:

Q1) main optical sources:

LED: advantage: cheap, low sensitivity to temp.

disadvantage: low coupling efficiency

LD: advantage: high coupling efficiency, small spectral width

disadvantage: high sensitivity to temp. expensive

main fibers:

single-mode step-index: advantage: high bandwidth, no intermodal disp.

disadvantage: cost and complexity, difficult to launch power

multimode step-index: advantage: easy to launch power since large core radius

disadvantage: very small bandwidth

multimode graded-index: advantage: larger bandwidth than step multi

disadvantage: more expensive to fabricate

main detectors:

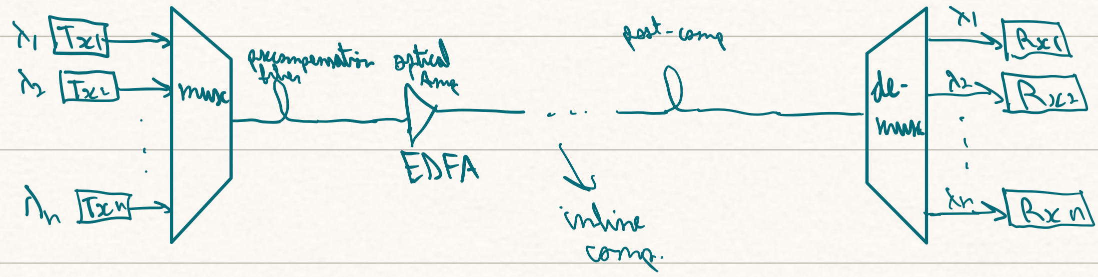
PIN photodiode: advantage: low bias voltage, stable with temp.

disadvantage: low responsivity

Avalanche PD: advantage: increased sensitivity, higher responsivity

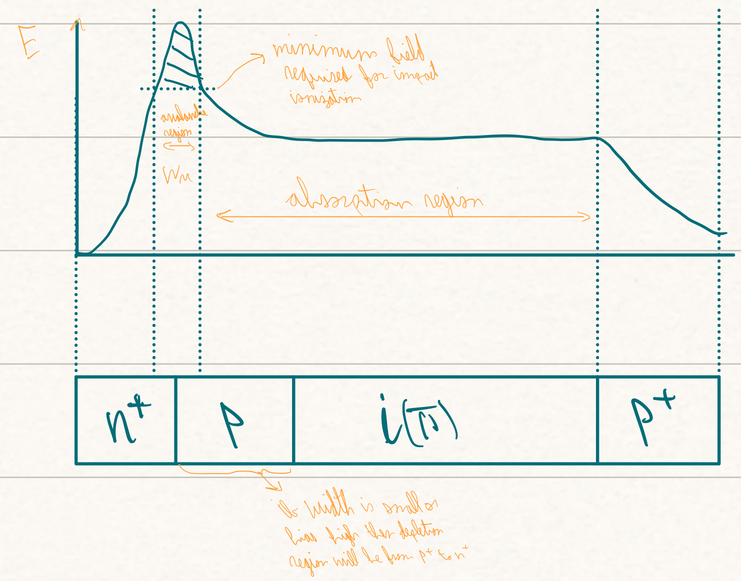
disadvantage: high bias voltage,

Q2)



- CWDM: coarse wavelength division multiplexing
- DWDM: dense " " "

Q3)



- in the avalanche region, the primary current generated in the intrinsic region is multiplied by a certain gain M. This is due to impact ionization wherein the generated carriers are accelerated by the high electric field in the avalanche region thereby ionizing atoms in their path upon collision.
- three materials used: Si ($0.4 \rightarrow 1.1\ \mu m$), Ge ($0.8 \rightarrow 1.8\ \mu m$), and InGaAs ($1.0 \rightarrow 1.7\ \mu m$)

Q4) $\because E_g \leq h\nu \rightarrow E_g \leq h\frac{c}{\lambda} \rightarrow \lambda \leq \frac{hc}{E_g}$

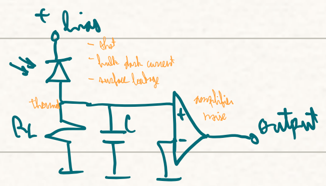
$\therefore \lambda_c = 1.6\ \mu m$ or $\lambda_c = \frac{1.24}{E_g(eV)} = 1.6\ \mu m$

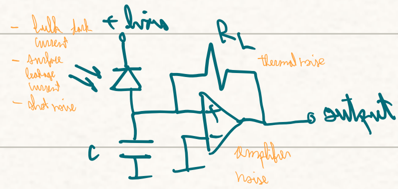
unity gain responsivity: $R = \frac{\eta q \lambda}{hc} = 1.123$

$\rightarrow I = P_{in} \cdot R \cdot M = 1.123 \cdot 50 \cdot 100 \times 10^{-9} = 5.615\ \mu A$

shot noise: $2 q I_m \cdot M \cdot F(M) \cdot B_e = \sigma_{shot}^2 = 6.352 \times 10^{-14}$
$= -101.97\ dBm$

Q5)





1- high impedance Amp. 2- transimpedance Amp.

- transimpedance is more commonly used as it can achieve G times the bandwidth increase of high impedance amps with only twice the noise.

Q6) $P_{sensitivity} = \frac{Q}{RM} \cdot \left[\frac{q M F(M) \cdot Q \cdot B}{2} + \sqrt{\frac{4 k_B T}{2 R_L} \cdot F_n \cdot B} \right]$

$\rightarrow P_{sensitivity} = \frac{Q}{RM} \cdot [5.122 \times 10^{-6} + 1.021 \times 10^{-6}]$

$\because BER = \frac{1}{4\pi} e^{-\frac{Q^2}{2}} \rightarrow Q = 6.4 \quad \therefore P_{sensitivity} = 4.37 \times 10^{-7}\ W$

& quantum limit: $\because 10^{-10} = e^{-N} \rightarrow N = 23$ photons

$E_{avg} = \frac{23 \cdot h\nu}{2}$ & $P = \frac{23 \cdot h \cdot c}{T_b\ \lambda \cdot 2} = \frac{23 \cdot h \cdot c}{2 \cdot \lambda} \cdot B =$

need λ

Q7) 1- distance | 1- bandwidth, wavelength, modes
2- bandwidth / data rate | 2- cost, refractive index, numerical aperture
3- BER | 3- responsivity, sensitivity

Q8) rise-time : $\sqrt{t_{tx}^2 + \left(\frac{440\, L^q}{B_0}\right)^2 + (D\,\sigma_\lambda\, L)^2 + \left(\frac{350}{B_{Rx}}\right)^2}$

→ 0 single mode

→ rise time $= \sqrt{(50\times10^{-3})^2 + (5\times10^{-3}\cdot 250\cdot 0.25)^2 + \left(\frac{350}{10\times10^3}\right)^2}$

∴ rise time = 0.3184 ns ≤ $\frac{0.7}{10}$ (in GHz) X

rise time limited

∴ $\sqrt{(50\times10^{-3})^2 + (5\times10^{-3}\cdot L\cdot 0.25)^2 + \left(\frac{350}{10\times10^3}\right)^2} \le \frac{0.7}{10}$

→ L = 27.4 km

∴ 9 repeaters needed

power - budget :

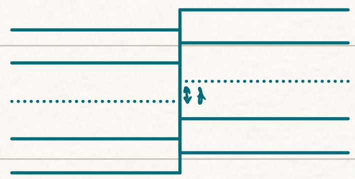
3.01 - -40 = $\alpha_f \cdot L + M\,l_c + N\,l_{sp} + 6$

power limited

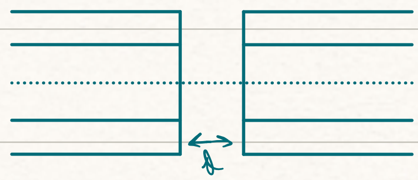
3.01 - -40 = $\alpha_f \cdot 27.4 + 2\cdot 2 + 6$ ✓

Second 2015

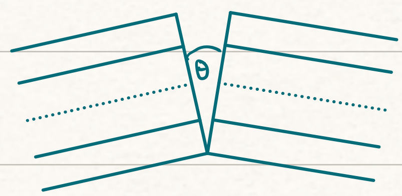
Q6: misalignment: 1- lateral offset :



2- longitudinal :

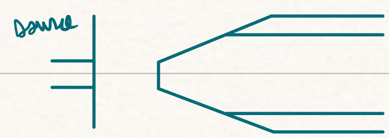


3- angular



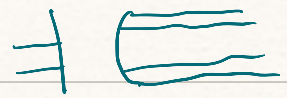
- lensing schemes :

1- taper-ended fiber :



2- non-imaging microsphere

3 - round-ended fiber



Q7: 1- coupling efficiency:

$$P_{LED, graded} = (NA)^2 \cdot P_s \cdot \left[1 - \left(\frac{\alpha}{\alpha+2}\right) \cdot \left(\frac{r_s}{a}\right)^{\alpha}\right]$$

$$r_s = a \rightarrow \eta = (0.2)^2 \cdot \left[1 - \frac{1}{2}\right] = 0.02$$

$$\rightarrow \eta = 2\%$$

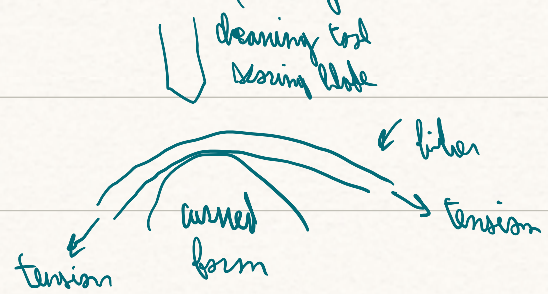
$$\wedge \; P_{LED, graded} = \eta \cdot \pi^2 \cdot r_s^2 \cdot B_0 = 0.02 \cdot 6.17 \, mW = 0.123 \, mW$$

2- single mode:

$$\eta = \left(\frac{a}{r_s}\right)^2 \cdot (NA)^2 = 0.04\%$$

$$\rightarrow P_{LED, step} = 2.47 \, \mu W$$

Q8: three splicing techniques: 1- fusion splicing, 2- V-groove mechanical splicing, 3- elastic-tube splicing



Q9: for a lambertian source: $B(\theta, \phi) = B_0 \cos(\theta)$

→ ~~half power~~ 3dB when $\cos(\theta) = \frac{1}{2}$ → $\theta = 60^\circ$

→ beamwidth = 120°

laser in lateral direction → $\phi = 0$

$$\therefore \frac{1}{B(\theta,\phi)} = \frac{1}{B_0 \cos^L \theta}$$

$$\rightarrow B(2.5°, 0) = B_0 \cos^L (2.5)$$

$$\rightarrow \cos^L(2.5) = 0.5 \rightarrow L = \log_{\cos(2.5)}(0.5)$$

$$\rightarrow L = 727.9 = 728 \text{ integers}$$

Q10: lateral displacement

$$A_{common} = 2a^2 \cos^{-1}\left(\frac{d}{2a}\right) - \left(a^2 - \frac{d^2}{4}\right)^{1/2}$$

take smaller radius → $A_{comm} = 7554 \ \mu m^2$

$$\rightarrow \eta_F = 0.962 \rightarrow L_{\eta_F} = 0.168 \text{ dB}$$

longitudinal: $\left(\frac{a}{a + s\tan(\theta_{0,max})}\right)^2 = \eta_F \quad \& \ \theta_{0,max} = 0.25$

$$\rightarrow \eta_F = 0.951$$

→ total losses = 0.384 dB

Second 2019:

Q8: $\frac{1}{B(\theta,\phi)} = \frac{\sin^2(\phi)}{B_0 \cos^T(\theta)} + \frac{\cos^2(\phi)}{B_0 \cos^L(\theta)}$

lateral → $\phi = 0$ → $B(\theta, 0) = B_0 \cos^L(\theta)$

$0.5 = \cos^{150}(\theta)$ → $\theta = 5.5°$ → beamwidth $= 11°$

transversal → $\phi = 90°$ → $\frac{1}{B(\theta, 90°)} = \frac{1}{B_0 \cos^T(\theta)}$ → $0.5 = \cos^{75}(\theta)$

→ $\theta = 7.77$ → beamwidth $= 15.55°$

Q9: $P_s = \pi^2 r_s^2 B_0 = 6.169$ mW

multimode: $\eta = (NA)^2 = 4\%$, $P_{coupled} = 0.247$ mW

single mode: $\eta = (NA)^2 \cdot \left(\frac{5}{25}\right)^2 = 0.04\%$, $P_{coupled} = 2.47$ uW

Q10: total loss = loss due to diameter difference (dB) + loss due to misalignment

→ loss due to diameter $= -10 \log \left(\frac{50}{62.5}\right)^2 = 1.938$ dB

→ loss due to misalignment: $-10 \log \left[\frac{2}{\pi} \cos^{-1}\left(\frac{d}{2a}\right) - \frac{d}{\pi a}\left[1 - \left(\frac{d}{2a}\right)^2\right]^{1/2}\right]$

→ 0.59 dB

∴ loss = 2.528 dB

$\theta_{0,max} = \sin^{-1}(NA)$

final 2022:

Q1) N/A

Q2: a) 3 dB beamwidth → $B(\theta,\phi) = \frac{B_0}{2}$ ∧ $\frac{1}{B(\theta,\phi)} = \frac{\sin^2(\phi)}{B_0 \cos^T(\theta)} + \frac{\cos^2(\phi)}{B_0 \cos^L(\theta)}$

lateral → $\phi = 0$ → $B(\theta, 0) = B_0 \cos^L(\theta)$

for $B(\theta, 0) = 0.5 B_0$ → $\cos(\theta) = (0.5)^{\frac{1}{L}}$

∴ $\theta = 3^\circ$ → beamwidth = 6°

transversal → $\phi = 90^\circ$ → $B(\theta, 90) = B_0 \cos^T(\theta)$

for 3 dB beamwidth: $2\theta = 2\cos^{-1}\left(0.5^{\frac{1}{100}}\right) = 13.5^\circ$

if $\theta = 3^\circ$ ∧ $\phi = 6^\circ$ → $B(\theta,\phi) = 50.61\ W/(cm^2 \cdot str)$

b) ∵ $P_s = \pi^2 \cdot r_s^2 \cdot B_0$ → $P_s = 29.86$ mW

same area units (e.g., cm²)

a) $P_{LED,\,step}\ \forall\ r_s > a = \left(\frac{a}{r_s}\right)^2 \cdot (NA)^2 \cdot P_s$ → $P_{coupled} = 1.45$ mW

b) $P_{LED,\,graded}\ \forall\ r_s < a = (NA)^2 \cdot \left[1 - \frac{\alpha}{\alpha+2}\left(\frac{r_s}{a}\right)^{\alpha}\right] \cdot P_s = 0.73$ mW

c) lateral (offset) loss = $\frac{2}{\pi}\cos^{-1}\left(\frac{d}{2a}\right) - \frac{d}{\pi a}\left[1 - \left(\frac{d}{2a}\right)^2\right]^{1/2} = 0.936$

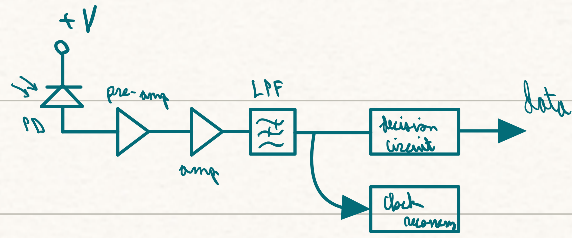
∴ lateral losses = $-10\log(0.9364) = 0.286$ dB

∵ Angular loss: $\eta_F \approx 1 - \frac{\theta}{\pi (NA)}$ = 0.8611 = 0.649 dB

→ radians!

∴ total loss = 0.935 dB

Q3: a)



- four types of noise generated:

1- shot, 2- dark bulk current, 3- surface leakage current, 4- thermal noise

b) cutoff wavelength: $\lambda_c (\mu m) = \frac{1.24}{E_g (eV)} = 1.653$

or ∵ $E > E_g \rightarrow \frac{hc}{\lambda} > E_g \rightarrow \lambda_c < \frac{hc}{E_g} = 1.656 \mu m$

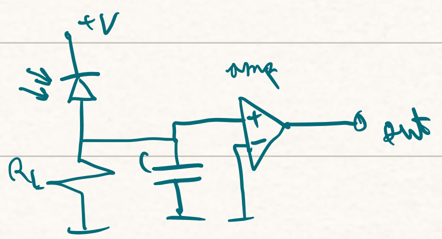
unity gain responsivity: $R = \frac{\eta q \lambda}{hc}$ (base units) $= 1.197$ A/W

generated current $= R \cdot M \cdot P_{in} = 5.98 \ \mu A = i_m$

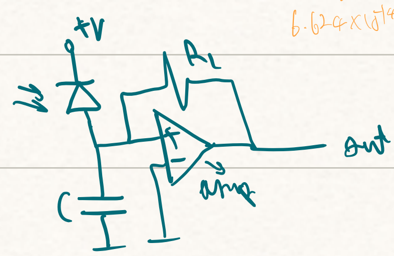
$$SNR = \frac{(i_m)^2}{\underbrace{2q(I_p + I_D) M^2 \cdot F(M) \cdot B_e}_{6.77 \times 10^{-12}} + 2q I_L B_e + \underbrace{\frac{4k_B T}{R_L} B_e}_{6.624 \times 10^{-14}}} = 5.23 = 7.19 \text{ dB}$$

($I_D \rightarrow 0$, $2qI_LB_e \rightarrow 0$)

Q4: a)



high impedance amplifier



Transimpedance amplifier

- Transimpedance amplifier is more commonly used since G x bandwidth gain can be achieved with only double the noise of high impedance amplifiers.

b) $P_{sensitivity} = \frac{Q}{RM} \cdot \left[\frac{qMF(M)QB}{2} + \sqrt{\frac{2k_B T}{R_L} F_n B} \right]$

∵ $BER = 10^{-11} = \frac{1}{\sqrt{2\pi}} e^{-\frac{Q^2}{2}} \rightarrow Q = 6.75$ ∴ $P_{sensitivity} = 559.3$ nW = −32.5 dBm

for $BER = 10^{-11} \rightarrow 25.33$ photons $\rightarrow P_{sensitivity} = B \cdot \frac{25.33 \cdot hc}{\lambda} = 32.5$ nW $= -44.9$ dBm

Q5: a) assuming three characteristics for each type

* sources:

+ LED:

- wide spectral width → low coupling efficiency
- low sensitivity to temperature
- low modulation bandwidth + low cost and complexity

+ LD:

- narrow spectral width → high coupling efficiency
- high sensitivity to temperature
- high modulation bandwidth + high cost and complexity

* fibers

+ single mode step-index:

- small core radius
- can support high bandwidths
- complex splicing + high cost

+ multimode graded index:

- larger core radius than single mode
- can support moderate bandwidths (lower than single)
- moderate splicing complexity + high cost (due to grading)

+ multimode step index:

- largest core radius (compared to previous two)
- supports low bandwidths
- simple slicing + low cost

* Receivers:

+ pin photodiode:

- low bias voltage required (less than 50 V)
- less variations with temperature
- simpler, less expensive

+ Avalanche photodiode:

- high bias voltage (usually several hundreds of volts)
- sensitive to temperature
- much higher sensitivity due to current multiplication

b) Rise-time budget: ∵ single mode → no modal delay (intermodal dispersion)

$$\rightarrow t_{sys} = \sqrt{\underbrace{t_{tx}^2 + 0 + (D\sigma_\lambda \cdot L)^2}_{ns} + \left(\frac{350}{B_{RX}(MHz)}\right)^2} = 0.257 \text{ ns}$$

Comparing t_{sys} with t_{NRZ} → $t_{sys} > t_{NRZ}$ → Repeaters needed

for $t_{sys} = t_{NRZ}$ → $\sqrt{\left(\frac{0.7}{10}\right)^2 - t_{tx}^2 - \left(\frac{350}{B_{RX}(MHz)}\right)^2} = D\sigma_\lambda L$

$\therefore L_{Rt} = 68.5$ km

power budget: $P_S - P_R = \alpha_f \cdot L + n \cdot l_\alpha + 0 + 6$

$\rightarrow \quad L = \frac{1}{\alpha}[30 - 2 - 6] = 88 \text{ km}$

$\therefore \quad L_{rt} < L_p \rightarrow$ rise-time limited system

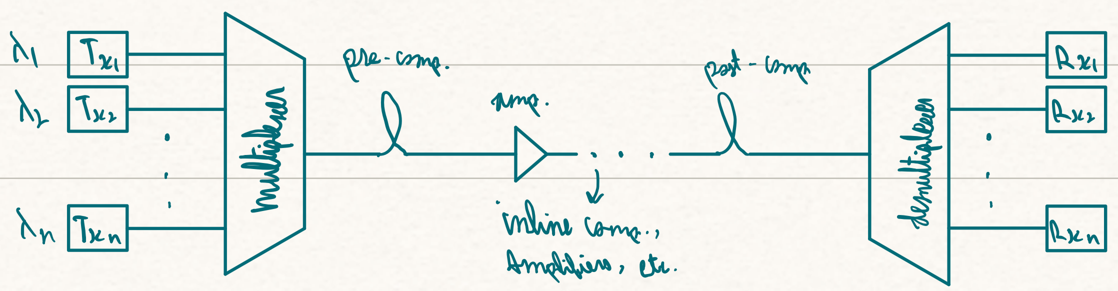
number of repeaters $= 7 \quad \left\lfloor \frac{L}{L_{rt}} \right\rfloor$ round down

final 2018:

Q1) N/A

Q2) not sure if it's included, but there is a block diagram for a WDM system in chapter 8

$$\text{bandwidth} = \frac{c}{\lambda_1} - \frac{c}{\lambda_2} = 43.3\,\text{THz}$$



Q3) N/A

Q4) $\lambda_c = \frac{1.24}{0.774} = 1.6\ \mu m$

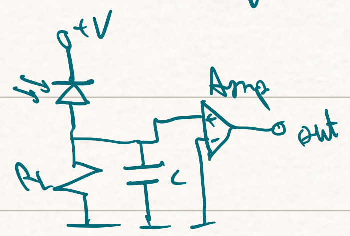
$$R = \frac{\eta \cdot q \cdot \lambda}{hc} = 1.123\ A/W$$

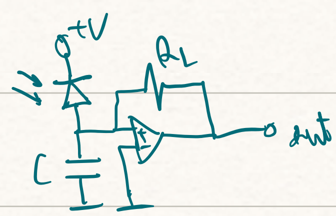
$$I_m = R \cdot M \cdot P_{in} = 561.5\ nA$$

−112 dBm

$$\sigma_{shot}^2 = 2 q I_m \cdot M F(M) \cdot B_e = 6.35 \times 10^{-15}\ W$$

Q5)





noise in photodiode: shot, dark (bulk current), surface leakage current

noise in amplifier: thermal in resistor + amplifier noise

Q6: ∵ $BER = 10^{-10} = \frac{1}{Q\sqrt{2\pi}} e^{-\frac{Q^2}{2}} \rightarrow Q = 6.4$

$\Rightarrow P_{sensitivity} = \frac{Q}{RM} \cdot \left[\frac{qMF(M)QB}{2} + \sqrt{\frac{2k_BT}{R_L} F_n B} \right]$

$\therefore P_{sensitivity} = 455.5\ nW = -33.4\ dBm$

Q7: key requirements: 1- distance, 2- bandwidth, 3- performance (BER)

+ source: type (LED or LD), wavelength, spectral width, number of modes

+ fiber: core and cladding diameters, refractive indices, refractive index profile of core

attenuation and dispersion (material)

+ detector: responsivity, response time, sensitivity, type (Pin or APD)

Q8) rise-time budget:

most likely needs repeaters so I'll skip to the next step

$\frac{0.7}{10} = \sqrt{t_{tx}^2 + 0 + (D\sigma_\lambda L)^2 + \left(\frac{350}{B_{rx}}\right)^2} \rightarrow L = 27.4\ km$

- note: since the calculated L is smaller than that given, our assumption is therefore correct.

power-budget:

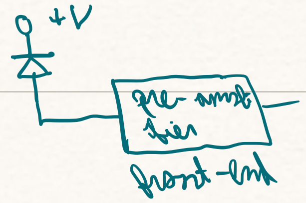
$3.01 + 40 = \alpha \cdot L + M \cdot l_a + 0 + 6\ dB$

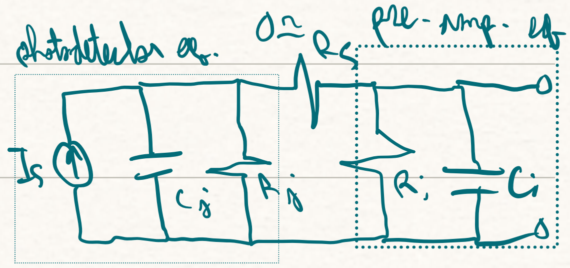
$\rightarrow L = 175\ km \quad \therefore$ rise-time limited

number of repeaters: $\left\lfloor \frac{260}{27.4} \right\rfloor = \underline{4}$

final 2012:

Q2) a)





b) Pin : advantages : low temperature sensitivity, low bias required
disadvantage : no gain, less sensitive to received power

APD : advantages : high sensitivity to received power, higher responsivity
disadvantage : expensive, complex, high bias, temperature instability

c) $\therefore \lambda_c = \frac{1.24}{0.75} = 1.653 \ \mu \rightarrow$ can be used for all three windows; however $\lambda = 1550$ nm → third window.

$I_n = M \cdot R \cdot P_{in} = M \cdot \frac{\eta \cdot q \lambda}{h c} \cdot P_{in} = 56.1 \ \mu A$

Q3) a) 1- distance, 2- bandwidth, 3- performance (BER)

b) source: 1- wavelength, 2- spectral width, 3- output power

fiber: 1- bandwidth, 2- core and cladding radii, 3- refractive indices ← profile + material

detector: 1- sensitivity, 2- responsivity, 3- wavelength

these characteristics determine the type for each.

c) 1) single-mode step index, LD, APD

2) multimode graded index, LED, Pin

note: step-multi → L < 1km ∧ B < 20 Mb/s

Q4) a) rise-time budget:

$$t_{sys} = \sqrt{t_{tx}^2 + \left(\frac{440 L^q}{B_0 (MHz \cdot km)}\right)^2 + (D \sigma_\lambda L)^2 + \left(\frac{350}{B_{rx}}\right)^2}$$

$\rightarrow t_{sys} = 0.461$ ns > 0.35 ns

∴ does not fulfill rise-time budget

power budget:

40 dB = 30 dB + 3 dB + 6 dB ✓

∴ fulfills power budget → rise-time limited

b) solution: repeaters

solve for L

$$0.35^2 = t_{tx}^2 + \left(\frac{440 L^q}{B_0 (MHz \cdot km)}\right)^2 + (D \sigma_\lambda L)^2 + \left(\frac{350}{B_{rx}}\right)^2$$

$$\rightarrow \left(\frac{440}{B_0}\right)^2 \cdot L^{2q} + (D \sigma_\lambda)^2 \cdot L^2 = 0.0819$$

on your calculator → $4.84 \times 10^{-4} L^{1.4} + 6.25 \times 10^{-6} \cdot L^2 - 0.0819 = 0$

$\rightarrow L = 36.225$ m

∴ 1 repeater required at half-way point

Second 2015:

Q7: $P_S = \pi^2 \cdot r_S^2 \cdot B_0 = 6.169$ mW

1- $\eta = (NA)^2 \cdot \left(1 - \frac{a}{a+2}\right) = 2\%$ → $P_{coupled} = 0.123$ mW

2- $\eta = 0.01\%$ → $P_{coupled} = 616.9$ nW

Q9: for lambertian $B(\theta, \phi) = B_0 \cos(\theta)$

→ $0.5 = \cos(\theta)$ → $2\theta = 120°$

Laser: $B(2.5°, 0) = 0.5 B_0 = B_0 \cos^L(\theta)$ → $L = 728$

Q10: losses: different radii, different NA, lateral and longitudinal misalignment

radii: $-20 \log\left(\frac{50}{62.5}\right)$

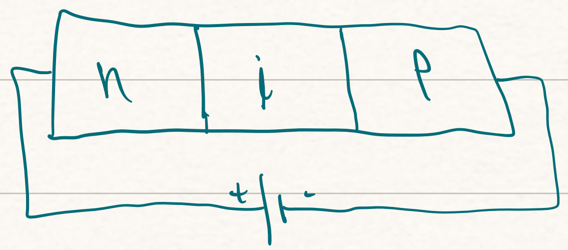
NA: 0 ∵ $NA_R > NA_E$

lateral: 0.2264

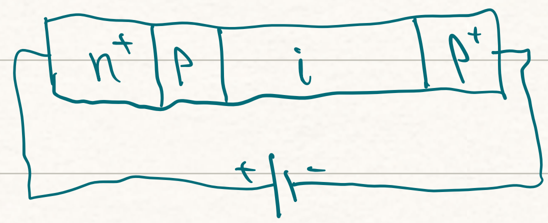
longitudinal:

final 2014:

Q2: a) pin



APD



- both can be used in the third window
- materials used are Si, Ge, InGaAs

b) $\lambda_c = \frac{1.24}{0.75} = 1.65\ \mu m$

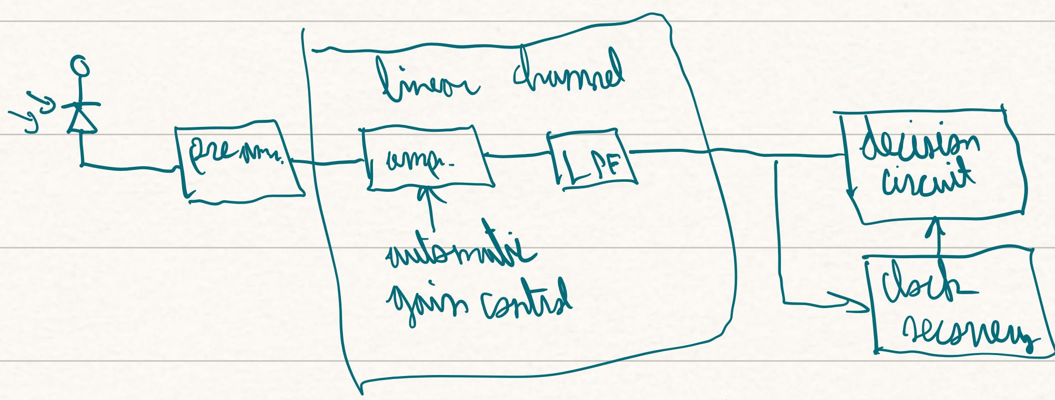
$$R = \frac{\eta q \lambda}{h c} = 1.123\ A/W$$

$$I_M = R M P_{in} = 112.3\ \mu A$$

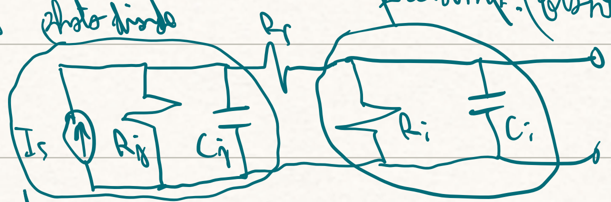
$$\sigma^2_{shot} = 2 q I_M \cdot M \cdot F(M) \cdot B_e = 1.8\ pA^2$$

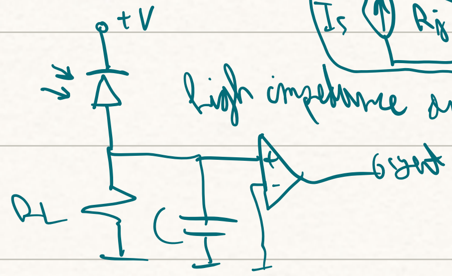
Q3: a)

→ optional amplifier

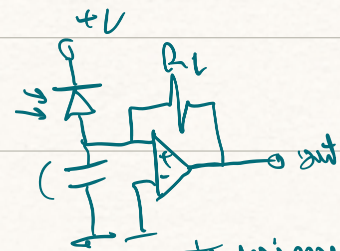


eq circuit





high impedance amp.



transimpedance

- transimpedance most commonly used because G times band width gain can be achieved with only double the thermal noise of high impedance Amp.

b) $P_{sensitivity} = \frac{Q}{RM}\left[\frac{qM F(M) Q B}{2} + \sqrt{\frac{2k_BT}{R_L} \cdot F_n B}\right]$

$\because$ $BER = 10^{-12} = \frac{1}{4\pi} e^{-\frac{Q}{2}}$ $\rightarrow Q = 7.085$

$\rightarrow P_{sensitivity} = 601.7\ nW = -32.2\ dBm$

Q4: a) optical source: 1- emission wavelength, 2- spectral width

3- bandwidth and switching speed, 4- number of modes

fiber: 1- core and cladding diameter, 2- bandwidth to be supported

3- material (refractive indices, attenuation, dispersion)

4- refractive index profile of core

receiver: 1- sensitivity, 2- lifetime

3- responsivity, 4- wavelength

+ types of all

b) assuming repeater needed

$\left(\frac{0.7}{10}\right)^2 = t_{tx}^2 + 0 + (D\sigma_\lambda L)^2 + \left(\frac{350}{B_{Rx}}\right)^2$

$\rightarrow L = 34.3\ km$

Raman :

$3 + 37 = 40 = 0.25 \cdot L + 2 \times 2 + 6 \rightarrow L = 120$

→ rise-time limited → 5 repeaters